Dynamics of Small Open Economies

Econ 4330 Open Economy Macroeconomics Spring 2008

Third lecture

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Budget constraints with infinite horizons

Period budget constraint

$$C_s + I_s + (B_{s+1} - B_s) = Y_s + rB_s - G_s, \quad s = t, t+1, t+2, \ldots. \qquad (2)$$

- Start with $s = t$.
- Use (2) for s=t+1 to eliminate B_{t+1}. B_{t+2} then enters the equation.
- Use (2) for s=t+2 to eliminate B_{t+2}.
- Continue to eliminate B_{t+s} until s=T, and you get

$$\sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} (C_s + I_s) + \left(\frac{1}{1+r}\right)^{T} B_{t+T+1}$$
$$= +(1+r)B_t + \sum_{s=t}^{t+T} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s) \qquad (3)$$

Take the limit as $t \to \infty$ and you get

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (C_s + I_s) = (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (Y_s - G_s) \qquad (4)$$

provided that all the limits exist and that

$$LIM = \lim_{T \to \infty} \left(\frac{1}{1+r}\right)^{T} B_{t+T+1} = 0 \qquad (5)$$

Why is (5) a reasonable assumption?

Suppose *LIM* < *0.* This means

- for large T, $B_{t+T+1} < 0$ and growing in absolute value at a rate greater than or equal to *r*.
- the country finances all interest payments by acquiring new debt.
- creditors will not accept that this goes on forever.
- *LIM* has to be greater than or equal to zero.

- $LIM = \lim_{T \to \infty} \left(\frac{1}{1+r}\right)^T B_{t+T+1} = 0 \qquad (5)$

Suppose *LIM > 0*. This means that

- - for large T, $B_{t+T+1} > 0$ and growing at a rate greater than or equal to *r.*
- the country is providing resources to others without getting anything in return.
- consumption can be increased without hitting.
- utility maximization demands *LIM* to be zero or negative.

Conclusion: LIM = 0.

Consistent with this:

- With an infinite horizon debt can be rolled over forever as long as some of the interest is paid from present income.
- To continue acquiring foreign assets forever can be consistent with utility maximization as long as some of the interest received is actually consumed.

A small economy model with infinite horizon

Utililty function

$$U_t = u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \beta^3 u(C_{t+3}) + \cdots = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s) \quad (6)$$

Production functions:

$$Y_t = A_t F(K_t), \qquad t = 1,2,\ldots \quad (7)$$

Definitional relationships

$$K_t = K_{t-1} + I_{t-1}, \qquad t = 1,2,\ldots. \quad (8)$$

$$CA_t = B_{t+1} - B_t = rB_t + Y_t - C_t - I_t - G_t, \qquad t = 1,2,\ldots\ldots \quad (9)$$

Budget constraint (4) or equivalently (5).

Optimization

Maximize

$U_t = \sum_{s=t}^{\infty} \beta^{s-t}\, u(C_s) \quad (10)$

with respect to B_{s+1} and K_{s+1}, $\; s = t, t+1, \ldots$given

$C_s = (1+r)B_s - B_{s+1} + A_s F(K_s) - (K_{s+1} - K_s) - G_s \qquad s = t, t+1, \ldots. \quad (11)$

and that *LIM=0.*

First order condition for B_{s+1}:

$$\frac{\partial U_t}{\partial B_{s+1}} = \beta^{s-t} u'(C_s) \frac{\partial C_s}{\partial B_{s+1}} + \beta^{s+1-t} u'(C_{s+1}) \frac{\partial C_{s+1}}{\partial B_{s+1}}$$
$$= \beta^{s-t} u'(C_s)(-1) + \beta^{s+1-t} u'(C_{s+1})(1+r) = 0$$

Hence, the consumption Euler equation

$$\beta(1+r)u'(C_{s+1}) = u'(C_s) \qquad (12)$$

First order condition for K_{s+1}:

$$\frac{\partial U_t}{\partial K_{s+1}} = \beta^{s-t}u'(C_s)\frac{\partial C_s}{\partial K_{s+1}} + \beta^{s+1-t}u'(C_{s+1})\frac{\partial C_{s+1}}{\partial K_{s+1}}$$
$$= \beta^{s-t}u'(C_s)(-1)+\beta^{s+1-t}u'(C_{s+1})(A_{s+1}F'(K_{s+1}) + 1) = 0$$

or

$$\beta u'(C_{s+1})(A_{s+1}F'(K_{s+1}) + 1) = u'(C_s)$$

Or after taking account of the Euler equation

$$A_{s+1}F'(K_{s+1}) + 1 = \frac{u'(C_s)}{\beta u'(C_{s+1})} = 1 + r$$

and surprise!

$$A_{s+1}F'(K_{s+1}) = r \qquad (13)$$

In addition to the first order conditions, we need the present value budget constraint to determine the levels of consumption and debt.

CES example again: Solving for C_t

The Euler equation reduces to

$$C_{s+1} = (1+r)^{\sigma}\beta^{\sigma}C_s = (1+v)C_s$$

where $v = (1+r)^{\sigma}\beta^{\sigma} - 1$ is the growth rate of consumption.

Hence, $C_s = (1+v)^{s-t}C_t$,

and the present value of consumption is

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}C_s = \sum_{s=t}^{\infty}\left(\frac{1+v}{1+r}\right)^{s-t}C_t = \frac{1}{1-\dfrac{1+v}{1+r}}C_t = \frac{1+r}{r-v}C_t \quad (14)$$

(Use formula for sum of infinite geometric series).

$r > v$ is necessary for convergence.

$r > v$ is always satsified when both $\sigma < 1$ and $\beta < 1$.

Recall the present value budget constraint

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} (Y_s - I_s - G_s) = W_t \quad (4')$$

W_t = total wealth

Replacing the lhs by,

$$\frac{1+r}{r-v}C_t$$

from (14), we find that

$$C_t = \frac{r-v}{1+r}W_t \qquad (15)$$

$v = 0$ Consume the permanent income from your total wealth.

$v > 0$ Consume less than your permanent income if you want a rising consumption path

Characterizing the solution for the current account

Define the *"permanent" value* of a variable

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\tilde{X}_t=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$$

Using the formula for the sum of an infinite geometric series:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}=\frac{1}{1-\frac{1}{1+r}}=\frac{1+r}{r}$$

Hence,

$$\frac{1+r}{r}\tilde{X}_t=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s \quad \Leftrightarrow \quad \tilde{X}_t=\frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$$

W_t can then be rewritten

$$W_t = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - \mathrm{I}_s - G_s) = (1+r)B_t + \frac{1+r}{r}(\tilde{Y}_t - \tilde{I}_t - \tilde{G}_t)$$

Hence, the solution for C_t can be rewritten

$$C_t = \frac{r-v}{1+r}W_t = rB_t + \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t - \frac{v}{1+r}W_t \qquad (16)$$

By definition

$$CA_t = rB_t + Y_t - C_t - I_t - G_t \qquad (17)$$

After inserting for C_t from (16)

$$CA_t = Y_t - \tilde{Y}_t - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t) + \frac{v}{1+r}W_t \qquad (8)$$

- Deviations between actual and permanent values of *Y, I and G.*
- Tilt factor related to growth rate.

A Stochastic Current Account Model

- Future levels of output, investment and government spending are stochastic
- Only financial asset is riskless bond which pays a constant interest rate *r*
- Rational expectations: Agent’s expectations are equal to the mathematical conditional expectations based on the economic model and all available information about current and past value of economic variables
- Current values of all exogenous variables are known by all decision makers before decisions are made

Optimization

Utility function

$$U_t = \mathbf{E}_\mathrm{t}\left\{\sum_{s=t}^{\infty} \beta^{s-t}\, u(C_s)\right\} \qquad (19)$$

Same budget equation and constraints, same initial conditions, same procedure.

First order condition with respect to B_{s+1} (compare slide 6):

$$\mathbf{E}_\mathrm{t}[\beta^{s-t}u'(C_s)(-1)+\beta^{s+1-t}u'(C_{s+1})(1+r)] = 0$$

or

$$\mathbf{E}_\mathrm{t}[u'(C_s)] = \mathbf{E}_\mathrm{t}[\beta(1+r)u'(C_{s+1})] \quad s = t, t+1, \ldots. \quad (20)$$

For *s*=*t* this specializes to

$$u'(C_t) = \mathbf{E}_\mathrm{t}[\beta(1+r)u'(C_{t+1})] \qquad (21)$$

First order condition with respect to K_{s+1} (compare slide 7):

$$\mathbf{E}_{\mathrm{t}}[\beta^{s-t}u'(C_s)(-1)+\beta^{s+1-t}u'(C_{s+1})(A_{s+1}F'(K_{s+1})+1)]=0$$

For *s*=*t* this specializes to

$$\mathbf{E}_{\mathrm{t}}\{\beta u'(C_{s+1})(A_{s+1}F'(K_{s+1})+1)\}=u'(C_s)$$

$$\mathbf{E}_{\mathrm{t}}\left\{\frac{\beta u'(C_{t+1})}{u'(C_t)}A_{t+1}F'(K_{t+1})\right\}+\mathbf{E}_{\mathrm{t}}\left\{\frac{\beta u'(C_{t+1})}{u'(C_t)}\right\}=1$$

Or, after inserting from the consumption Euler equation

$$\mathbf{E}_{\mathrm{t}}\left\{\frac{\beta(1+r)u'(C_{t+1})}{u'(C_t)}A_{t+1}F'(K_{t+1})\right\}=r \qquad (22)$$

The linear-quadratic example

Exogenous endowments (Y_t), no investment.

No trend growth in consumption: $\beta(1+r) = 1$

Quadratic utility function

$$u(C) = C - \frac{a_0}{2}C^2, \quad a_0 > 0 \qquad (23)$$

Euler equation $\mathbf{E}_{\mathrm{t}}[u'(C_s)] = \mathbf{E}_{\mathrm{t}}[\beta(1+r)u'(C_{s+1})]$

$$\mathbf{E}_{\mathrm{t}}[1 - a_0 C_s] = \mathbf{E}_{\mathrm{t}}[\beta(1+r)(1 - \mathrm{a}_0 \mathrm{C}_{\mathrm{s+1}})]$$

$$1 - a_0 \mathbf{E}_{\mathrm{t}} C_s = 1 - a_0 \mathbf{E}_{\mathrm{t}} \mathrm{C}_{\mathrm{s+1}}$$

$$\mathbf{E}_{\mathrm{t}} C_{s+1} = \mathbf{E}_{\mathrm{t}} C_s \quad s = t, t+1, \ldots . (24)$$

For *s=1* we get Robert Hall's random walk result:

$$\mathbf{E}_{\mathrm{t}} C_{t+1} = C_t \quad (25)$$

Taking expectations on both sides of the budget constraint, we find

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathbf{E}_{\mathrm{t}}C_s = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathbf{E}_{\mathrm{t}}(Y_s - G_s) = W_t$$

Since $\mathbf{E}_{\mathrm{t}}C_{s+1} = C_t$ for all *s*>*t*, the lhs is (compare (14))

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathbf{E}_{\mathrm{t}}C_s = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}C_t = C_t\frac{1+r}{r}$$

Hence (compare (15))

$$C_t = \frac{r}{1+r}W_t = rB_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathbf{E}_{\mathrm{t}}(Y_s - G_s) \qquad (26)$$

Certainty equivalence: Act as if the expected values were certain to be realized.

Constraints and first-order conditions are linear in all the stochastic variables.

Necessary: Quadratic utility function (dubious) and non-stochastic *r*.

Response-impulse relations for output shocks

C_t is determined by $\mathbf{E}_t Y_s$, s=t+1,t+2,….How, are these expectations formed?

Example: Consumers believe income follows the stochastic process

$$Y_{s+1} - \bar{Y} = \rho(Y_s - \bar{Y}) + \varepsilon_{s+1} \qquad (27)$$

where $0 \leq \rho \leq 1$, $\mathbf{E}_t \varepsilon_s = 0$ for s=t+1, t+2,… ,and ε_t is serially uncorrelated.

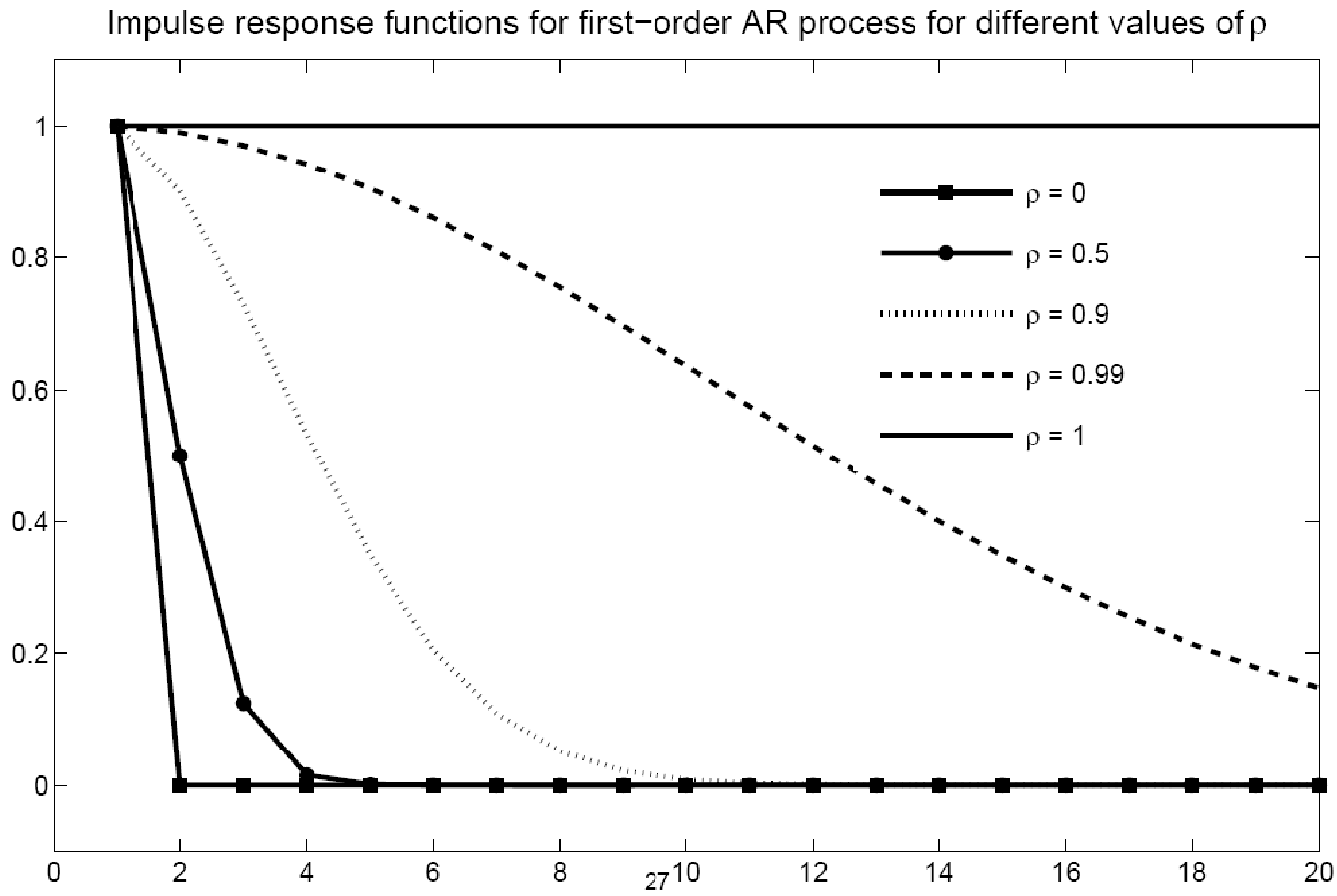
ρ is the coefficient of autocoregression.

ρ measures the degree of persistence of the process

$\rho = 0$ Y_s varies randomly around $\bar{Y}$. No serial correlation.

$0 < \rho < 1$ Y_s returns gradually towards $\bar{Y}$ after a shock. Positive serial corr.

$\rho = 1$ Y_s random walk, no tendency to return to $\bar{Y}$, $Y_{s+1} - Y_s = \varepsilon_{s+1}$


Impulse response functions for first−order AR process for different values of ρ
ρ = 0
ρ = 0.5
ρ = 0.9
ρ = 0.99
ρ = 1
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20
27

By successive insertions in (27) we find (details on slide 22)

$$Y_s - \bar{Y} = \rho^{s-t}(Y_t - \bar{Y}) + \sum_{i=t+1}^{s} \rho^{i-t}\varepsilon_i \qquad (28)$$

Take expectations on both sides of (28):

$$\mathbf{E}_\mathrm{t}[Y_s - \bar{Y}] = \rho^{s-t}(Y_t - \bar{Y}) \qquad (29)$$

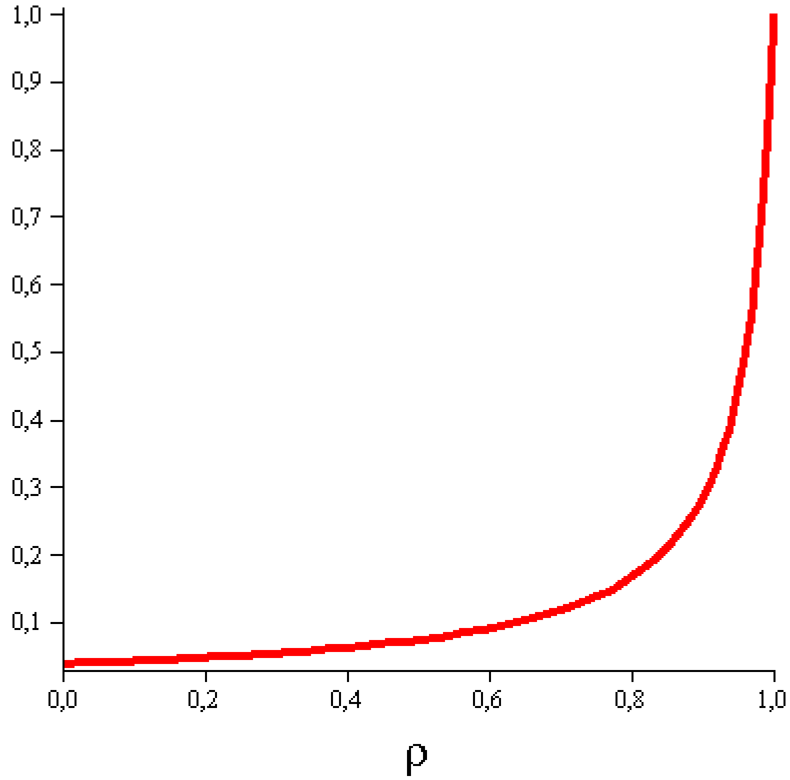
Insert the expectations from (29) in the consumption function (26) and you find (details on slide 23)

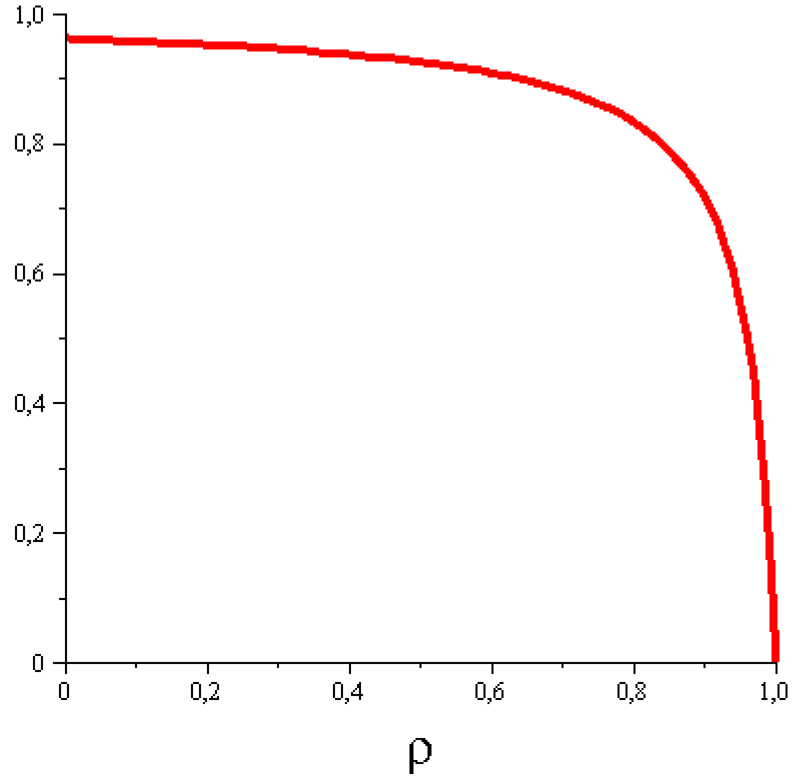
$$C_t = rB_t + \bar{Y} + \frac{r}{1+r-\rho}(Y_t - \bar{Y}) \qquad (30)$$

By definition $CA_t = rB_t + Y_t - C_t$. After inserting for C_t:

$$CA_t = \frac{1-\rho}{1+r-\rho}(Y_t - \bar{Y}) \qquad (31)$$

CA does not depend on B_t.





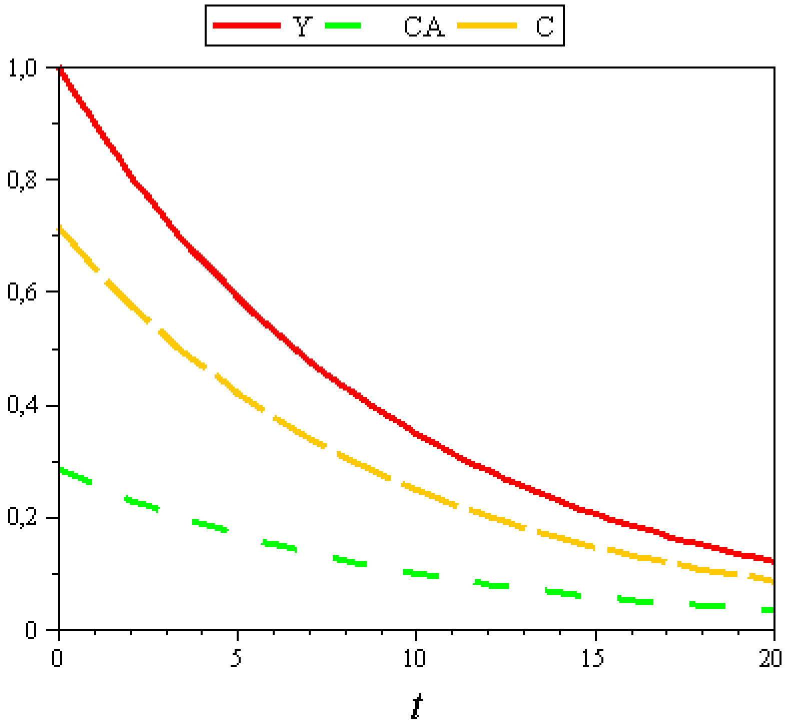
r = 0.04

Effect of Y_t on C_t. (MPC)

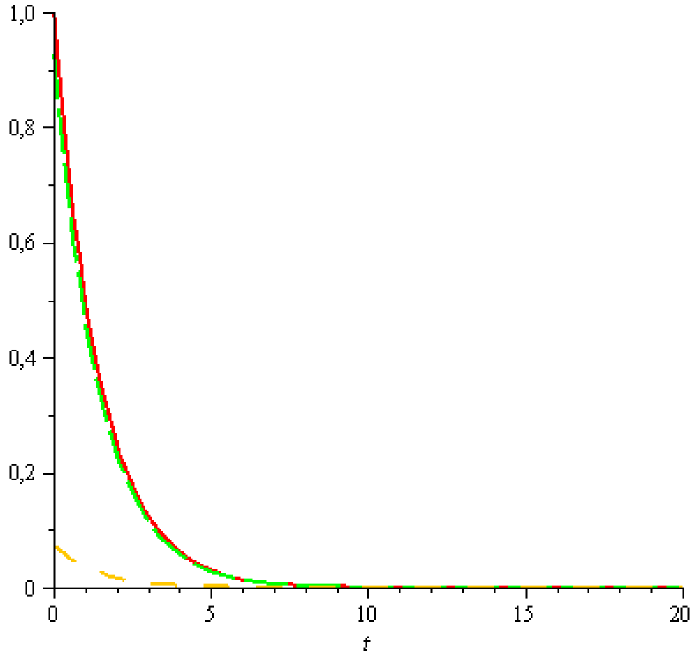
$$\frac{r}{1+r-\rho}$$

Effect of Y_t on CA_t

$$\frac{1-\rho}{1+r-\rho}$$



Implpuse response ρ = 0.9, r = 0.04



Impulse response ρ = 0.5, r = 0.04

Derivation of (28)

Start from (27) with s=t:

$$Y_{t+1} - \bar{Y} = \rho(Y_t - \bar{Y}) + \varepsilon_{t+1}$$

Move forward 1 period:

$$Y_{t+2} - \bar{Y} = \rho(Y_{t+1} - \bar{Y}) + \varepsilon_{t+2}$$

Insert for $Y_{t+1} - \bar{Y}$:

$$Y_{t+2} - \bar{Y} = \rho^2(Y_t - \bar{Y}) + \rho\varepsilon_{t+1} + \varepsilon_{t+2}$$

Move forward 1 period:

$$Y_{t+3} - \bar{Y} = \rho(Y_{t+2} - \bar{Y}) + \varepsilon_{t+3}$$

Insert:

$$Y_{t+3} - \bar{Y} = \rho^3(Y_t - \bar{Y}) + \rho^2\varepsilon_{t+1} + \rho\varepsilon_{t+2} + \varepsilon_{t+3}$$

Continue until *s* and you end up with (28).

Derivation of (30)

Start with the consumption function

$$C_t = rB_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \mathbf{E}_{\mathrm{t}} Y_s$$

Add and subtract $\bar{Y}$.

$$C_t = rB_t + \bar{Y} + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} (\mathbf{E}_{\mathrm{t}} Y_s - \bar{Y})$$

Insert for the expectations from (29)

$$C_t = rB_t + \bar{Y} + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \rho^{s-t}(Y_t - \bar{Y})$$

Use the formula for the sum of an infinite geometric series

$$C_t = rB_t + \bar{Y} + \frac{r}{1+r-\rho}(Y_t - \bar{Y})$$