Seminar 4 – ECON 4325

Wednesday 6 April and Friday 8 April, 2016

The excel sheet exercises from seminar 3

We start the seminar with a go through of the excel sheet exercises from seminar 3.

Seminar presentation

- Cassandra (Wednesday) and Jørgen (Friday) present the Fed decision from 16 March.

Seminar problem 1

Suppose that the economy can be represented by the following New Keynesian model:

$$\pi_t = \beta E_t \pi_{t+1} + \kappa x_t + u_t \quad , \qquad 0 < \beta, \kappa < 1 \tag{1}$$

$$x_t = E_t x_{t+1} - \frac{1}{\sigma}(i_t - E_t \pi_{t+1} - \rho) + e_t \quad , 0 < \sigma < 1 \tag{2}$$

where equation (1) is a forward-looking NK Phillips curve that explains optimal price-setting behavior, and equation (2) is the dynamic IS-equation. x_t is the output gap, defined as in the CGG-paper: $x_t = y_t - z_t$, where y_t is the actual level of output and z_t is potential output, both measured as log-deviations from a deterministic trend. π_t is the inflation rate, measured as deviation from a trend (i.e. the inflation target) and i_t is the nominal interest rate. e_t is the demand shock in period t and u_t is the supply shock in period t. The shocks are AR(1)-processes:

$$u_t = \rho_u u_{t-1} + \hat{u}_t \quad , \quad 0 < \rho_u < 1 \tag{3}$$

$$e_t = \rho_e e_{t-1} + \hat{e}_t \quad , \quad 0 < \rho_e < 1 \tag{4}$$

where $\hat{e}_t$ and $\hat{u}_t$ are i.i.d. random variables with zero mean and variances σ_e^2 and σ_u^2 respectively.

Suppose that the central bank sets the interest rate according to the following *Taylor rule*:

$$i_t = \rho + \phi_\pi \pi_t + \phi_x x_t \quad , \qquad \phi_\pi , \phi_x > 0 \tag{5}$$

where we assume *the Taylor-principle* to hold and where ρ is the discount rate.

Problem 1A

Discuss advantages and disadvantages with simple interest rules like equation (5).

Problem 1B

Let's assume that equation 1, 2 and 5 together yield a unique equilibrium solution, which is the divine coincidence. Solve for equilibrium inflation and output gap in this economy and interpret the solutions.

Hint 1: The first step when solving for equilibrium inflation and output gap should always be to insert for the nominal interest rate.

Hint 2: *The 5-step method of undetermined coefficients* will probably be a good choice in order to solve this problem further since we have two expectation terms and it therefore is more difficult to solve the equations forward. A difference from what we have been doing in class, however, is that there are two shocks present in this model (a demand shock *and* a supply shock). The challenge lies at *step 3* – where you have to construct four equations, one for each guess-coefficient. Step 1 goes as follows:

$$\pi_t = \psi_e e_t + \psi_u u_t$$

$$x_t = \alpha_e e_t + \alpha_u u_t$$

$$E_t \pi_{t+1} = E_t[\psi_e e_{t+1} + \psi_u u_{t+1}] = \psi_e \rho_e e_t + \psi_u \rho_u u_t$$

$$E_t x_{t+1} = E_t[\alpha_e e_{t+1} + \alpha_u u_{t+1}] = \alpha_e \rho_e e_t + \alpha_u \rho_u u_t$$

You should be able to find the following four guess-coefficients:

$$\psi_e = b\kappa\sigma \qquad \text{(i)}$$

$$\psi_u = d[\sigma(1-\rho_u) + \phi_x] \qquad \text{(ii)}$$

$$\alpha_e = b\sigma(1-\beta\rho_e) \qquad \text{(iii)}$$

$$\alpha_u = -d(\phi_\pi - \rho_u) \qquad \text{(iv)}$$

where
$$b = \frac{1}{\kappa(\phi_\pi - \rho_e) + (1-\beta\rho_e)[\sigma(1-\rho_e) + \phi_x]} > 0$$

and
$$d = \frac{1}{\kappa(\phi_\pi - \rho_u) + (1-\beta\rho_u)[\sigma(1-\rho_u) + \phi_x]} > 0$$

and where ψ_e and ψ_u determine the effects on inflation from a demand shock and a supply shock respectively and where α_e and α_u determine the effects on the output gap from a demand shock and a supply shock respectively, when monetary policy follows equation (5). How can we be sure that both *b* and *d* are positive coefficients?

Hint: To get a better grip of the interpretation: Update your excel sheet.

Problem 1C

Now, suppose that the central bank's loss function is:

$$L_t = \frac{1}{2} E_t \sum_{k=0}^{\infty} \beta^k [\pi_{t+k}^2 + \lambda x_{t+k}^2] \quad (6)$$

and that the central bank uses discretion to conduct optimal monetary policy instead of the monetary policy rule. We still assume that equation (1) and (2) hold, but that e_t and u_t now are white noise processes, so that $E_t x_{t+1} = E_t \pi_{t+1} = 0$. Derive optimal monetary policy under discretion.

Problem 1D

From lecture 8, we know that the equilibrium solutions for inflation and the output gap under discretion (without commitment) when the loss function is like equation (6) and the shocks are AR(1) processes are equal to:

$$\Rightarrow \pi_t = \left(\frac{\lambda^*}{\kappa^2 + \lambda^*(1-\beta\rho)} \right) u_t \quad (30)$$

- Inserting the reduced form expression for inflation in equation (27) yields the optimal reduced form expression for output:

$$x_t = -\frac{\kappa}{\lambda^*}\pi_t = -\frac{\kappa}{\lambda^*}\left(\frac{\lambda^*}{\kappa^2 + \lambda^*(1-\beta\rho)} \right) u_t = -\left(\frac{\kappa}{\kappa^2 + \lambda^*(1-\beta\rho)} \right) u_t \quad (31)$$

Compare these equilibrium solutions with the solutions found in problem 1B. Can the central bank achieve optimal policy by using a rule like equation (5)? Explain. Your answer to problem 1D, should be in words only, no derivations.