EXAM
ECON 4330

Covid-19 Real Exchange Rates

Consider the model with tradable and non-tradable goods as in class but with two modifications. There is no capital and the production function in the non-tradable sector has decreasing returns to scale. To remind you: The country produces Y_T tradables and Y_N nontradables. Tradables can be imported and exported without any costs, while nontradables are impossible to export/import. Labor is mobile across sectors, but not across countries The tradable good is the numeraire. p is the relative price of nontradables. w is the wage rate. Output is assumed to be given by two production functions:

$$Y_T = A_T L_T \tag{1}$$

$$Y_N = A_N L_N^{\xi}, \tag{2}$$

where L_T and L_N are labor inputs into the tradable sector and non-tradable sector respectively and $0 < \xi < 1$. We assume a representative agent who chooses consumption C_T and C_N to maximize utility, for $0 < \epsilon < 1$,

$$C_T^{\epsilon} C_N^{1-\epsilon}$$

subject to the budget constraint $C_T + pC_N = wL + \Pi$, where L is inelastically supplied labor and Π are profits. In equilibrium $L = L_T + L_N$.

Now assume that (I know it is a crazy assumption) that a pandemic (aka as Corona virus) hits the home country (and only the home country) so that the productivity of non-tradables is reduced to $A_N\theta$ with $\theta < 1$. Assume $C_T = Y_T$ if necessary.

1. Derive the real exchange rate before and during the pandemic.

2. Now assume that the foreign country is also hit by the virus and productivity of non-tradables is reduced from A_N^* to $A_N^*\theta^*$. Derive the real exchange rate before and during the pandemic. Denote foreign variables by $*$.

Important: You have to explain your results. Just writing down equations without clear, short and sensible explanations is **not** sufficient.

Hint: Define the real exchange rate, use the household's optimization problem, derive the demand for tradable and nontradable goods, the price of non-tradables and the allocation of labor across sectors.

Answer Guide:

1. The real axchange rate is defines as $Q = P^*/P$.

2. The household's optimization problem, the first order condition and the demand for tradable and nontradable goods are

   Max
   $$C_T^{\epsilon} C_N^{1-\epsilon}$$
   subject to the budget constraint $C_T + pC_N = wL + \Pi$.
   $$\epsilon C_T^{\epsilon-1} C_N^{1-\epsilon} = \lambda \tag{3}$$
   $$(1-\epsilon) C_T^{\epsilon} C_N^{-\epsilon} = p\lambda \tag{4}$$
   for Lagrange multiplier λ.

3. The firm optimization for the traded and non-traded sector and the equilibrium wage and the profits in the tradable and non-tradable sector are:

   Firm Problem: Profits = Max Y - wL

4. The price p (as a function of L_N) is

   $pMPL^{NT} = MPL^T$

5. The domestic demand for the non-tradable good is
   $$p = \frac{(1-\epsilon)C_T}{\epsilon C_N}. \tag{5}$$

6. All non-tradable goods have to be produced at home, $C_N = Y_N = A_N L_N^{\xi}$. Consider now the long run equilibrium where imports and exports of tradable good are zero as well so that $C_T = Y_T = A_T L_T$. The labor allocation L_T and L_N is
   $$\frac{L_T}{L_N} = \frac{\epsilon}{\xi(1-\epsilon)} \tag{6}$$

and

$$L_N = L\frac{\xi(1-\epsilon)}{\epsilon + \xi(1-\epsilon)}. \tag{7}$$

7. The price p is

$$p = \frac{(1-\epsilon)A_T L_T}{\epsilon A_N \theta L_N^{\xi}}. \tag{8}$$

8. L_T/L_N and L_N are

$$\frac{L_T}{L_N} = \frac{\epsilon}{\xi(1-\epsilon)} \tag{9}$$

and

$$L_N = L\frac{\xi(1-\epsilon)}{\epsilon + \xi(1-\epsilon)}. \tag{10}$$

9. The pandemic leaves L_N unchanged and increases the price of non-tradables and the real exchange rate Q falls?

10. If A_N^* is reduced to $A_N^*\theta^*$, p^*/p changes by θ/θ^*. Q falls (increases) if θ/θ^* falls (increases).

ECON4330 Exam Part B

May 11, 2020

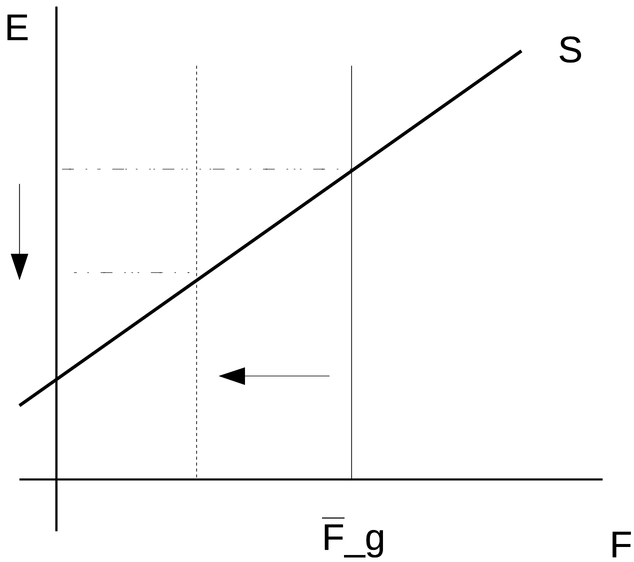
Guidelines: Explain in your own words. Do not simply copy paste formulas or text from the lecture notes/books. You can include formulas, but you have to explain them in words with the goal to convince the reader that you have 100% understood what the formula means. You do not have to write down the definition of the variables in an equation as long as you are using the notation from the corresponding lecture notes/book chapters.

Equilibrium in the foreign exchange (forex) market

1. Explain what "uncovered interest rate parity" means and how it is related to capital mobility.
   Solution: Uip describes equilibrium in currency markets under perfect capital mobility. When capital mobility is perfect, currency markets can only be in equilibrium if investors get the same expected nominal return in terms of one of the currencies for investing in either currency. The expected nominal return of investing in foreign currency measured in the domestic currency is the foreign nominal interested rate plus expected depreciation.

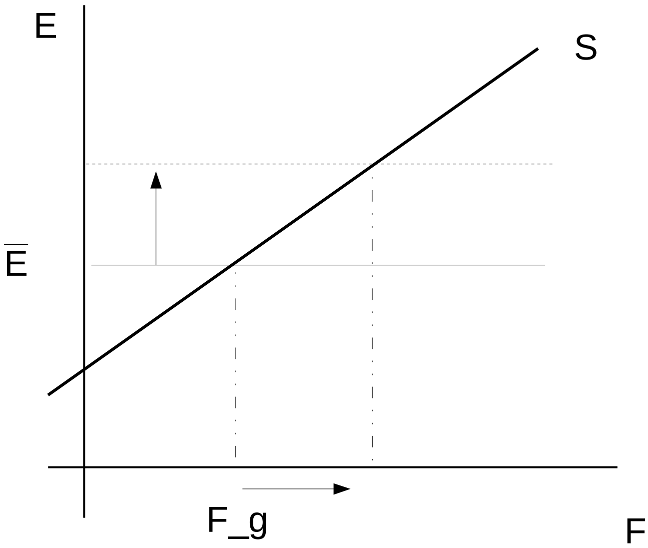
2. Consider the equilibrium in the forex market $F_g = S(E) = -F_p(E) - F_*(E)$ and assume that the supply curve is well behaved. Show graphically how we find the

   (a) new E if the CB adjusts F_g under the floating exchange rate regime
   Solution:



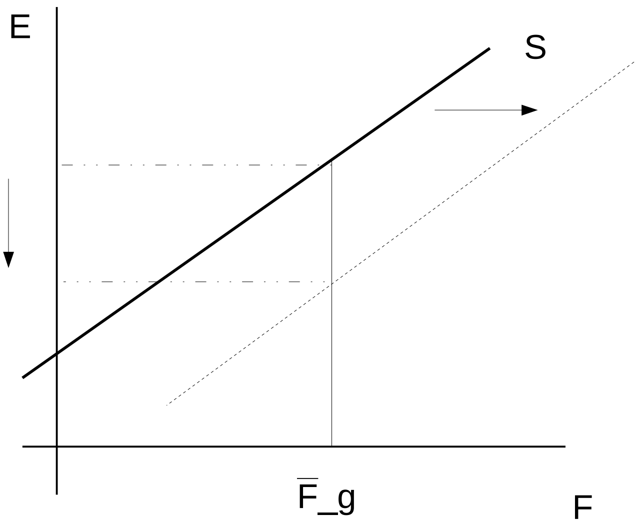
The CB reduces its reserves. This leads to a decrease in the exchange rate since the private agents are only willing to buy more foreign currency from the central bank if the price of foreign currency falls. [Alternative answer/graph: The CB increases its reserves and the opposite happens.]

(b) new F_g if the CB devalues the domestic currency under the fixed exchange rate regime.
Solution:



A devaluation means the CB increases the targeted level of E. This means private agents sell foreign currency to the CB as they now get more in return than before and, therefore, the CB's reserves go up.

(c) new E if there is a shock to the supply curve which leads the public to suddenly demand less \$ under the floating exchange rate regime.
Solution:



Private agents demanding less \$ means supply curve shifts to the right. Excess supply of \$ brings down the exchange rate (the price of the \$.)

Explain in a few words what your graphs a)-c) are showing. Make sure to label axes and curves.

Uncertainty, global risk sharing, and non-traded goods

3. Consider a world consisting of two small open economies, indexed by $h = H, F$, producing two goods, $g = T, N$. Good T can be traded, good N is non-tradeable. There are two periods and in period two there are two states of nature ($s = 1, 2$) occurring with probability $\pi(s)$. In country h, output (consumption) of good $g = T, N$ in period one is $Y^h_{g,1}$ ($C^h_{g,1}$) and in period two state s it is $Y^h_{g,2}(s)$ ($C^h_{g,2}(s)$). The traded good is the numeráire. The price of the non-traded good is $p^h_{N,1}$ ($p^h_{N,2}(s)$) in period one (period two state s).

   There are Arrow Debreu securities (ADSs) that pay out one unit of the *tradable* good in either state s. The price of such an ADS for state s is $p(s)/(1 + r)$.

   (a) Write down country H's intertemporal budget constraint and explain its components. [hint: To that end it is useful to express output and consumption values in the different periods and states in units of the traded good in period one.]
   Solution:

$$C^H_{T,1} + p^H_{N,1} C^H_{N,1} + \sum_s \frac{p(s)}{1+r} C^H_{T,2}(s) + \sum_s \frac{p(s)}{1+r} p^H_{N,2}(s) C^H_{N,2}(s) \tag{1}$$
$$= Y^H_{T,1} + p^H_{N,1} Y^H_{N,1} + \sum_s \frac{p(s)}{1+r} Y^H_{T,2}(s) + \sum_s \frac{p(s)}{1+r} p^H_{N,2}(s) Y^H_{N,2}(s)$$

   On the left hand side is the value of lifetime consumption, summed over the two goods, two periods, and two states in period two in units of the traded good in period one (numeráire.). On the right hand side is the value of total production, calculated the same way.

(b) Expected lifetime utility of a representative agent from country h is

$$U = u\left(C_{T,1}^h, C_{N,1}^h\right) + \beta \sum_{s=1}^{2} \pi(s) u\left(C_{T,2}^h(s), C_{N,2}^h(s)\right) \quad \text{with} \quad u'() > 0, u''() < 0.$$

Write down the maximization problem of the representative agent from country H.
Solution:

$$\max_{C_{T,1}^H, C_{N,1}^H, C_{T,2}^H(1), C_{N,2}^H(1), C_{T,2}^H(2), C_{N,2}^H(2)} U \quad \text{s.t.} \quad (1) \tag{2}$$

(c) Derive and explain the first-order conditions for country H that guide

i. optimal relative consumption of the two goods in period one
Solution: Use Lagrange method to find FOC $C_{N,1}^H$:

$$\frac{\partial \mathcal{L}}{\partial C_{N,1}^H} = \frac{\partial u\left(C_{T,1}^H, C_{N,1}^H\right)}{\partial C_{N,1}^H} - \lambda p_{N,1}^H = 0 \tag{3}$$

and FOC $C_{T,1}^H$:

$$\frac{\partial \mathcal{L}}{\partial C_{T,1}^H} = \frac{\partial u\left(C_{T,1}^H, C_{N,1}^H\right)}{\partial C_{T,1}^H} - \lambda = 0 \tag{4}$$

dividing the two FOCs gives

$$\frac{\frac{\partial u\left(C_{T,1}^H, C_{N,1}^H\right)}{\partial C_{N,1}^H}}{\frac{\partial u\left(C_{T,1}^H, C_{N,1}^H\right)}{\partial C_{T,1}^H}} = p_{N,1}^H$$

Interpretation: The marginal rate of substitution of consumption of the traded and non-traded good in period 1 equals the relative price.

ii. optimal relative consumption of the traded good across states 1 and 2 in period 2.
Solution: FOC $C_{T,2}^H(s)$ for $s = 1, 2$:

$$\frac{\partial \mathcal{L}}{\partial C_{T,2}^H(s)} = \beta \pi(s) \frac{\partial u\left(C_{T,2}^H(s), C_{N,2}^H(s)\right)}{\partial C_{T,2}^H(s)} - \lambda \frac{p(s)}{1+r} = 0 \tag{5}$$

Dividing the conditions for the two states gives

$$\frac{\frac{\partial u\left(C_{T,2}^H, C_{N,2}^H(1)\right)}{\partial C_{N,2}^H(1)}}{\frac{\partial u\left(C_{T,2}^H, C_{N,2}^H(2)\right)}{\partial C_{T,2}^H(2)}} = \frac{p(1)}{p(2)}$$

Interpretation: The marginal rate of substitution of consumption of the traded good in states one and two equals the relative price of one safe unit of consumption in the two states, i.e. the relative ADSs prices.

(d) Suppose the countries are in complete autarky. What will be the price of an ADS for state 1 in country H ?
Solution: Divide (5) by (4) to eliminate λ and solve for

$$\frac{p^H(1)}{1+r} = \beta\pi(1)\frac{\frac{\partial u\left(C^H_{T,2}(1),C^H_{N,2}(1)\right)}{\partial C^H_{T,2}(1)}}{\frac{\partial u\left(C^H_{T,1},C^H_{N,2}\right)}{\partial C^H_{T,1}}}.$$

Then, plug in $C^H_{N,1} = Y^H_{N,1}$, $C^H_{T,1} = Y^H_{T,1}$, $C^H_{N,2}(1) = Y^H_{N,2}(1)$, and $C^H_{T,2}(1) = Y^H_{T,2}(1)$ to arrive at

$$\frac{p^H(1)}{1+r} = \beta\pi(1)\frac{\frac{\partial u\left(Y^H_{T,2}(1),Y^H_{N,2}(1)\right)}{\partial C^H_{T,2}(1)}}{\frac{\partial u\left(Y^H_{T,1},Y^H_{N,1}\right)}{\partial C^H_{T,1}}}$$

(e) Under what conditions will the ADS for state 1 be more expensive in country H than in country F under autarky? [Hint: multiple combinations of output levels across states and goods imply this result. Find and describe one of them.]
Solution: One possibility is a situation where non-tradable output is the same in both countries in all periods and times, tradable output is the same in period one, but country H produces less tradables in period 2 state 1. Then consumption in this state is relatively scarce in H and the corresponding ADS is expensive.