EXAM
ECON 4330

Real Exchange Rates and Labor Supply

Consider the model with tradable and non-tradable goods as in class but with two modifications. There is no capital and the production function in the non-tradable sector has decreasing returns to scale. To remind you: The country produces Y_T tradables and Y_N nontradables. Tradables can be imported and exported without any costs, while nontradables are impossible to export/import. Labor is mobile across sectors, but not across countries The tradable good is the numeraire. p is the relative price of nontradables. w is the wage rate. Output is assumed to be given by two production functions:

$$Y_T = A_T L_T \tag{1}$$

$$Y_N = A_N L_N^{\xi}, \tag{2}$$

where L_T and L_N are labor inputs into the tradable sector and non-tradable sector respectively and $0 < \xi < 1$. We assume a representative agent who chooses consumption C_T and C_N to maximize utility, for $0 < \epsilon < 1$,

$$C_T^{\epsilon} C_N^{1-\epsilon}$$

subject to the budget constraint $C_T + pC_N = wL + \Pi$, where L is inelastically supplied labor and Π are profits. In equilibrium $L = L_T + L_N$.

Now assume that a shock hits the home country (and only the home country) so that the production of non-tradables chnages to $Y_N = A_N L_N^{\hat{\xi}}$. Assume $C_T = Y_T$.

1. Derive the labor allocation before and after the shock.

2. Derive the real exchange rate before and after the shock.

Important: You have to explain your results. Just writing down equations without clear, short and sensible explanations is **not** sufficient.

Hint: Define the real exchange rate, use the household's optimization problem, derive the demand for tradable and nontradable goods, the price of non-tradables and the allocation of labor across sectors.

Answer Guide:

1. The real axchange rate is defines as $Q = P^*/P$.

2. The household's optimization problem, the first order condition and the demand for tradable and nontradable goods are

   Max
   $$C_T^{\epsilon} C_N^{1-\epsilon}$$
   subject to the budget constraint $C_T + pC_N = wL + \Pi$.

   $$\epsilon C_T^{\epsilon-1} C_N^{1-\epsilon} = \lambda \tag{3}$$
   $$(1-\epsilon) C_T^{\epsilon} C_N^{-\epsilon} = p\lambda \tag{4}$$

   for Lagrange multiplier λ.

3. The firm optimization for the traded and non-traded sector and the equilibrium wage and the profits in the tradable and non-tradable sector are:

   Firm Problem: Profits = Max Y - wL

4. The price p (as a function of L_N) is
   $pMPL^{NT} = MPL^T$

5. The domestic demand for the non-tradable good is

   $$p = \frac{(1-\epsilon)C_T}{\epsilon C_N}. \tag{5}$$

6. All non-tradable goods have to be produced at home, $C_N = Y_N = A_N L_N^{\xi}$. Consider now the long run equilibrium where imports and exports of tradable good are zero as well so that $C_T = Y_T = A_T L_T$. The labor allocation L_T and L_N is

   $$\frac{L_T}{L_N} = \frac{\epsilon}{\xi(1-\epsilon)} \tag{6}$$

and

$$L_N = L\frac{\xi(1-\epsilon)}{\epsilon+\xi(1-\epsilon)}. \tag{7}$$

7. The price p is

$$p = \frac{(1-\epsilon)A_T L_T}{\epsilon A_N \theta L_N^{\xi}}. \tag{8}$$

8. L_T/L_N and L_N are

$$\frac{L_T}{L_N} = \frac{\epsilon}{\xi(1-\epsilon)} \tag{9}$$

and

$$L_N = L\frac{\xi(1-\epsilon)}{\epsilon+\xi(1-\epsilon)}. \tag{10}$$

ECON4330 Exam Part B

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Guidelines: Explain in your own words. Do not simply copy paste formulas or text from the lecture notes/books. You can include formulas, but you have to explain them in words with the goal to convince the reader that you have 100% understood what the formula means. You do not have to write down the definition of the variables in an equation as long as you are using the notation from the corresponding lecture notes/book chapters.

Mean-variance model of portfolio choice

1. Consider the mean-variance model of portfolio choice discussed in lecture 11. The investor's optimal portfolio share of foreign currency can be expressed as the sum of two portfolios, a return-driven portfolio and a minimum-variance portfolio.

   (a) Show analytically that the minimum-variance portfolio minimizes the variances of the real return.
   Solution: Optimization problem:

   $$\max_f Var(\pi)$$

   FOC:

   $$2f\sigma_{ee} - 2\sigma_{ep} = 0 \quad \Rightarrow \quad f = \frac{\sigma_{ep}}{\sigma_{ee}}$$

   Extra point if someone checks the SOC. But since we never did this in the lecture, it's not expected here.

   (b) Why can investment in foreign currency help to hedge domestic inflation risk?
   Solution: Since domestic inflation under plausible circumstances leads to a nominal depreciation, foreign nominal returns increase when the domestic inflation rate increases. The same is not true for domestic nominal returns.

(c) What is the relationship between exchange-rate risk and inflation risk when the real exchange rate is constant?
Solution: When the real exchange rate is constant, $e = p - p_*$. Hence,

$$\sigma_{ee} = \sigma_{pp} + \sigma_{p_*p_*} - 2\sigma_{pp_*}$$

(d) Suppose that the real exchange rate is constant and inflation is uncorrelated across countries. Use the optimal investment in the minimum variance portfolio to show that hedging domestic inflation risk through investment in foreign currency is more attractive when domestic inflation risk is high and less attractive when foreign inflation risk is high.
Solution: When the real exchange rate is constant and inflation is uncorrelated

$$f_M = \frac{\sigma_{ep}}{\sigma_{ee}} = \frac{\sigma_{ep}}{\sigma_{pp} + \sigma_{p_*p_*}}$$

$$\frac{\partial f_M}{\partial \sigma_{pp}} = \frac{\sigma_{p_*p_*}}{(\sigma_{pp} + \sigma_{p_*p_*})^2} > 0$$

$$\frac{\partial f_M}{\partial \sigma_{pp}} = \frac{-\sigma_{pp}}{(\sigma_{pp} + \sigma_{p_*p_*})^2} < 0$$

The mimimum variance portfolio increases in σ_{pp} and decreases in $\sigma_{p_*p_*}$.

Imperfections in international capital markets

2. Explain why imperfect observability of countries' output can lead to a breakdown of the international market for risky assets.
Solution: If countries could sign contracts specifying a payment in bad times, i.e., times of low output, they will always optimally claim to be in the bad state in order to trigger the payment. No matter the actual state of the economy. Everybody can anticipate this behaviour and, therefore, no one is willing to sell such contracts. [extra point for pointing out that sellers would demand a price at least as high as $1/(1+r)$ because they will pay back with certainty. Just as if they had sold a risk-free asset.]

3. Which country suffers from the breakdown of the international market for risky assets and why?
Solution: All countries. If the trade is possible, both the sellers and the buyers in such contracts are better or equally well off. Otherwise, they would not engage in the trade in the first place. The buying country would like to commit to telling the truth about its state, it just has no credible means of doing so in this setup.

4. Why may borrowing countries be better off if they can be sanctioned for defaulting on their debt?
Solution: The borrowing country suffers from a breakdown of international asset markets due

to possibility of deliberate sovereign default, just like the lending countries. Both sides forgo the gains from consumption smoothing. The possibility of sanctions makes lenders willing to give an amount up to the level that can recover through sanctions. Any lending is better than no lending in terms of consumption smoothing.

5. In the model with sovereign default, sanctions, and investment, why do lenders always prefer that the borrowing country invests more of the borrowed funds?
   Solution: Investment increases the borrowing country's future output, in contrast to the alternative use of the borrowed funds, which is consumption. Higher output increases the pain from being sanctioned for defaulting and, hence, repayment becomes more likely.

6. What is the debt overhang problem?
   Solution: The debt overhang problem describes circumstances under which a high initial level of debt discourages borrowing countries from investing the efficient amount. It can explain why poor and indebted countries may remain poor, rather than grow. It can occur if future output depends both on investment and on an uncertainty productivity level. When choosing the optimal investment level, borrowing countries internalize that they will default with a certain probability, in which case the marginal gain from investment benefits only the creditors. The higher is the initial debt level, the more likely becomes default and the smaller is the expected marginal benefit from investment for the borrowing country. Hence, higher debt leads to less investment.

7. Explain under what circumstances it can be optimal for lenders to forgive a part of the debt
   Solution: When high debt levels discourage borrowers from investment, forgiving debt can increase investment and hence the probability that the remaining debt is repaid. The current value of the debt is the outstanding amount times the probability of default. If the probability of repayment increases enough, it can overcompensate the loss from a smaller outstanding amount.