ECON4330

Exam questions

Spring 2023

1 Export Boom [45 Points]

Consider an economy (think of Norway) with a current account surplus. Now assume that the price of the exported good increases. The price of the imported good is unaffected. What happens to the NFA, the current account and consumption in this economy?

Explain your results. Keep your answer concise. There is no benefit from long but imprecise answers.

Guide

Follow the logic in the lecture.

Note first that Norway is a small open economy. Note furthermore that the NFA is positive since it is the accumulated positive current accounts of the past. An increase in export prices is an increase in the value of exports and thus an increase of the wealth of the nation which is the discounted sum of future income. This leads to an increase in consumption. In the current period and due to consumption smoothing also in all future periods.

2 International Finance [15 Points]

For this part, please write short and precise answers. Introducing key equations can be helpful but the answers should primarily be described in words.

- Explain what a *forward contract* is (in the context of currency markets).
- Explain the two concepts *covered interest rate parity* and *uncovered interest rate parity.* Which (or both) of the two are implied by *no arbitrage*?
- Explain what is meant by the empirical regularity known as the *forward premium puzzle.*

- A forward conctract is an agreement at a time t to trade a given quantity of one currency for a given quantity of another at a specified time t' in the future. No transaction occurs at time t, but the contract is determined at time t.
- Covered interest rate parity holds when $(1+i_t) = \frac{F_t}{\mathcal{E}_t}(1+i_t^*)$, where i_t is the domestic nominal interest rate, $\mathcal{E}_t$ is the exchange rate (domestic currency/foreign currency) and F_t is the forward rate (domestic currency/foreign currency). CIP equalizes the return from saving at a domestic bank account and saving at a foreign bank account with the exchange risk hedged away using the forward contract. Uncovered interest rate parity holds when $(1+i_t) = \frac{\mathbb{E}_t \mathcal{E}_{t+1}}{\mathcal{E}_t}(1+i_t^*)$, i.e., when the expected return from exchanging to the foreign currency, saving at a foreign interest rate, and exchanging back to domestic currency is equal to the return from saving at a domestic interst rate. Covered interest rate parity is implied by no arbitrage, since it equalizes the returns of two safe investment strategies (i.e., two strategies with the same payoffs in all states of the world), while uncovered interest rate parity is not since it only equalizes expected returns.

- The forward premium puzzle is the following: when $F_t/\mathcal{E}_t > 1$ (a currency is trading at a premium in the forward market), the currency tends to depreciate (and not appreciate, as predicted by UIP)-

3 International Risk Sharing [40 Points]

Consider an endowment economy with two periods, two countries, uncertainty, and complete markets. The output of Home is Y_1 in period 1 and $Y_2(s)$ in period 2 when the period-2 state is $s \in \{1, \ldots, S\}$. The output of Foreign is Y_1^* in period 1 and $Y_2^*(s)$ in period 2. Let $p(s)/(1+r)$ denote the price of an Arrow-Debreu security $B_2(s)$ paying out 1 unit in period 2 if the period-2 state is s. Let C_1 denote consumption of Home in period 1 and let $C_2(s)$ denote consumption of Home in period 2 if the period-2 state is s (correspondingly, let C_1^* and $C_2^*(s)$ denote the consumption of Foreign).

- Write down the first-period and second-period budget constraints for Home, and then derive the consolidated budget constraint for Home. Assume that neither Home nor Foreign have any outstanding assets/debt. (The budget constraint for Foreign is symmetrical)

In period 1, Home can purchase Arrow-Debreu securities $B_2(s)$ at price $p(s)/(1+r)$ (for $s = 1, \ldots, S$) and consume. Therefore, the period 1 budget constraint is

$$C_1 + \sum_{s=1}^{S} \frac{p(s)}{1+r} B_2(s) = Y_1.$$

In the second period, Home redeems their Arrow-Debreu security and has the budget constraint

$$C_2(s) = Y_2(s) + B_2(s).$$

Substituting in the second-period budget constraint into the first, we arrive at

$$C_1 + \sum_{s=1}^{S} \frac{p(s)}{1+r} C_2(s) = Y_1 + \sum_{s=1}^{S} \frac{p(s)}{1+r} Y_2(s).$$

Let the preferences for Home be given by $\log C_1 + \beta \sum_{s=1}^{S} \pi(s) \log C_2(s)$ (where $\pi(s)$ is the probability that the period-2 state is s).

- State Home's maximization problem given preferences and the budget constraint.
- Compute Home's first-order conditions. Home's first-order conditions, together with Home's budget constraint, should constitute a system of $s+2$ in $s+2$ unknowns (with the Lagrange multiplier) or $s+1$ equations in $s+1$ unknowns (if you eliminated the Lagrange multiplier).

$$\max_{C_1, \{C_2(s)\}_{s=1,\ldots,S}} \log C_1 + \beta \sum_{s=1}^{S} \pi(s) \log C_2(s) \qquad s.t. \qquad C_1 + \sum_{s=1}^{S} \frac{p(s)}{1+r} C_2(s) = Y_1 + \sum_{s=1}^{S} \frac{p(s)}{1+r} Y_2(s).$$

The first order conditions are

$$\frac{1}{C_1} = \lambda,$$

$$\beta \pi(s) \frac{1}{C_2(s)} = \frac{p(s)}{1+r} \lambda, \qquad s = 1, \ldots, S.$$

The first-order conditions, together with the budget constraint, are $S+2$ equations in C_1, $C_2(s)$ for $s = 1, \dots, S$, and λ ($S+2$ unknowns).

Eliminating the Lagrange multiplier, we obtaind the S first-order conditions,

$$\beta\pi(s)\frac{1}{C_2(s)} = \frac{p(s)}{1+r}\frac{1}{C_1}, \qquad s = 1, \dots, S.$$

Assume that Foreign have identical preferences to Home (and that, thus, the corresponding first-order conditions and budget constraint characterize the solution to Foreign's problem). In equilibrium, we have the aggregate resource constraints $C_1 + C_1^* = Y_1 + Y_1^*$ and $C_2(s) + C_2^*(s) = Y_2(s) + Y_2^*(s)$.

- Summarize equilibrium by writing down the first-order conditions and consolidated budget constraints for Home and Foreign, as well as the aggregate resource constraints. Count the number of equations and unknowns (and recall that, by Walras's law, one of the resource constraints is redundant).
- Show that, in equilibrium, Home consumption is proportional to world output. That is, show that $C_1 = \gamma(Y_1 + Y_1^*)$ and $C_2(s) = \gamma(Y_2 + Y_2^*)$ for some constant γ. Hint: this can be shown by "guess and verify" by the following steps:
  - Guess that $C_1 = \gamma(Y_1 + Y_1^*)$ and $C_2(s) = \gamma(Y_2 + Y_2^*)$. Solve for C_2^* and $C_2^*(s)$ using the resource constraint.
  - Plug the guess into the first-order conditions for Home and solve for Arrow-Debreu prices $p(s)/(1+r)$.
  - Check that those prices also satisfy Foreign's first-order conditions.
  - Plug the prices and the consumption guess into Home's budget constraint and solve for γ.
  - (By Walras's law, it is not necessary to check that Foreign's budget constraint also is satisfied.)
  - Conclude that we have found prices $p(s)/(1+r)$ and consumption (C_1, $C_2(s)$, C_1^*, and $C_2^*(s)$) consistent with both Home's and Foreign's consumption problems and with the resource constraints. We have thus found an equilibrium of the model.

- Given that $C_1 + C_1^* = Y_1 + Y_1^*$, $C_1 = \gamma(Y_1 + Y_1^*)$ implies $C_1^* = (1-\gamma)(Y_1 + Y_1^*)$. Correspondingly, $C_2^*(s) = (1-\gamma)(Y_2(s) + Y_2^*(s))$.
- We plug the guess into Home's first-order conditions. We get

$$\beta\pi(s)\frac{1}{\gamma(Y_2(s) + Y_2^*(s))} = \frac{p(s)}{1+r}\frac{1}{\gamma(Y_1 + Y_1^*)}, \qquad s = 1, \dots, S.$$

or, solving for $p(s)/(1+r)$,

$$\frac{p(s)}{1+r} = \beta\pi(s)\frac{(Y_1 + Y_1^*)}{Y_2(s) + Y_2^*(s)}, \qquad s = 1, \dots, S.$$

We plug the guess intor Foreign's first-order conditions and obtain

$$\beta\pi(s)\frac{1}{(1-\gamma)(Y_2(s) + Y_2^*(s))} = \frac{p(s)}{1+r}\frac{1}{(1-\gamma)(Y_1 + Y_1^*)}, \qquad s = 1, \dots, S.$$

which is satisfied if $\frac{p(s)}{1+r} = \beta\pi(s)\frac{(Y_1+Y_1^*)}{Y_2(s)+Y_2^*(s)}$.

- Home's consolidated budget constraint is

$$C_1 + \sum_{s=1}^{S} \frac{p(s)}{1+r} C_2(s) = Y_1 + \sum_{s=1}^{S} \frac{p(s)}{1+r} Y_2(s).$$

  Plugging in the expressions for C_1 and $C_2(s)$, as well as for $p(s)/(1+r)$, we get

$$\gamma \left((Y_1 + Y_1^*) + \beta \sum_{s=1}^{S} \pi(s) \frac{Y_1 + Y_1^*}{Y_2(s) + Y_2(s)^*} (Y_2(s) + Y_2^*(s)) \right) = Y_1 + \beta \sum_{s=1}^{S} \pi(s) \frac{Y_1 + Y_1^*}{Y_2(s) + Y_2^*(s)} Y_2(s)$$

  or

$$\gamma = \frac{1}{1+\beta} \left(\frac{Y_1}{Y_1 + Y_1^*} + \beta \sum_{s=1}^{S} \pi(s) \frac{Y_2(s)}{Y_2(s) + Y_2^*(s)} \right).$$

- (Not necessary)
- We have thus found prices

$$\frac{p(s)}{1+r} = \beta \pi(s) \frac{(Y_1 + Y_1^*)}{Y_2(s) + Y_2^*(s)}, \qquad s = 1, \ldots, S.$$

  and quantities $C_1 = \gamma(Y_1 + Y_1^*)$, $C_2(s) = \gamma(Y_2 + Y_2^*)$, $C_1^* = (1-\gamma)(Y_1 + Y_1^*)$, $C_2^*(s) = (1-\gamma)(Y_2 + Y_2^*)$ such that Home's and Foreign's consumption problems are solved and the resource constraints are satisfied.