Monetary Policy

(Advanced Monetary Economics)

ECON 4325

Nina Larsson Midthjell - Lecture 8 - 18 March 2016

Outline

- Monetary Policy in the NKM (CGG (2000) and Gali ch.4.3 and 4.4.1. Ch. 4.4.2 can be dropped)
  - Implementation of interest rate rules
- Introduction to optimal monetary policy (commitment versus discretion)
- The New Keynesian Model – The CGG (1999) version
- The policy objective
- Optimal Monetary Policy without commitment (= time-consistent discretion)

At lecture 9:

- Gains from commitment
- Optimal Monetary Policy with commitment
- Some practical complications

Monetary Policy in the NKM
Implementation of interest rate rules

- What do we want?

- A monetary policy rule that combined with the following two equations yield a solution to the model consistent with the desired equilibrium outcome:

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \kappa \tilde{y}_t \quad (1)$$

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\left(\underbrace{i_t - E_t\{\pi_{t+1}\}}_{r_t} - r_t^{natural}\right) \quad (2)$$

$$i_t = something$$

- Given the rule, we insert for the nominal interest rate in the DIS-equation and then solve the equation system.

Monetary Policy in the NKM
Implementation of interest rate rules

- Candidate 1: $i_t = r_t^{natural}$ (3)

- At lecture 7, we saw that this rule yields the desired outcome, i.e. **the divine coincidence** $\left(\tilde{y}_t = \pi_t = 0\right)$

- Inserting for the rule (equation 3) in the DIS-equation and solving the equation system to get the system of difference equations yields:

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 1 & 1/\sigma \\ \kappa & \beta + \dfrac{\kappa}{\sigma} \end{bmatrix} \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} = \mathrm{A}_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}, \tag{4}$$

- One solution to equation (4) is: $\tilde{y}_t = \pi_t = 0$, which is the desired equilibrium.

- However, the solution is **not unique.**

- Recall that in order to ensure uniqueness of the solution, then both eigenvalues of A_T must lie within the unit circle. Here, one lies in the interval (0,1) and one is strictly greater than unity for all parameter values.

Multiple Equilibria

Monetary Policy in the NKM

Implementation of interest rate rules 3/13

- Candidate 2: $$i_t = r_t^{natural} + \phi_\pi \pi_t + \phi_y \tilde{y}_t \qquad \phi_\pi, \phi_y > 0 \qquad (5)$$

where the natural real interest rate is defined as in previous lectures:

$$r_t^{natural} = \rho - \sigma(1-\rho_a)\psi_{ya} a_t + (1-\rho_z) z_t$$

- Inserting for this rule in the IS-equation and solving the equation system yields a solution very similar to what we found last week:

$$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = \Gamma \begin{bmatrix} \sigma & 1-\beta\phi_\pi \\ \sigma\kappa & \kappa + \beta(\sigma+\phi_y) \end{bmatrix} \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} = \mathrm{A}_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix}, \quad \Gamma \equiv \frac{1}{\sigma + \phi_y + \kappa\phi_\pi} \qquad (6)$$

- One solution to equation (6) is: $\tilde{y}_t = \pi_t = 0$, which is the desired equilibrium.

- The solution is unique when coefficients are chosen by the central bank so that (same as at lecture 7!):

$$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0 \qquad (7)$$

CB credibility and actions crucial for the outcome

Monetary Policy in the NKM
Implementation of interest rate rules

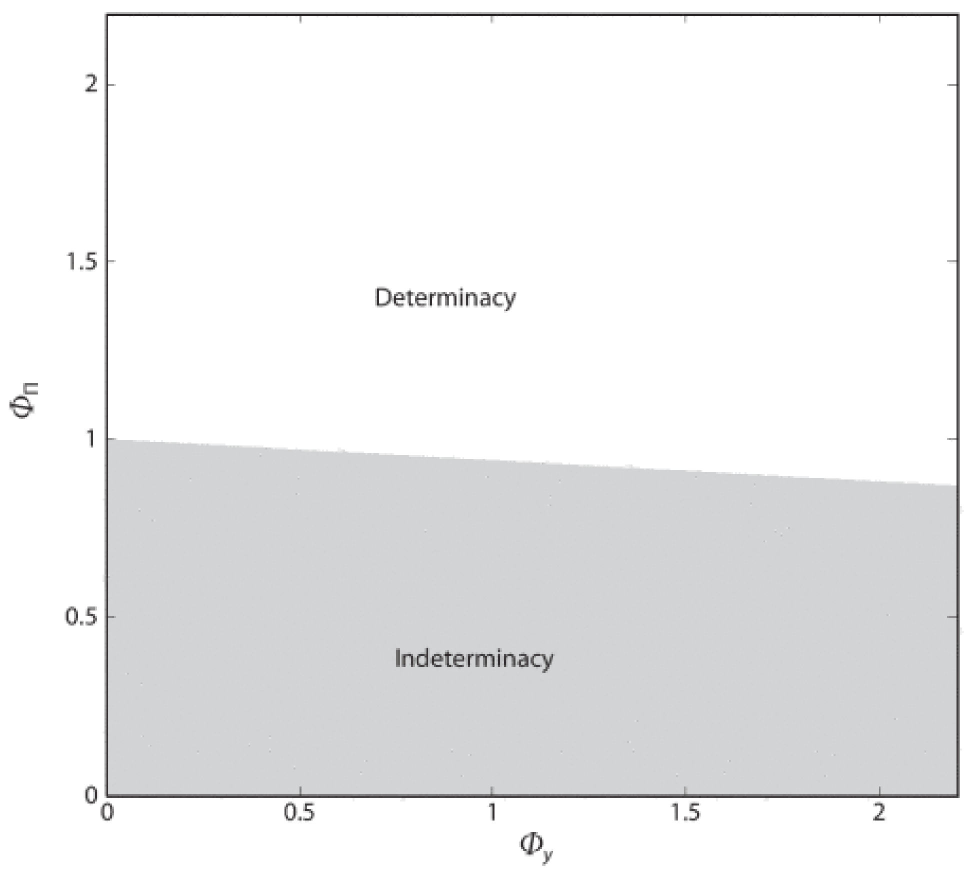

Figure 4.1. Determinacy and Indeterminacy Regions: Standard Taylor Rule.

Gali (2015)

Monetary Policy in the NKM
Implementation of interest rate rules

- What is the economic intuition behind equation (7): $\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0$?

- Let's consider what would be the eventual implication of the rule (eq. 5) for the nominal rate and the Phillips curve if inflation ***permanently*** increased by $d\pi$ (assuming that the natural rate does not change):

$$
\begin{aligned}
di &= \phi_\pi d\pi + \phi_y d\tilde{y}, && (8)\\
d\pi &= \beta d\pi + \kappa d\tilde{y}, && (9)
\end{aligned}
$$

Combining implies:

$$di = \left[\phi_\pi + \phi_y \frac{(1-\beta)}{\kappa}\right] d\pi, \quad (10)$$

- For the system in eq. (6) to have a unique solution equal to the desired solution, eq. (7) must be satisfied.
- When the terms in brackets in eq. (10) is greater than unity, then eq. (7) ***is*** satisfied and we will have a unique solution!

Interpretation:

- What does eq. 7 indicate?

$$\boxed{di = \left[\phi_\pi + \phi_y \frac{(1-\beta)}{\kappa}\right] d\pi,} \qquad (10)$$

- The Taylor principle:

"Adjust the nominal interest rate more than 1-for-1 with changes in inflation"

What happens if the central bank does not follow the Taylor principle?

- Suppose households permanently increase consumption, without any change in economic fundamentals. Production increases and thereby the marginal cost. Inflation increases.
- In response, the CB increases the nominal interest rate but less than the increase in inflation. The real interest rate falls.
- The reduction in the real interest rate justifies even higher consumption!
- If the CB increases the nominal rate by more than 1-for-1, then the real interest rate increases. (**Active** monetary policy)
- The Taylor principle is important in the design of monetary policy rules. Avoids that the central bank becomes a source of unnecessary fluctuations in economic activity.
- **Clarida, Galí, and Gertler (2000):** A change from **passive** to **active** monetary policy in the early 1980s can explain the observed stabilization of macroeconomic outcomes in the US.

Section 4.3.1.3: Study yourself

Monetary Policy in the NKM
Implementation of interest rate rules

- So now we know that a rule aiming for stabilization of inflation and output gap movements with a more than 1-for-1 response of the nominal interest rate to changes in inflation will yield the desired equilibrium outcome.

- So why can we, as central bankers, not make use of the rule: $i_t = r_t^{natural} + \phi_\pi \pi_t + \phi_y \tilde{y}_t$? (5)

- Because it asks us to adjust the nominal interest rate 1-for-1 with the *natural* interest rate and the natural interest rate is <u>not observable</u> unless we have:
  1) exact knowledge of the true model of the economy
  2) exact knowledge of all parameter values
  3) exact knowledge of realized values of all shocks

- In order to deal with this: Use **"simple" rules** based on <u>observable variables only</u>, with no requirement of precise knowledge of model or parameter values.

- It will turn out that these simple rules are robust across models and choice of parameter values.

What about the output gap?

Monetary Policy in the NKM

Implementation of interest rate rules

- How to choose the correct simple rule?

- It can be shown (not by you, unless you are very interested, then see the appendix☺) that a second order approximation to household's welfare in the case of an employment subsidy is the right way to go. The approximation yields the following welfare loss function:

$$\mathbb{W}_t = \frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^k \left[\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha} \right) \tilde{y}_{t+k}^2 + \frac{\epsilon}{\lambda} \pi_{t+k}^2 \right], \quad (11)$$

where $\lambda \equiv \frac{(1-\theta)(1-\beta\theta)}{\theta} \frac{1-\alpha}{1-\alpha+\alpha\epsilon}$.

- The optimal simple rule will minimize the welfare loss of output gap and inflation deviations from their target levels.

- The loss function is increasing in the variance of the output gap and inflation. Why?

- The more fluctuation the higher is the welfare loss.

Monetary Policy in the NKM

Implementation of interest rate rules

The average welfare loss per period is given by a linear combination of the variances of the output gap and inflation

$$\mathbb{L} = \frac{1}{2}\left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha}\right) var(\widetilde{y}_t) + \frac{\epsilon}{\lambda}\, var(\pi_t)\right] \qquad (12)$$

- Fluctuations in the output gap is more costly if:
  - σ is high (high "risk-aversion")
  - φ is high (labor supply elasticity is low)
  - α is high (more decreasing returns to scale in production).
- Fluctuations in inflation is more costly if:
  - ϵ is high (substitution between goods is high)
  - λ is low (more price stickiness)

Parameter values crucial

Monetary Policy in the NKM

Implementation of interest rate rules

- We finish up our use of the Gali book by:
  - introducing a commonly used simple interest rate rule
  - calibrate the model's parameters
  - determine what the implied variances of inflation and the output gap and the corresponding welfare losses will be by using that rule, relative to the optimal allocation of no welfare loss.

$$\rho = -\log \beta$$

$$\phi_\pi, \phi_y > 0$$

- The simple Taylor rule: $$i_t = \rho + \phi_\pi \pi_t + \phi_y \hat{y}_t, \qquad (13)$$

- The central bank decision coefficients are set to satisfy equation (7), to ensure a unique solution: $$\kappa(\phi_\pi - 1) + (1-\beta)\phi_y > 0 \qquad (7)$$

- Rewriting the Taylor rule in terms of the output gap (add and subtract $\phi_y \hat{y}_t^{natural}$):

$$i_t = \rho + \phi_\pi \pi_t + \phi_y \tilde{y}_t + v_t, \qquad (14)$$

$$v_t \equiv \phi_y \hat{y}_t^{natural},$$

- Equation (13) corresponds to our rule from last week, but the interpretation of the shock parameter is now different (How?).

- The equilibrium solution is the same as at lecture 7: $$\begin{bmatrix} \tilde{y}_t \\ \pi_t \end{bmatrix} = A_T \begin{bmatrix} E_t\{\tilde{y}_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + B_T \left[\hat{r}_t^{natural} - v_t\right] \quad [6]$$

Monetary Policy in the NKM
Implementation of interest rate rules

- Inserting for $\hat{r}_t^{natural} - v_t$, using the same logic as at lecture 7, and assuming that the discount factor and technology follows AR(1) processes, the following equality holds:

$$\hat{r}_t^{natural} - v_t = -\psi_{ya}\left(\phi_y + \sigma(1-\rho_a)\right)a_t + (1-\rho_z)z_t \qquad (15)$$

- Equation (15) shows that the more the central bank reacts to changes in the output gap in order to stabilize output (i.e. the larger ϕ_y is):
  - The more will the variance of inflation and output gap increase and;
  - The higher is the welfare loss of fluctuations in the output gap.

- Now, let's look at some results!

- NB: Section 4.4.2 on constant money growth: Can be skipped

New results in the 2015-edition. We use those (as presented on this slide), not the results in the 2008-edition.

Monetary Policy in the NKM

Implementation of interest rate rules

TABLE 4.1
Evaluation of Simple Rules: Taylor Rule

	Technology				*Demand*			
ϕ_π	1.5	1.5	5	1.5	1.5	1.5	5	1.5
ϕ_y	0.125	0	0	1	0.125	0	0	1
$\sigma(y)$	1.85	2.07	2.25	1.06	0.59	0.68	0.28	0.31
$\sigma(\tilde{y})$	0.44	0.21	0.03	1.23	0.59	0.68	0.28	0.31
$\sigma(\pi)$	0.69	0.34	0.05	1.94	0.20	0.23	0.09	0.10
L	1.02	0.25	0.006	7.98	0.10	0.13	0.02	0.02

Galí (2015): Table 4.1

- When technology shock: The more response to output fluctuations the larger is the variances and the higher is the welfare loss!
- A Taylor rule that responds aggressively to inflation gets us pretty close to the optimal policy (in terms of small welfare loss)

Monetary Policy in the NKM

Implementation of interest rate rules

Target Horizons: $k = 1$, $q = 1$	$\hat{\beta}$	$\hat{\gamma}$
1960:1-1979:2		
GDP	0.80	0.44
	(0.09)	(0.11)
Unemployment	0.73	0.78
	(0.06)	(0.12)
1979:3-1996:4		
GDP	1.80	0.12
	(0.19)	(0.13)
Unemployment	1.77	0.12
	(0.17)	(0.24)

Clarida, Galí and Gertler (2000): Table 1

- Clarida, Gali and Gertler (2000): Big difference between pre- and post-Volker/Greenspan in the US.
- During pre-Volker era: Fed rose nominal interest rates by less than the change in expected inflation.
- What are the implications of doing so?
- Clear change of policy targets

Paul Volker is the man who is willing to do whatever it takes

Marvin Goodfriend

Added in the 2015-edition.

The discussion of table 4.1 in the 2015-edition:

Table 4.1 reports the standard deviation of output, the output gap and inflation for different configurations of coefficients ϕ_π and ϕ_y in rule (19). The analysis is conducted conditional on technology and demand/preference shocks separately. The standard deviation of the innovations in both the technology and preference processes is set to one percent. For each shock, the first column reports results based on the original calibration proposed by Taylor (1993). The second and third columns are based on a rules involving no response to output fluctuations, with a very aggressive anti-inflation stance in the case of the third rule ($\phi_\pi = 5$). Finally, the fourth rule assumes a strong output-stabilization motive ($\phi_y = 1$). The remaining parameters are calibrated at their baseline values, introduced in chapter 3.

For each version of the Taylor rule, table 4.1 shows the implied standard deviations of output, the output gap, and (quarterly) inflation, all expressed in percentage terms, as well as the welfare losses resulting from the deviations from the efficient allocation, expressed as a percentage of steady state consumption. Several results stand out. First, when technology shocks are the source of fluctuations, a tradeoff emerges between stabilization of output on the one hand, and stabilization of inflation and the output gap on the other: increasing the value of coefficient ϕ_y leads to a reduction in the volatility of output, but to higher volatility in the output gap and inflation and, hence, larger welfare losses. Those losses increase substantially when the output coefficient ϕ_y is set to unity, relative to Taylor's original calibration. Second, the smallest welfare losses are attained when the monetary authority responds to changes in inflation only. Furthermore, those losses (as well as the underlying fluctuations in the output gap and inflation) become smaller as the strength of that response increases.

When the analysis is conditioned on demand shocks being the source of fluctuations, the previous tradeoff vanishes: the natural level of output remains unchanged, implying that output stabilization is equivalent to output gap stabilization. Thus, increases in either ϕ_π or ϕ_y appear to be effective at stabilizing the welfare-relevant variables and reducing welfare losses.

Taking all the findings above into account, it can be concluded that in the context of the basic New Keynesian model considered here, a simple Taylor-type rule that responds aggressively to movements in inflation can approximate arbitrarily well the optimal policy.[10]

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Optimal policy in the NKM
Introduction

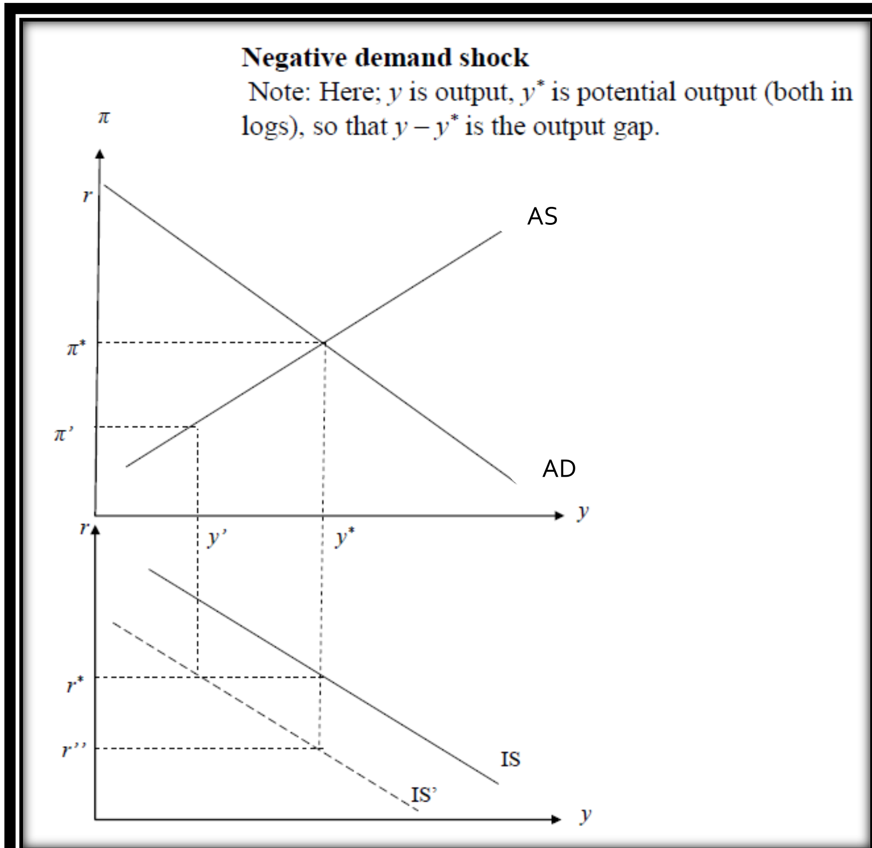
"Having looked at monetary policy from both sides now, I can testify that central banking in practice is as much art as science. Nonetheless, while practicing this dark art, I have always found the science quite useful".

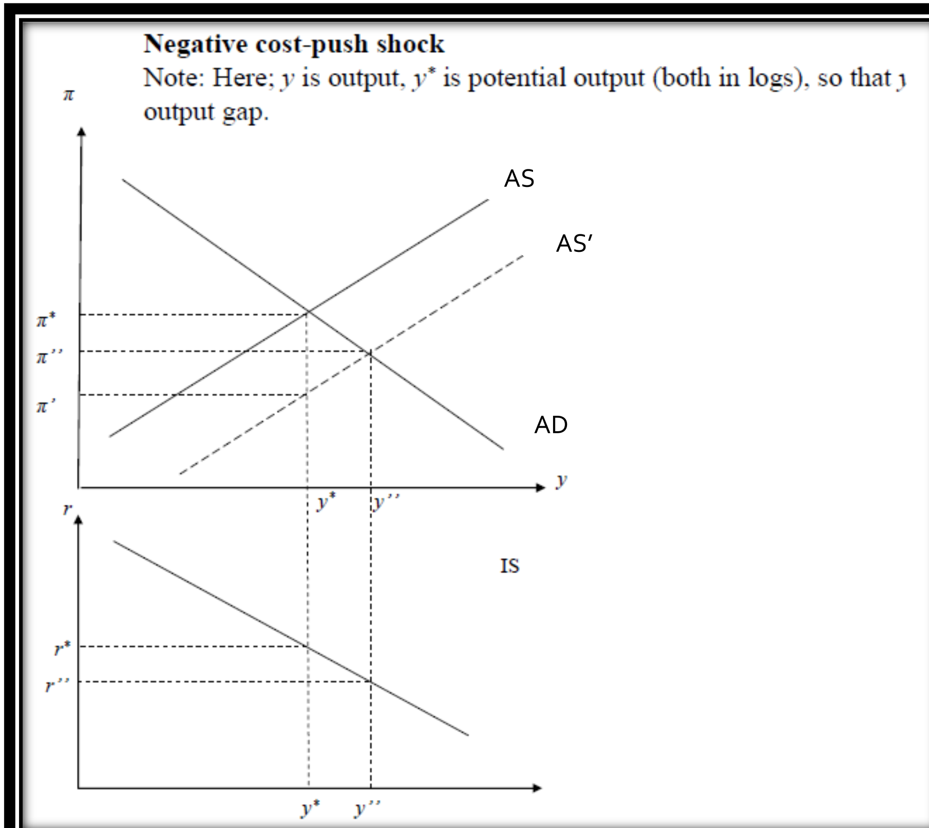
Alan S. Blinder

- The key stumbling block for policy-formation is limited knowledge of the way the macroeconomy works.
- Results that are highly model-specific are of limited use.
- If the results are robust across a variety of macroeconomic frameworks:
- Key friction: New Keynesian Perspective = Temporary Nominal Price Stickiness
- Key policy instrument: The nominal interest rate (+ credibility)
- Optimal monetary policy depends on the degree of persistence in inflation and output.

"Science of monetary policy"

- Optimal central bank response to a shock will depend the origin of the shock
  - How will a negative demand shock affect inflation and output?
  - How will a negative supply shock (a cost-push shock) affect inflation and output?





Optimal policy in the NKM
Introduction

- Credibility of monetary policy is relevant because private sector behavior depends on the expected course of monetary policy

- How to increase credibility?
  - The central bank commits to a policy rule
  - Institutional reforms

- Remember from lecture 2 (i.e. The Barro Gordon model):

  An inefficiently high inflation rate may arise in the absence of commitment if the central bank's output target exceeds the market clearing level

  Inflation bias

Gain 1 from commitment: Eliminates the inflation bias

Gain 2 from commitment: Credible commitment to fight inflation in the future can improve the current output/inflation trade-off faced by the central bank also when no inflation bias

- Will address this over the next two lectures by examining optimal policy in the NKM both with and without commitment!

Optimal policy in the NKM
Introduction

- Some practical problems can complicate the policy-making:

Imperfect information

Non-smooth preferences over inflation and output

Model uncertainty

How should we use the intermediate targets?

Outside and inside lags

What should we choose as our monetary policy instrument?

Should we smooth the changes in the interest rate?

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The New Keynesian Model

The CGG version 1/5

- The model is a dynamic general equilibrium model with money and temporary nominal price rigidities.
- Monetary policy affects the real economy in the short run
- Aggregate behavioral equations evolve explicitly from hh and firm optimization
- Current economic behavior depends critically on expectations about future course of monetary policy, as well as current policy.
- In the CGG-paper the key equations are introduced directly
  - Fortunately we know how to derive them!
  - To refresh your memory, study lecture 5 one more time.
- The output gap is defined in the following way: $x_t \equiv y_t - z_t$
- Recall from lecture 5 that the output gap is non-zero when firm marginal costs associated with excess demand deviate from steady state

The New Keynesian Model

The CGG version – the DIS curve

2/5

- Recall the dynamic IS equation derived at lecture 5 (slide 33):

The DIS-eq is **THE SECOND KEY EQUATION OF THE NKM!**

$$\tilde{y}_t = E_t\{\tilde{y}_{t+1}\} - \frac{1}{\sigma}\left(i_t - E_t\{\pi_{t+1}\} - r_t^{natural}\right)$$

- The dynamic IS equation used in the CGG-paper:

Demand shock!

$$x_t = -\eta\left[i_t - E_t\pi_{t+1}\right] + E_t x_{t+1} + g_t \qquad (16)$$

where $g_t = \mu g_{t-1} + \hat{g}_t, \qquad 0 \le \mu \le 1 \qquad \hat{g}_t \sim iid\,(0, \sigma_g^2)$ (17)

Interpretation:

- Recall the NK Phillips curve derived at lecture 5 (slide 29):

$$\pi_t = \beta E_t\{\pi_{t+1}\} + \underbrace{\lambda\left(\sigma + \frac{\varphi+\alpha}{1-\alpha}\right)}_{\kappa}\tilde{y}_t = \beta E_t\{\pi_{t+1}\} + \kappa\tilde{y}_t$$

The NKPC is **THE FIRST KEY EQUATION OF THE NKM!**

- The NK Phillips curve used in the CGG-paper:

Cost-push shock!

$$\pi_t = \kappa x_t + \beta E_t \pi_{t+1} + u_t \tag{18}$$

where $$u_t = \rho u_{t-1} + \hat{u}_t, \qquad 0 \le \rho \le 1 \qquad \hat{u}_t \sim iid(0, \sigma_u^2) \tag{19}$$

Interpretation:

The New Keynesian Model
The CGG version

- Iterating equation (16) forward yields:

$$x_t = E_t \sum_{k=0}^{\infty} \left\{ -\eta \left[i_{t+k} - \pi_{t+k+1} \right] + g_{t+k} \right\} + \underbrace{E_t x_{t+\infty}}_{=0} \qquad (20)$$

because $E_t x_{t+1} = E_t \left\{ -\eta \left[i_{t+1} - E_t \pi_{t+2} \right] \right\} + E_t x_{t+2} + E_t g_{t+1}$

- Iterating equation (18) forward yields:

$$\pi_t = E_t \sum_{k=0}^{\infty} \beta^k \left\{ \kappa x_{t+k} + u_{t+k} \right\} \qquad (21)$$

Contrast to "standard" Phillips curve:
No inflation inertia or lagged dependence

because $E_t \pi_{t+1} = \kappa E_t x_{t+1} + \beta E_t \pi_{t+2} + E_t u_{t+1}$

- The output gap and the inflation rate depend on both current and expected future stance of monetary policy and on current and expected future values of shocks!

- Having these two equilibrium equations established, what do we need to close the model??

The New Keynesian Model
The CGG version 5/5

- In order to close the model we must introduce monetary policy somehow. Choose to take the nominal interest rate as the policy instrument.
- When the nominal interest rate is the instrument, the central bank adjusts money supply to hit the interest rate target.
  - Not necessary to specify a money market equilibrium condition
- Because of nominal rigidities changes in monetary policy will have an effect on the real interest rate both today and in future periods.
- Beliefs about how the central bank will set the interest rate in the future also matter, since both households and firms are forward looking.
- Forward looking agents create a challenge for monetary policy; how monetary policy should respond to disturbances in the economy is a non-trivial question!
  - The more central bankers know about the effects of shocks and about the transmission mechanisms of monetary policy the better

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The New Keynesian Model
The policy objective 1/2

- Central Bank target variables: x_t and π_t
- The central bank will try to maximize welfare by minimizing the following loss function:

$$L_t = \frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^k \left\{ \lambda^* x_{t+k}^2 + \pi_{t+k}^2 \right\} \tag{22}$$

- Why target inflation?
  - The uncertainty generated for lifetime financial planning and for business planning is a major cost of inflation variability.
- Why target the natural level of output?
  - Since it is the level obtained if no price or wage frictions
  - If distortions exist (e.g. monopolistic competition, taxes etc.) the welfare maximizing level of output might exceed the natural level (Can this level be reached in optimum when expectations are rational and monetary policy is discretionary?)
- What is the optimal relative weight on the output gap?

The New Keynesian Model
The policy objective 2/2

- What is the optimal level of inflation?
  - Should be positive due to:
    - Measurement errors
    - DNWR

Price stability when inflation no longer is a public concern

- In the following we take the inflation target and the coefficients as given.
- Remember: We have not yet introduced a rule, we study optimal monetary policy in the NKM *prior to* the introduction of a rule.
- So how should the central bank act in order to maximize welfare?

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Optimal Monetary Policy
Discretion (time-consistent) 1/12

- Why discuss time-consistent discretion?
  - Accords best with reality
  - Must fully understand it to see the benefits of commitment

- The central bank's optimal choice of $\{\pi_t, x_t, i_t\}$ every period can be divided into two steps:

Step 1

Minimize the loss function (eq.22) with respect to inflation and the output gap, subject to the NKPC (eq.18)

Step 2

Conditional on optimal values of inflation and output gap, determine the value of the nominal interest rate implied by the DIS equation (eq. 16)

Optimal Monetary Policy
Discretion (time-consistent) 2/12

- Since the central bank cannot credibly manipulate private sector beliefs in the absence of commitment (due to rational expectations), it must take private sector expectations as given.

- The period minimization problem (step 1):

$$\underset{x_t,\pi_t}{Min}\, L_t = \frac{1}{2}\left\{\lambda^* x_t^2 + \pi_t^2\right\} + F_t, \qquad F_t \equiv \frac{1}{2} E_t \sum_{k=1}^{\infty} \beta^k \left\{\lambda^* x_{t+k}^2 + \pi_{t+k}^2\right\} \qquad (23)$$

subject to $\pi_t = \kappa x_t + f_t,$

$$f_t \equiv \beta E_t \pi_{t+1} + u_t \qquad (24)$$

- Why reformulate it this way?
  - Under discretion, future inflation and output are not affected by today's actions (since the central bank can re-optimize every period!)
  - The central bank cannot directly manipulate expectations (since expectations are rational)

- FOCs: $$x_t = -\frac{\kappa}{\lambda^* + \kappa^2} f_t = -\frac{\kappa}{\lambda^* + \kappa^2}\left[\beta E_t \pi_{t+1} + u_t\right], \qquad (25)$$

$$\pi_t = \frac{\lambda^*}{\lambda^* + \kappa^2} f_t = \frac{\lambda^*}{\lambda^* + \kappa^2}\left[\beta E_t \pi_{t+1} + u_t\right], \qquad (26)$$

- The optimal choices of inflation and output gap both depend on private agents' inflation expectations.
  - **NBNB:** What is the main difference from the Barro Gordon discussion at lecture 2?
- Combining the two FOCs yields: $$x_t = -\frac{\kappa}{\lambda^*}\pi_t, \qquad (27)$$
- Interpretation:

Leaning against the wind!

- How to find the reduced form solutions?
  - There are two first order conditions explaining optimal central bank behavior
  - There are rational expectations
  - There are two ways to find the reduced form solutions
    - Method of undetermined coefficients
    - Solving equation (26) forward

Solving equation (26) forward:

- The private agents know the central bank optimality conditions and therefore form the following expectations for inflation in period t+1:

$$E_t\pi_{t+1} = \frac{\lambda^*}{\lambda^* + \kappa^2}\left[\beta E_t\pi_{t+2} + \underbrace{E_t u_{t+1}}_{\rho u_t}\right], \tag{28}$$

- Inserting for the inflation expectations in the FOC (eq.26):

$$\pi_t = \frac{\lambda^*}{\lambda^* + \kappa^2}\left[\beta \underbrace{\frac{\lambda^*}{\lambda^* + \kappa^2}\left[\beta E_t\pi_{t+2} + \rho u_t\right]}_{E_t\pi_{t+1}} + u_t\right] = \left(\frac{\lambda^*\beta}{\lambda^* + \kappa^2}\right)^2 E_t\pi_{t+2} + \frac{\lambda^*}{\lambda^* + \kappa^2}\left(u_t + \frac{\lambda^*\beta\rho}{\lambda^* + \kappa^2}u_t\right) \tag{29}$$

Optimal Monetary Policy
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- Iterating equation (29) forward:

$$\pi_t = \left(\frac{\lambda^*\beta}{\lambda^*+\kappa^2}\right)^2 \left[\underbrace{\frac{\lambda^*}{\lambda^*+\kappa^2}\left[\beta E_t\pi_{t+3} + \rho^2 u_t\right]}_{E_t\pi_{t+2}}\right] + \frac{\lambda^*}{\lambda^*+\kappa^2}\left(u_t + \frac{\lambda^*\beta\rho}{\lambda^*+\kappa^2}u_t\right) = \left(\frac{\lambda^*\beta}{\lambda^*+\kappa^2}\right)^3 E_t\pi_{t+3} + \frac{\lambda^*}{\lambda^*+\kappa^2}\left(u_t + \frac{\lambda^*\beta\rho}{\lambda^*+\kappa^2}u_t + \left(\frac{\lambda^*\beta\rho}{\lambda^*+\kappa^2}\right)^2 u_t\right)$$

$$\Rightarrow \pi_t = \underbrace{\left(\frac{\lambda^*\beta}{\lambda^*+\kappa^2}\right)^n E_t\pi_{t+n}}_{\to 0} + \frac{\lambda^*}{\lambda^*+\kappa^2}\left(\frac{1}{1-\frac{\lambda^*\beta\rho}{\lambda^*+\kappa^2}}\right)u_t = \frac{\lambda^*}{\lambda^*+\kappa^2}\left(\frac{\lambda^*+\kappa^2}{\kappa^2+\lambda^*(1-\beta\rho)}\right)u_t$$

$$\Rightarrow \pi_t = \left(\frac{\lambda^*}{\kappa^2+\lambda^*(1-\beta\rho)}\right)u_t \tag{30}$$

- Inserting the reduced form expression for inflation in equation (27) yields the optimal reduced form expression for output:

$$x_t = -\frac{\kappa}{\lambda^*}\pi_t = -\frac{\kappa}{\lambda^*}\left(\frac{\lambda^*}{\kappa^2+\lambda^*(1-\beta\rho)}\right)u_t = -\left(\frac{\kappa}{\kappa^2+\lambda^*(1-\beta\rho)}\right)u_t \tag{31}$$

- Interpretation:

Method of undetermined coefficients

- Difference from lecture 7: Now, the central bank optimizes welfare by minimizing a loss function. Previously, we closed the model with a rule and aimed at choosing the rule that minimized the loss function.

- As a result, instead of focusing on the NKPC and the DIS equation, we must now focus on the central bank's period FOCs (eq. 25 and 26). Combining them in an equation system yields:

$$\begin{bmatrix} x_t \\ \pi_t \end{bmatrix} = \begin{bmatrix} 0 & -\dfrac{\kappa\beta}{\lambda^* + \kappa^2} \\ 0 & \dfrac{\lambda^*\beta}{\lambda^* + \kappa^2} \end{bmatrix} \begin{bmatrix} E_t\{x_{t+1}\} \\ E_t\{\pi_{t+1}\} \end{bmatrix} + \begin{bmatrix} -\dfrac{\kappa}{\lambda^* + \kappa^2} \\ \dfrac{\lambda^*}{\lambda^* + \kappa^2} \end{bmatrix} u_t \qquad (32)$$

- Solving the system by the 5-step method of undetermined coefficients introduced at lecture 7 will yield the same result (i.e. will yield equations 30 and 31).
  - Give it a shot and try to understand why you can use this method!

Optimal Monetary Policy

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Recall:

- The central bank's optimal choice of $\{\pi_t, x_t, i_t\}$ every period can be divided into two steps:

Step 1

Minimize the loss function (eq.22) with respect to inflation and the output gap, subject to the NKPC (eq.18)

Step 2

Conditional on optimal values of inflation and output gap, determine the value of the nominal Interest rate implied by the DIS equation (eq.16):

$$x_t = -\eta\left[i_t - E_t\pi_{t+1}\right] + E_t x_{t+1} + g_t \qquad (16)$$

- We know that the desired values of the inflation rate and the output gap are:

$$\pi_t = \left(\frac{\lambda^*}{\kappa^2 + \lambda^*(1-\beta\rho)}\right)u_t \qquad x_t = -\left(\frac{\kappa}{\kappa^2 + \lambda^*(1-\beta\rho)}\right)u_t$$

- Rational expectations yield:

$$E_t\pi_{t+1} = \left(\frac{\lambda^*}{\kappa^2 + \lambda^*(1-\beta\rho)}\right)E_t u_{t+1} = \left(\frac{\rho\lambda^*}{\kappa^2 + \lambda^*(1-\beta\rho)}\right)u_t \qquad E_t x_{t+1} = -\left(\frac{\rho\kappa}{\kappa^2 + \lambda^*(1-\beta\rho)}\right)u_t$$

Optimal Monetary Policy

Discretion (time-consistent) 8/12

- The optimal feed-back policy of the interest rate is then found by inserting the desired values in the DIS-equation:

$$i_t = \frac{1}{\eta}\left[\frac{\eta\rho\lambda^* + \kappa(1-\rho)}{\kappa^2 + \lambda^*(1-\beta\rho)}\right]u_t + \frac{1}{\eta}g_t \Rightarrow i_t = \left(1 + \frac{\kappa(1-\rho)}{\eta\rho\lambda^*}\right)E_t\pi_{t+1} + \frac{1}{\eta}g_t \qquad (33)$$

- How does the central bank react to a demand shock?
- How does the central bank react to changes in inflation expectations?

- Interpretation of eq.(33):

- Let's now look at some key results that emerge from this model

Key result 1
Trade-off between output and inflation variability when cost-push shock

The efficient policy frontier (σ_x, σ_π) is found:
For the output gap:

$$Var(x_t) = \sigma_x^2 = E_t(x_t^2) - [E_t(x_t)]^2$$

$$\Rightarrow \sigma_x^2 = \left(\frac{\kappa}{\kappa^2 + \lambda^*(1-\beta\rho)}\right)^2 E_t(u_t^2) - \left[\left(\frac{\kappa}{\kappa^2 + \lambda^*(1-\beta\rho)}\right) E_t u_t\right]^2$$

$$\Rightarrow \sigma_x^2 = \left(\frac{\kappa}{\kappa^2 + \lambda^*(1-\beta\rho)}\right)^2 \sigma_u^2$$

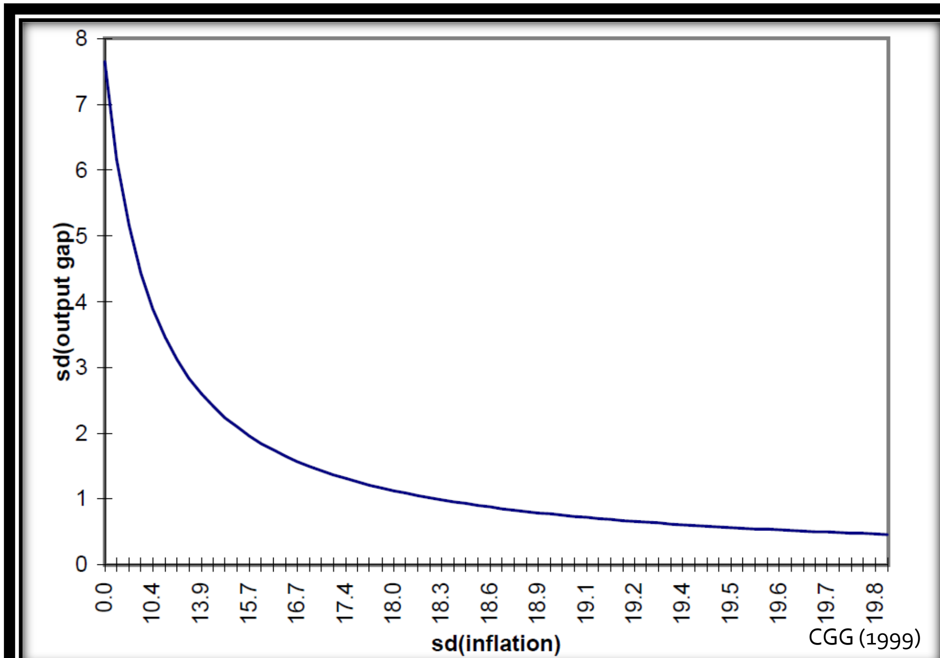
$$\Rightarrow \sigma_x = \left(\frac{\kappa}{\kappa^2 + \lambda^*(1-\beta\rho)}\right) \sigma_u \qquad (34)$$

The corresponding standard deviation for inflation:

$$\sigma_\pi = \left(\frac{\lambda^*}{\kappa^2 + \lambda^*(1-\beta\rho)}\right) \sigma_u \qquad (35)$$

The larger relative weight on stabilizing output, the lower is output volatility and the higher is inflation volatility.

- What happens when $\lambda^* = 0$?
- Will there still be a trade-off if no cost-push shock?
- Why do output and inflation volatility disappear in the absence of a cost-push shock?



CGG (1999)

Key result 2

- Optimal monetary policy incorporates inflation targeting. How?
- The optimal policy aims for convergence of inflation to its target over time when cost-push shocks are present and the relative weight on output stabilization is non-zero.
  - Leaning against the wind
  - "You gain some and you lose some"
- Extreme inflation targeting (adjusting policy to immediately reach target, so that $\sigma_\pi = 0$) is only optimal under two circumstances:
  - When cost-push inflation is absent (i.e. $\sigma_u = 0$)
  - When weight on output stabilization is zero (i.e. $\lambda^* = 0$)

Optimal Monetary Policy
Discretion (time-consistent) – Key results

Key result 3

- From equation (33) we know that optimal policy requires that the nominal interest rate should be adjusted by more than one-to-one to changes in inflation expectations.

Recall equation (33):

$$i_t = \frac{1}{\eta}\left[\frac{\eta\rho\lambda^* + \kappa(1-\rho)}{\kappa^2 + \lambda^*(1-\beta\rho)}\right]u_t + \frac{1}{\eta}g_t \Rightarrow i_t = \left(1 + \frac{\kappa(1-\rho)}{\eta\rho\lambda^*}\right)E_t\pi_{t+1} + \frac{1}{\eta}g_t$$

- Because of this result one should make sure that the coefficient on expected inflation in an interest rate rule is larger than unity.

- What happens to inflation if the coefficient is less than unity and inflation expectations increase?

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Key result 4

- From equation (33) we know that optimal policy requires that the nominal interest rate should be adjusted to perfectly offset demand shocks and not react to shocks to potential output z_t

Recall equation (33): $$i_t = \frac{1}{\eta}\left[\frac{\eta\rho\lambda^* + \kappa(1-\rho)}{\kappa^2 + \lambda^*(1-\beta\rho)}\right]u_t + \frac{1}{\eta}g_t \Rightarrow i_t = \left(1 + \frac{\kappa(1-\rho)}{\eta\rho\lambda^*}\right)E_t\pi_{t+1} + \frac{1}{\eta}g_t$$

- No trade-off when demand shock, so can be adjusted for right away

- No trade-off from productivity shocks either!
  - A permanent rise in productivity raises potential output and actual output by the same amount (due to impact on permanent income)
  - No change in the output gap $x_t \equiv y_t - z_t$ or in the inflation rate so no need to change the interest rate

- The central bank should therefore not adjust the interest rate when there are productivity movements
  - Challenge for monetary policy: Distinguish between the **sources** of business cycle shocks

Next two weeks:

Next week: Easter break
Suggestion: Catch up and read the CGG (1999) paper

Lecture 9 on 1 April

- Gains from commitment
- Optimal Monetary Policy with commitment
- Some practical complications

- The rest of the CGG (1999) paper (sections 4 and 5)

Section 6 in the CGG paper can be dropped, so can section 7.

Monetary Policy
(Advanced Monetary Economics)

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