# Monetary Policy

(Advanced Monetary Economics)

ECON 4325

Nina Larsson Midthjell - Lecture 9 - 1 April 2016

# Outline

#### Optimal monetary policy in a New Keynesian Model

- Optimal monetary policy under discretion (lecture slides 32-43 from lecture 8)
- Introduction to commitment
- Gains from commitment: Optimum within a family of rules (CGG 1999, section 4.2.1)
- Gains from commitment: The Unconstrained Optimum (CGG 1999, section 4.2.2)
- Some practical complications (CGG 1999, section 5)

• CGG (1999), sections 6 and 7 may be dropped.

## NB:

We will start with lecture slides 32-43 from lecture 8, so do not forget to bring your slides!

# Introduction

- 1/4
- In the presence of cost-push shocks, there is a **trade-off** for monetary policy:

$$\pi_{t} = \beta E_{t} \{\pi_{t+1}\} + \kappa \widetilde{y}_{t} + u_{t}$$
$$\widetilde{y}_{t} = E_{t} \{\widetilde{y}_{t+1}\} - \frac{1}{\sigma} \left(\underbrace{i_{t} - E_{t} \{\pi_{t+1}\}}_{r_{t}} - \rho\right) + g_{t}$$

- Three ways of specifying **optimal monetary policy**:
  - 1. Under discretion (i.e. the central bank re-optimizes period by period. Lecture 8 slides discussed today)
  - 2. Commitment to a simple rule
    - a) Instrument rule (lecture 8)
    - b) Targeting rule (today)
  - 3. Full state-contingent commitment (today)

## Two main gains from commitment

- Gains related to time-inconsistency and resulting inflation bias when the central bank tries to push output above its potential level (Recall the Barro-Gordon model from lecture 2).
- Gains emerging even with inflationary bias absent, because of price-setting being dependent on expectations about the future in the model (what we will discuss today).

#### Gains from commitment related to the classic inflationary bias problem

$$L_{t} = \frac{1}{2} E_{t} \sum_{i=0}^{\infty} \beta^{i} \left\{ \lambda^{*} \left[ y_{t+i} - (z_{t+i} + k) \right]^{2} + \pi_{t+i}^{2} \right\}, \qquad k > 0$$
<sup>(1)</sup>

#### Key result 5 (CGG 1999)

If the central bank desires to push output above potential, then under discretion a suboptimal equilibrium may emerge with inflation persistently above target and no gain in output

• See lecture 2 for a thorough discussion of the time-inconsistency problem

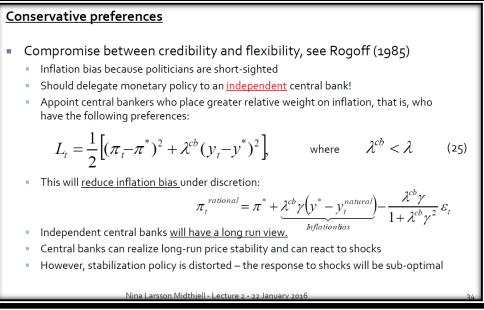
## Introduction Gains from commitment

## Key result 6 (CGG 1999)

• Appointing a conservative central bank chairman who assigns a higher relative cost to inflation than society as a whole, reduces the inefficient inflationary bias that is obtained

under discretion when k > 0

Remember from lecture 2 (slide 34):



• **Cost from conservative preferences:** Reduction in inflation variance come at the cost of higher output variance, as eq. (34) and (35) at lecture 8 emphasize. Recall:

$$\sigma_{x} = \left(\frac{\kappa}{\kappa^{2} + \lambda^{*}(1 - \beta\rho)}\right)\sigma_{u} \qquad \sigma_{\pi} = \left(\frac{\lambda^{*}}{\kappa^{2} + \lambda^{*}(1 - \beta\rho)}\right)\sigma_{u}$$

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## Introduction Gains from commitment

- So do central bankers today aim at a higher output target than the potential output level?
- If this is true, so that k = 0, will there still be any gains from increasing credibility?

k = 0

#### Gains from commitment when

- To the extent that price setting today depends on beliefs about future economic conditions, a central bank that is able to signal a clear commitment to controlling inflation may face an improved short run output/inflation trade-off.
- The central bank hence no longer takes private sector expectations as given, but recognizes that its policy choice effectively determines these expectations.
- Two types of monetary policy under commitment when k = 0 is discussed in the paper:
  - The optimum within a simple family of policy rules (section 4.2.1), including the optimality condition from lecture 8, equation (33) (i.e. the optimal rule under discretion):

$$i_{t} = \frac{1}{\eta} \left[ \frac{\eta \rho \lambda^{*} + \kappa (1 - \rho)}{\kappa^{2} + \lambda^{*} (1 - \beta \rho)} \right] u_{t} + \frac{1}{\eta} g_{t} \Rightarrow i_{t} = \left( 1 + \frac{\kappa (1 - \rho)}{\eta \rho \lambda^{*}} \right) E_{t} \pi_{t+1} + \frac{1}{\eta} g_{t}$$

The unconstrained optimum (section 4.2.2)

vel?

Not likely

Likely

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• CGG (1999), sections 6 and 7 may be dropped.

 Recall from our discussion without commitment (lecture 8 slides) that it is optimal for the central bank to adjust the output gap in the following way:

$$x_{t} = -\left(\frac{\kappa}{\kappa^{2} + \lambda^{*}(1 - \beta\rho)}\right)u_{t}$$
(2)

Now, let's consider a targeting rule for the target variable X<sub>t</sub> that is contingent on the fundamental cost-push shock in the following way:

$$x_t^c = -\omega u_t, \qquad \omega > 0 \tag{3}$$

- Each value of the coefficient  $\omega > 0$  will relate to one particular rule, so that when:  $\omega = \left(\frac{\kappa}{\kappa^2 + \lambda^*(1 - \beta \rho)}\right)$ , we have the optimum under discretion as a special case.
- Inflation under this rule can be found from the iterated version of the NK Phillips curve:

$$\pi_{t}^{c} = E_{t} \sum_{k=0}^{\infty} \beta^{k} \left\{ \kappa x_{t+k}^{c} + u_{t+k} \right\} = E_{t} \sum_{k=0}^{\infty} \beta^{k} \left\{ -\kappa \omega u_{t+k} + u_{t+k} \right\} = E_{t} \sum_{k=0}^{\infty} \beta^{k} \left\{ (1 - \kappa \omega) u_{t+k} \right\}$$

Here  $E_t u_{t+k} = \rho^k u_t$ , hence this reduces to:  $\pi_t^c = \sum_{\substack{k=0\\k=0}}^{\infty} (\rho\beta)^k \{(1-\kappa\omega)u_t\} = \frac{(1-\kappa\omega)}{1-\rho\beta}u_t$  (4) Nina Larsson Midthjell - Lecture 9 - 1 April 2016

- Under discretion, when the central bank re-optimizes every period , reducing the output gap with one percentage point today will create a reduction in inflation today with  $\kappa$
- When the policy rule in equation (3) is imposed, the relationship is:  $\pi_t = E_t \sum_{k=0}^{\infty} \beta^k \{ \kappa x_{t+k} + u_{t+k} \}$  (5)

• Since 
$$\pi_t^c = \frac{\kappa}{1 - \rho\beta} x_t^c + \frac{1}{1 - \rho\beta} u_t, \qquad \frac{\kappa}{1 - \rho\beta} > \kappa$$
 (6)

, inflation falls more in period t under such a rule. Why?

- Because the policy will have an impact on the future course of the output gap as well!
- If the central bank reports a high omega, they communicate an aggressive response to a
  persistent supply shock.
- Since inflation depends on the future course of excess demand, commitment to the tough policy rule leads to a magnified drop in inflation per unit of output loss, relative to the case of discretion.
- What is the optimal value of omega?

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- In order to maximize welfare, the central bank now faces the following minimization problem:

$$\min_{x_{i}^{c},\pi_{i}^{c}} L_{t} = \frac{1}{2} E_{t} \sum_{k=0}^{\infty} \beta^{k} \left\{ \lambda^{*} \left( x_{t+k}^{c} \right)^{2} + \left( \pi_{t+k}^{c} \right)^{2} \right\} \Leftrightarrow \min_{x_{i}^{c},\pi_{i}^{c}} L_{t} = \frac{1}{2} \left\{ \lambda^{*} \left( x_{t}^{c} \right)^{2} + \left( \pi_{t}^{c} \right)^{2} \right\} E_{t} \left[ \sum_{k=0}^{\infty} \beta^{k} \left\{ \left( \frac{u_{t+k}}{u_{t}} \right)^{2} \right\} \right] \quad (7)$$
Subject to
$$\pi_{t}^{c} = \frac{\kappa}{1 - \rho\beta} x_{t}^{c} + \frac{1}{1 - \rho\beta} u_{t} \quad (6)$$

(In order to see that the multiple in equation (7) can be constructed like that, insert for the output gap and inflation from equations (3) and (4) and open up the sum.)

• The optimality condition for the output gap when inserted for inflation in equation (7) is:  $\frac{dL_t}{dx_t^c} = \lambda^* x_t^c \Omega_t + \Omega_t \left(\frac{\kappa}{1-\rho\beta} x_t^c + \frac{1}{1-\rho\beta} u_t\right) \frac{\kappa}{1-\rho\beta} = 0$   $\left[ \left[ \left(-\kappa - \frac{1}{2}\right)^2 \right] - \left(-\kappa - \frac{1}{2}\right)^2 -$ 

$$\Rightarrow \left[\lambda^* + \left(\frac{\kappa}{1 - \rho\beta}\right)^{-}\right] x_t^c = -\frac{\kappa}{\left(1 - \rho\beta\right)^2} u_t \Rightarrow \left[\lambda^* (1 - \rho\beta)^2 + \kappa^2\right] x_t^c = -\kappa u_t \Rightarrow x_t^c = \frac{-\kappa}{\left[\kappa^2 + \lambda^* (1 - \rho\beta)^2\right]} u_t \tag{8}$$

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- Inserting for optimal output gap in inflation equation (6) yields optimality condition for inflation equal to:

$$\pi_t^c = \frac{\kappa}{1 - \rho\beta} \left[ \frac{-\kappa}{\left[\kappa^2 + \lambda^* (1 - \rho\beta)^2\right]} u_t \right] + \frac{1}{1 - \rho\beta} u_t = \frac{\left[\kappa^2 + \lambda^* (1 - \rho\beta)^2\right] - \kappa^2}{(1 - \rho\beta) \left[\kappa^2 + \lambda^* (1 - \rho\beta)^2\right]} u_t$$
$$\Rightarrow \pi_t^c = \frac{\lambda^* (1 - \rho\beta)^2}{(1 - \rho\beta) \left[\kappa^2 + \lambda^* (1 - \rho\beta)^2\right]} u_t = \frac{\lambda^* (1 - \rho\beta)}{\left[\kappa^2 + \lambda^* (1 - \rho\beta)^2\right]} u_t \qquad (9)$$

• Comparing these optimality conditions with the ones we got without commitment:

Without  
commitment
$$\pi_t = \left(\frac{\lambda^*}{\kappa^2 + \lambda^*(1 - \beta\rho)}\right) u_t$$
 $x_t = -\left(\frac{\kappa}{\kappa^2 + \lambda^*(1 - \beta\rho)}\right) u_t$ Indicating a  
more  
negative  
output gap  
with  
commitmentWith  
commitment $\pi_t^c = \left(\frac{\lambda^*(1 - \beta\rho)}{\kappa^2 + \lambda^*(1 - \beta\rho)^2}\right) u_t < \pi_t$  $x_t^c = -\left(\frac{\kappa}{\kappa^2 + \lambda^*(1 - \beta\rho)^2}\right) u_t < x_t$ Indicating a  
more  
more  
negative  
output gap  
with  
commitment

- If the central bank chooses weight on output equal to  $\lambda^c = \lambda^* (1 \beta \rho) < \lambda^*$ , it will yield the optimal omega.
- The central bank put less weight on output and will react more aggressively to inflation deviations from target.
- The output cost of lowering inflation declines from  $\lambda^*$  to  $\lambda^c$  per unit, since reducing inflation a given amount only requires a fraction  $(1-\beta\rho)$  of the output loss required under discretion.

$$\pi_t^c = \left(\frac{\lambda^* (1 - \beta \rho)}{\kappa^2 + \lambda^* (1 - \beta \rho)^2}\right) u_t < \pi_t$$
$$x_t^c = -\left(\frac{\kappa}{\kappa^2 + \lambda^* (1 - \beta \rho)^2}\right) u_t < x_t$$

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Cost-benefit analysis using this targeting rule:

- Period t: Assume a cost-push shock hits the economy. What is the effect of decreasing  $\lambda^c$ ?
  - If  $\lambda^c$  is reduced, the output gap becomes more negative and inflation goes down with a  $\kappa$ -part of the reduction in the output gap.
- Period t+1 (the cost-push shock is smaller because  $0 < \rho_u < 1$ ):
  - If  $\lambda^c$  is reduced from period t, then we'll also have  $x_{t+1} \downarrow$  and  $\pi_{t+1} \downarrow$
  - If  $\pi_{t+1} \downarrow$  , then  $\pi_t \downarrow$  by  $\beta_{\mathcal{K}}$
- Period t+2 (the cost-push shock is even smaller):
  - If  $\lambda^c$  is reduced from period t, then we'll also have  $x_{t+2} \downarrow$  and  $\pi_{t+2} \downarrow$

If 
$$\pi_{t+2} \downarrow$$
, then  $\pi_{t+1} \downarrow$  and  $\pi_t \downarrow$  by  $(\beta \kappa)^2$ 

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Cost-benefit analysis using this targeting rule:

The trade-off between output gap and inflation in period t is therefore:

$$\lambda^* x_t = -\kappa \Big( 1 + \rho_u \beta + (\rho_u \beta)^2 + (\rho_u \beta)^3 + \dots \Big) \pi_t$$

or:

1

$$x_t = \frac{-\kappa}{\lambda^* (1 - \rho_u \beta)} \pi_t$$

• What about all the other periods? They imply similar costs and benefits:

$$\beta \lambda^* x_{t+1} = -\beta \kappa \left( 1 + \rho_u \beta + (\rho_u \beta)^2 + (\rho_u \beta)^3 + \ldots \right) \pi_{t+1}$$

$$x_{t+1} = \frac{-\kappa}{\lambda^* (1 - \rho_u \beta)} \pi_{t+1}$$

• Optimality condition:

$$x_{t} = \frac{-\kappa}{\lambda^{*}(1 - \rho_{u}\beta)}\pi_{t} \quad \text{or} \quad \lambda^{c} = \lambda^{*}(1 - \beta\rho) < \lambda^{*}$$
(10)

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# Gains from commitment

## Optimum within a family of policy rules

The slide has been updated on 3 April 2016 with the info in red

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- Commitment to a rule over time results in a higher relative weight on inflation.
- Doing this increases welfare. How do we know that?
  - The optimal choice under commitment is <u>not</u> the optimal choice under discretion, hence the commitment solution is a welfare improvement because the optimal choice under discretion falls within the class of rules that the central bank can choose from
- The idea is that because price-setting depends on expected future economic conditions, the central bank would like to convince private agents that it will be tough in the future, without having to contract output by too much today.
- Under discretion, the central bank has an incentive to try to re-convince the agents about this
  policy choice every period and will always (optimally) choose to contract the output gap by
  less than under commitment.
- A private agent with rational expectations will recognize this incentive and not expect contractions in the future either. Leads to higher inflation under discretion.
- If the central bank is free to deviate from the rule, it will always choose the optimal policy under discretion, which calls for a smaller contraction of output, relative to the case of commitment

- As a result the cost-push shock generates higher inflation rates in the absence of commitment:
  - A cost for private agents trying to plan future investment and consumption decisions
- Remember, this is <u>not</u> tied to a central bank desire of higher output levels than the potential/natural level! The "modern" central banks do not try to "fool" agents.
- What is the behavior of the nominal interest rate in the case of commitment? Explain.

#### Key result 7 (CGG 1999)

If price-setting depends on expectations of future economic conditions, then a central bank that can credibly commit to a rule faces an improved short run trade-off between inflation and output.

In this case, the solution under commitment perfectly resembles the solution that would obtain for a central bank with discretion that assigned a higher cost to inflation than the true social cost.

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• CGG (1999), sections 6 and 7 may be dropped.

- We now look at full state-contingent commitment.
- The central bank specifies some action for **all possible future states**.
- We solve this by using the Lagrangian in the standard way:

$$L_{t} = \frac{1}{2} \sum_{i=0}^{\infty} \beta^{i} E_{t} \left\{ \left( \pi_{t+i}^{2} + \lambda^{*} x_{t+i}^{2} \right) - 2\Lambda_{t+i} \left( \pi_{t+i} - \beta \pi_{t+i+1} - \kappa x_{t+i} - u_{t+i} \right) \right\}, \quad (11)$$

Optimality conditions:  $x_t = -\frac{\kappa}{\lambda^*} \pi_t$  (12)  $x_{t+i} - x_{t+i-1} = -\frac{\kappa}{\lambda^*} \pi_{t+i}$  for i = 1, 2, 3.... (13)

What are the differences and similarities with discretion?

 No longer restrict the choice of X<sub>t</sub> to depend on the contemporaneus value of the shock, but allow instead for rules that are a function of the entire history of shocks (history dependence).

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Cost-benefit analysis for state-contingent commitment:

- How much should the central bank reduce  $X_t$  in periods t, t+1, t+2..., in order to fight inflation?
  - In period t, if the output gap becomes more negative, then inflation goes down with a *K*-part of the reduction in the output gap.
  - In period t+1: if  $x_{t+1} \downarrow$  , then  $\pi_{t+1} \downarrow$  by  $\kappa$  , discounted by  $\beta$
  - If  $\pi_{t+1} \downarrow$  , then  $\pi_t \downarrow$  by  $\beta \kappa$
  - There is an additional gain because inflation falls in period t

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Cost-benefit analysis for state-contingent commitment:

How does this fit with our first order conditions?

• For period t: 
$$x_t = -\frac{\kappa}{\lambda^*} \pi_t \Leftrightarrow \lambda^* x_t = -\kappa \pi_t$$
 (14)

LHS: marginal cost of opening up the output gap.
RHS: marginal benefit – a reduction of inflation of κ per unit of output.

For period t+1: 
$$x_{t+1} = x_t - \frac{\kappa}{\lambda^*} \pi_{t+1} = -\frac{\kappa}{\lambda^*} \pi_t - \frac{\kappa}{\lambda^*} \pi_{t+1}$$
  
 $\beta \lambda^* x_{t+1} = -\beta \kappa \pi_{t+1} - \beta \kappa \pi_t$ 
(15)

LHS: again the discounted marginal cost of opening up the output gap.
 RHS: marginal benefit – a reduction in inflation in both periods of κβ and κ units, where the latter must be discounted.

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The optimality conditions can be written as:

$$x_{t+i} = -\frac{\kappa}{\lambda^*} \hat{p}_{t+i}$$
 for i = 0,1,2,3..... (16)

where  $\hat{p}_{t+i} = p_{t+i} - p_{t-1}$  and  $p_t = \log P_t$ 

- The central bank commits to bring the price level back to the pre-commitment level.
- Looks like price level targeting!

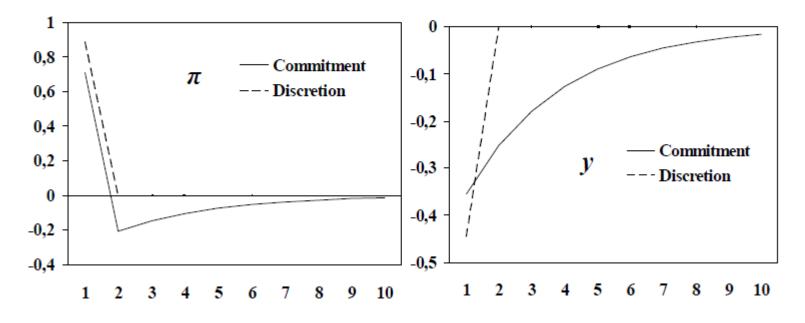
#### Key result 8 (CGG 1999)

The globally optimal policy rule under commitment has the central bank partially adjust demand in response to inflationary pressures.

The idea is to exploit the dependence of current inflation on expected future demand.



# Gain from commitment. Standard NK model





#### Properties of commitment:

- History dependence
  - Today's action depends on what happened in the past
- Inflation over-shooting
  - Stationary price-level
  - Bygones are not bygones
    - Under discretion, bygones are bygones
- Improved trade-off
  - More stable inflation
  - Not necessarily more stable output
    - Income vs. substitution effects.

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# **Commitment in practice**

## Gains from commitment exaggerated?

- New Keynesian DSGE-models very forward-looking
- But: Enough with a forward-looking exchange rate
  - $e_t = E_t e_{t+1} (r_t r_t^*) + \varepsilon_t$
  - Leitemo, Røisland and Torvik (2002)
- Gain from commitment if credible promises about future actions have positive effect today

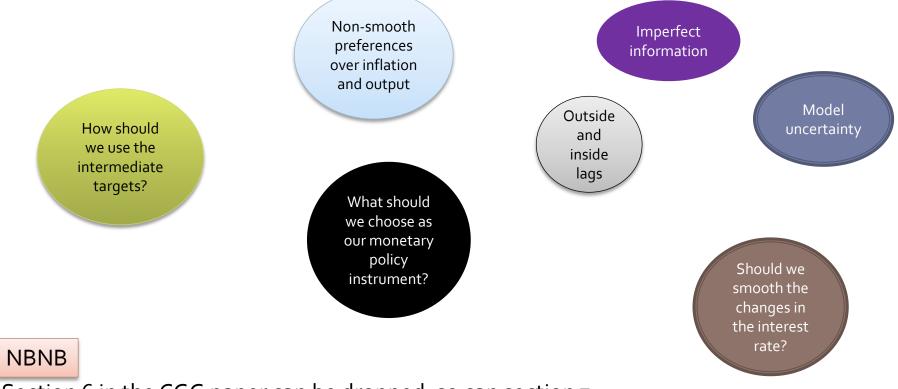
Forward Guidance

- Commitment particularly important when ZLB bites
  - Eggertson and Woodford (2003)
- Loose commitment
  - The CB re-optimizes with a certain probability
  - Schaumburg and Tambalotti (2007), Debortoli and Nunes (2010)
- Mixed empirical evidence on CBs ability/willingness to commit

# Some practical complications

Section 5 in the CGG paper – study yourself<sup>©</sup>

You are expected to know what the possible complications are



Section 6 in the CGG paper can be dropped, so can section 7.

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# Next week:

#### Lecture

- Markov Switching in a New Keynesian Framework
- Davig and Leeper (2007)

#### Seminar 4

- More interpretation of shocks in the NKM
- The seminar exercise is on the web

## Seminar 5

More on commitment

## Seminar 6

Exam exercises

# Monetary Policy

(Advanced Monetary Economics)

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