EXAM ECON4330

Spring 2024

1 Real exchange rates [50 Points]

To obtain the real exchange rate we need more than our one homogeneous good. There are two sectors producing traded and non-traded goods.

1. Output in the two sectors are (subscripts T and N is for traded and non-traded):

$$Y_T = A_T K_T^{\alpha} L_T^{1-\alpha} \tag{1}$$

$$Y_N = A_N K_N^{\alpha} L_N^{1-\alpha} \tag{2}$$

where $0 < \alpha, \alpha < 1$. Capital can be rented in an international market at price r. Labor is mobile between sectors at home, the wage rate is w. The price of the non-traded good is p. The price of the traded good is p_T..

(a) Set up the maximization problems and find the first order conditions of the firms optimization problem in the two sectors. Use capital intensities,

$$k_T = \frac{K_T}{L_T} \quad \text{and} \quad k_N = \frac{K_N}{L_N},$$

in the conditions.

Answer:

The firm problems are as follows (note that the traded good numeraire, p_T, and price of non-traded good is p:

$$\max_{K_T, L_T} p_T A_T K_T^{\alpha} L_T^{1-\alpha} - wL_T - rK_T \tag{3}$$

$$\max_{K_N, L_N} pA_N K_N^{\alpha} L_N^{1-\alpha} - wL_N - rK_N \tag{4}$$

The FOCs wrt K_T, L_T, K_N, L_N is

$$K_T: \quad \alpha p_T A_T k_T^{\alpha-1} = r \tag{5}$$

$$L_T: \quad (1-\alpha) p_T A_T k_T^{\alpha} = w \tag{6}$$

$$K_N: \quad p\alpha A_N k_N^{\alpha-1} = r \tag{7}$$

$$L_N: \quad p(1-\alpha) A_N k_N^{\alpha} = w \tag{8}$$

(b) Solve for the wage rate in the traded sector. How does it depend on the world interest rate?

Answer:

To solve for the wage rate in the traded sector, solve eq. (5) for k_T.

$$\alpha p_T A_T k_T^{\alpha-1} = r \tag{see 5}$$

$$k_T = \left(\frac{\alpha p_T A_T}{r} \right)^{\frac{1}{1-\alpha}}$$

Then insert k_T into eq. (6).

$$w = (1-\alpha) p_T A_T k_T^{\alpha} \tag{see 6}$$

$$w = (1-\alpha) p_T A_T \left(\frac{\alpha A_T}{r}\right)^{\frac{\alpha}{1-\alpha}}$$

$$w = (1-\alpha) p_T A_T (p_T A_T)^{\frac{\alpha}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}$$

$$w = (1-\alpha)(p_T A_T)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}$$

(c) Solve for the price of the non-traded good, p.

Answer:

To find price p, solve both eq. (7) and (8) for p and set the equations equal to each other,

$$p\alpha A_N k_N^{\alpha-1} = r \qquad \text{(see 7)}$$

$$\Leftrightarrow$$

$$p = k_N^{1-\alpha} \frac{r}{A_N \alpha}$$

and

$$p(1-\alpha) A_N k_N^{\alpha} = w \qquad \text{(see 8)}$$

$$\Leftrightarrow$$

$$p = \frac{w}{A_N(1-\alpha)} k_N^{-\alpha},$$

which implies that

$$k_N^{1-\alpha} \frac{r}{A_N \alpha} = p = \frac{w}{A_N(1-\alpha)} k_N^{-\alpha}.$$

Now, solve for the capital intensity,

$$k_N = \frac{w}{r} \frac{\alpha}{1-\alpha}$$

and insert the capital intensity back into one of the expressions for p,

$$p = k_N^{1-\alpha} \frac{r}{A_N \alpha}$$

$$p = \left(\frac{w}{r} \frac{\alpha}{1-\alpha}\right)^{1-\alpha} \frac{r}{A_N \alpha}$$

$$p = \frac{r}{A_N \alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} r^{\alpha-1} w^{1-\alpha}$$

Then insert for w from (b),

$$p = \frac{r^{\alpha}}{A_N \alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left((1-\alpha)(p_T A_T)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}\right)^{1-\alpha} \qquad (9)$$

(d) Let the real exchange rate be defined as $Q = \frac{EP^*}{P}$ and the price index as $P = (p_T)^{\lambda} p^{1-\lambda}$, where λ is the weight on traded good in the index. Assume $E = 1$ and $P^* = 1$. Assume p_T increases. What happens to the real exchange rate Q and to the real wage, $\frac{w}{P}$?

Answer:

Note that λ is the size of the tradeable sector relative to the non-tradeable sector. Let's rewrite p to work with simpler expressions for price and wage,

$$p = \frac{r^{\alpha}}{A_N \alpha} \left(\frac{\alpha}{1-\alpha}\right)^{1-\alpha} \left((1-\alpha)(p_T A_T)^{\frac{1}{1-\alpha}} \left(\frac{\alpha}{r}\right)^{\frac{\alpha}{1-\alpha}}\right)^{1-\alpha} \qquad \text{(see 9)}$$

$$p = \frac{1}{A_N} (p_T A_T)^{\frac{1-\alpha}{1-\alpha}} r^{\frac{\alpha-\alpha}{1-\alpha}} \frac{1}{\alpha} \left(\frac{\alpha}{1-\alpha} (1-\alpha) \alpha^{\frac{\alpha}{1-\alpha}}\right)^{1-\alpha}$$

And introduce

$$B = \frac{1}{\alpha}\Big(\frac{\alpha}{1-\alpha}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}}\Big)^{1-\alpha}$$

Thus we have

$$p = B\frac{1}{A_N}(p_T A_T)^{\frac{1-\alpha}{1-\alpha}} r^{\frac{\alpha-\alpha}{1-\alpha}}$$

To find the effect on real exchange rate, insert P into Q (recall that $E = 1$ and $P^* = 1$ (both are exogenous))

$$Q = \frac{EP^*}{P} = \frac{1}{P} = \frac{1}{p^{1-\lambda}} = p^{\lambda-1} = \Big(B\frac{1}{A_N}(p_T A_T)^{\frac{1-\alpha}{1-\alpha}} r^{\frac{\alpha-\alpha}{1-\alpha}}\Big)^{\lambda-1}$$

and real wage (where w is from (b)),

$$\frac{w}{P} = (1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} B^{\lambda-1} A_N^{1-\lambda} r^{\frac{(\alpha-\alpha)(\lambda-1)-\alpha}{1-\alpha}} (p_T A_T)^{\frac{(1-\alpha)(\lambda-1)+1}{1-\alpha}}$$

Take the first derivative wrt p_T,

$$\frac{\partial Q}{\partial p_T} = \frac{(1-\alpha)(\lambda-1)}{1-\alpha} A_T (p_T A_T)^{\frac{(1-\alpha)(\lambda-1)-1}{1-\alpha}} \Big(B\frac{1}{A_N} r^{\frac{\alpha-\alpha}{1-\alpha}}\Big)^{\lambda-1} < 0$$

since $0 < \alpha, \lambda < 1$.

$$\frac{\partial \frac{w}{P}}{p_T} = \frac{(1-\alpha)(\lambda-1)+1}{1-\alpha}(1-\alpha)\alpha^{\frac{\alpha}{1-\alpha}} B^{\alpha-1} A_N^{1-\alpha} r^{\frac{(\alpha-\alpha)(\lambda-1)-\alpha}{1-\alpha}} A_T (p_T A_T)^{\frac{(1-\alpha)(\lambda-1)+1}{1-\alpha}-1} > 0$$

Second Question [20 Points]

a) What is a difference between uncovered and covered interest parities?

Answer:

CIP = covered interest rate parity: $(1+i_t) = (1+i_t^*)\left(\frac{F_t}{\varepsilon_t}\right)$

UIP = uncovered interest rate parity: $(1+i_t) = (1+i_t^*)E\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)$

Difference in: $\left(\frac{F_t}{\varepsilon_t}\right)$ and $E\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)$

CIP takes forward exchange rate while UIP expected exchange rate

b) How would you test for free capital mobility (financial integration)?

Answer:

Using covered interest rate differential: the closer value to zero is, the more integrated the market is

$$(1+i_t) - (1+i_t^*)\left(\frac{F_t}{\varepsilon_t}\right) = 0$$

c) Show using "fundamental equation of asset pricing" that UPI does not hold in theory.
Where "fundamental equation of asset pricing" is:

$$E_1[m_2(r_2^P - i)] = 0$$

where
$m_2 = \beta\frac{u'(C_2)}{u'(C_1)}$ – discount factor
r_2^P – return on a portfolio P
i – safe return

Answer:

$$E_1\left[m_2((1+i^*)\left(\frac{\varepsilon_2}{\varepsilon_1}\right) - (1+i))\right] = 0 \implies (1+i) = (1+i^*)E_1\left[\frac{m_2\varepsilon_2}{E[m_2]\varepsilon_1}\right]$$

$$(1+i) = (1+i^*)\left[E_1\frac{\varepsilon_2}{\varepsilon_1} + Cov\left(\frac{m_2}{E[m_2]}, \frac{\varepsilon_2}{\varepsilon_1}\right)\right]$$

Recall that **UIP**: $(1+i_t) = (1+i_t^*)E\left(\frac{\varepsilon_{t+1}}{\varepsilon_t}\right)$. This holds iff the exchange rate is uncorrelated with the stochastic discount factor m_2. This in not in general true

Third Question [30 Points]

There are 2 periods and 2 goods (traded and nontraded). Domestic price of tradable goods, denoted as P_t^T obeys the law of one price:

$$P_t^T = \varepsilon_t P_t^{T*}$$

where P_t^T – domestic price of the tradable good

P_t^{T*} – foreign price of the tradable good t

ε_t – nominal exchange rate defined as the domestic currency price of one unit of foreign currency — so an increase in ε_t is a depreciation of the domestic currency.

Output of tradable goods of period T, denoted as Y_t^T is considered as a countries endowment.

Output of nontraded goods, denoted Y_t^N , is produced by perfectly competitive firms using labor, h_t, as only input using Cobb-Douglas production function with decreasing returns to scale. Price of non-tradable goods, denoted as P_t^N Assume that wages are fully fixed.

Representative household has no assets in period 0, gets wage and profit incomes in both periods, and can save in a foreign bond, B_t yielding a return of $(1 + r^*)$ (with the traded good as numeraire). Households utility function depends on consumption of both tradables C_t^T and non-tradables C_t^N

$$logC_1 + \beta logC_2$$

where $C_t = (C_t^T)^\gamma (C_t^N)^{1-\gamma}$

a) Derive households intertemporal budget constraint

Answer:

$$C_1^T + p_1 C_1^N + \frac{C_2^T + p_2 C_2^N}{1 + r^*} = Y_1^T + \frac{W_1}{\varepsilon_1 P_1^{T*}} h_1 + \frac{\Pi_1}{\varepsilon_1 P_1^{T*}} + \frac{Y_2^T + \frac{W_2}{\varepsilon_2 P_2^{T*}} h_2 + \frac{\Pi_2}{\varepsilon_2 P_2^{T*}}}{1 + r^*}$$

b) Set up firms and households maximization problem and derive supply and demand schedules in equilibrium:

Answer:

Firm:

$$max_{h_t}\ \Pi = P_t^N Y_t^N - W_t h_t$$

Household:

$$max_{C_t^T, C_t^N}\ log((C_1^T)^\gamma (C_1^N)^{1-\gamma}) + \beta log((C_2^T)^\gamma (C_2^N)^{1-\gamma})$$

s.t.

$$C_1^T + p_1 C_1^N + \frac{C_2^T + p_2 C_2^N}{1 + r^*} = Y_1^T + \frac{W_1}{\varepsilon_1 P_1^{T*}} h_1 + \frac{\Pi_1}{\varepsilon_1 P_1^{T*}} + \frac{Y_2^T + \frac{W_2}{\varepsilon_2 P_2^{T*}} h_2 + \frac{\Pi_2}{\varepsilon_2 P_2^{T*}}}{1 + r^*}$$

Supply schedule:

$$p_1 = \frac{W_1/(\varepsilon_1 P_1^{T*})}{F'(h_1)}$$

Demand schedule:

$$p_1 = \frac{1-\gamma}{\gamma} \frac{\frac{1}{1+\beta}\left(Y_1^T + \frac{Y_2^T}{1+r^*}\right)}{F(h_1)}$$

c) Assume a fixed exchange rate regime: The exchange rate is constant, $\varepsilon_t = \varepsilon$

How does the economy adjust to different shocks under a fixed exchange rate regime and downward nominal wage rigidity?

i) World interest rate r^*increases

Answer:

Shifts demand inwards. Because the nominal wage W is sticky and the exchange rate ε_1 is fixed, the supply schedule is unaffected. I.e., the increase in the interest rate lowers both the price of non-tradables and the hours worked. *The increase in the interest rate results in less production/increased unemployment*

ii) Experiment (imported inflation): The world cost of the tradable good, P_1^{T*}, increases

Answer:

Shifts demand inwards. Shifts supply inwards. Because the nominal wage W is sticky and the exchange rate ε_1 is fixed, the increase in the foreign price shifts the supply schedule. *The demand schedule is unaffected.* Higher price of non-tradables, i.e., core inflation, and lower output/higher unemployment.

iii) Experiment (more oil): The NPV output of the tradeable good, $\left(Y_1^T + \frac{Y_2^T}{1+r^*}\right)$, increases

Answer:

Shifts demand outwards. Because the nominal wage W is sticky and the exchange rate ε_1 is fixed, the supply schedule is unaffected. I.e., the increase in the interest rate increases both the price of non-tradables and the hours worked. I.e., the increased oil production results in core inflation and in more production/less unemployment.