Exchange rate determination under inflation targeting

Econ 4330 Lecture 9

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Inflation targeting

Welfare the ultimate goal

Inflation closer to the ultimate goal than money supply or exchange rate

Inflation also further from the instruments of monetary policy

Strict inflation targeting - focus on inflation rate alone

Flexible inflation targeting - allowing for other goals as long as inflation is reasonably stable around a target

Not a uniquely defined policy, but a broad category / framework

Modeling inflation target regimes

Two main approaches

1. Central banks as optimizing agents
2. Central bank reaction functions (e.g. Taylor rule)

Lag structure important

Discrete time

Simulation models

Gross simplifications below

The core of the model

IS-curve

Interest parity condition

Phillips-curve

Interest setting equation

Home and foreign goods

Interest rate parity except for stochastic disturbance

Focus on real exchange rates, real interest rates

Return to nominal rates later

Inflation target assumed to be credible

Key variables (in logs):

π_{t+1} rate of inflation from period t to $t+1$

y_t output

r_t real exchange rate

ρ_t real interest rate from period t to $t+1$

$\rho_{*,t}$ foreign real interest rate

Values of these variables in a stationary state are: $\bar{\pi}$, $\bar{y}$, $\bar{r}$, $\bar{\rho}$, $\bar{\rho_*}$

$\bar{y}$ = capacity output, $\bar{\pi}$ = inflation target

Due to capital mobility: $\bar{\rho} = \bar{\rho_*}$

IS-curve

Demand relation

$$y_t = \alpha_0 - \alpha_1 \rho_t + \alpha_2 r_t + u_{y,t}$$

$u_{y,t}$ = demand shock, $\alpha_1 > 0$, $\alpha_2 > 0$
Long run equilibrium

$$\bar{y} = \alpha_0 - \alpha_1 \bar{\rho} + \alpha_2 \bar{r}$$

Difference yields:

$$y_t - \bar{y} = -\alpha_1(\rho_t - \bar{\rho}) + \alpha_2(r_t - \bar{r}) + u_{y,t} \tag{1}$$

Note that $\bar{r}$, is determined by $\bar{y}$ and $\bar{\rho}$.

Real interest rate parity

$$\mathbf{E}_t r_{t+1} - \bar{r} = (r_t - \bar{r}) + (\rho_t - \bar{\rho}) - (\rho_{*,t} - \bar{\rho}) + u_{e,t} \tag{2}$$

$u_{e,t}$ = risk premium shock

Since $r_t = e_t + p_{*,t} - p_t$,

$$\begin{aligned}
\mathbf{E}_t r_{t+1} - r_t &= \mathbf{E}_t(e_{t+1} + p_{*,t+1} - p_{t+1}) - (e_t + p_{*,t} - p_t) \\
&= (\mathbf{E}_t e_{t+1} - e_t) + (\mathbf{E}_t p_{*,t+1} - p_{*,t}) - (\mathbf{E}_t p_{t+1} - p_t) \\
&= i_t - i_{*,t} + u_{e,t} + (\mathbf{E}_t p_{*,t+1} - p_{*,t}) - (\mathbf{E}_t p_{t+1} - p_t) \\
&= \rho - \rho_* + u_{e,t}
\end{aligned}$$

Phillips curve

$$\pi_{t+1} = \bar{\pi} + \beta(y_t - \bar{y}) + u_{\pi,t+1} \tag{3}$$

$u_{\pi,t}$ = cost-push shock, $\beta > 0$

Expected inflation equal to the target

Slightly inconsistent, but simplifies

Interest rate setting

$$\rho_t = \bar{\rho} + \phi(\mathbf{E}_t \pi_{t+1} - \bar{\pi}) \tag{4}$$

$\phi > 0$ Real interest rate is increased when inflation exceeds target

$$i = \rho_t + \mathbf{E}_t \pi_{t+1} = \bar{\rho} + \bar{\pi} + (1 + \phi)(\mathbf{E}_t \pi_{t+1} - \bar{\pi})$$

Resembles Taylor-rule, but

- Forward looking interest rate setting
- Output gap left out (simplifies)
- Import prices left out (simplifies)
- Empirical reaction functions more complex

The exogenous shocks

$u_{y,t}$, $u_{\pi,t}$, $u_{e,t}$, $\rho_{*,t} - \bar{\rho}_*$
Assume:

- Unconditional expectation of shocks is zero
- Shocks are stationary (constant variance, not random walk)
- Contemporary shocks are observed by central bank

Relating the interest rate to the state of the economy

Take expectation of the Phillips-curve (3):

$$\mathbf{E}_t \pi_{t+1} = \bar{\pi} + \beta(y_t - \bar{y}) + \mathbf{E}_t u_{\pi,t+1}$$

Insert this in the interest rule (4) and then insert from the IS-curve (1):

$$\begin{aligned}
\rho_t - \bar{\rho} &= \phi(\mathbf{E}_t \pi_{t+1} - \bar{\pi}) \\
&= \phi\beta(y_t - \bar{y}) + \phi \mathbf{E}_t u_{\pi,t+1} \\
&= \phi\beta(-\alpha_1(\rho_t - \bar{\rho}) + \alpha_2(r_t - \bar{r}) + u_{y,t}) + \phi \mathbf{E}_t u_{\pi,t+1}
\end{aligned}$$

Interest rate setting forward looking

CB takes account of effect of interest on inflation

Solve for $\rho_t - \bar{\rho}$

$$\rho_t - \bar{\rho} = \frac{\phi\beta\alpha_2}{1+\phi\beta\alpha_1}(r_t - \bar{r}) + \frac{1}{1+\phi\beta\alpha_1}(\phi\beta u_{y,t} + \phi \mathbf{E}_t u_{\pi,t+1}) \qquad (5)$$

CB raises real interest rate in response to:

- Depreciated (real) exchange rate
- Positive demand shock
- Positive expected cost push shock

Dynamics of the expected real exchange rate

Insert for real interest rate from (5) in real interest parity (2):

$$
\begin{aligned}
\mathbf{E}_t r_{t+1} - \bar{r} &= (r_t - \bar{r}) + (\rho_t - \bar{\rho}) - (\rho_{*,t} - \bar{\rho}) + u_{e,t} \\
&= (r_t - \bar{r}) + \frac{\phi\beta\alpha_2}{1+\phi\beta\alpha_1}(r_t - \bar{r}) \\
&\quad + \frac{1}{1+\phi\beta\alpha_1}(\phi\beta u_{y,t} + \phi \mathbf{E}_t u_{\pi,t+1}) - (\rho_{*,t} - \bar{\rho}) + u_{e,t} \\
&= \epsilon(r_t - \bar{r}) - z_t
\end{aligned}
$$

where

$$\epsilon = \frac{1+\phi\beta(\alpha_1+\alpha_2)}{1+\phi\beta\alpha_1} > 1$$

and

$$z_t = -\frac{1}{1+\phi\beta\alpha_1}(\phi\beta u_{y,t} + \phi \mathbf{E}_t u_{\pi,t+1}) + (\rho_{*,t} - \bar{\rho}) - u_{e,t}$$

$$\mathbf{E}_t r_{t+1} - \bar{r} = \epsilon(r_t - \bar{r}) - z_t \tag{6}$$

Depreciated real exchange rate

- Demand pressure, inflationary pressure
- High real interest rate
- Further real depreciation
- Instability

Confidence in CBs ability to reach inflation target leads to choice of saddle path leading to stationary state.

Deriving the saddle path

From forwarding the equation for the real exchange rate (6):

$$\mathbf{E}_t r_{t+s+1} - \bar{r} = \epsilon(\mathbf{E}_t r_{t+s} - \bar{r}) - \mathbf{E}_t z_{t+s}, \qquad s = 0, 1, 2, ... \tag{7}$$

Solution:

$$r_t - \bar{r} = \frac{1}{\epsilon} \sum_{i=0}^{\infty} \epsilon^{-i} \mathbf{E}_t z_{t+i} \tag{8}$$

Proof

Simplify the expressions by temporarily leaving out the expectations operator:

$$r_{t+s+1} - \bar{r} = \epsilon(r_{t+s} - \bar{r}) - z_{t+s},$$

Iterate from $s = 0$:

$$\begin{aligned}
r_{t+1} - \bar{r} &= \epsilon(r_t - \bar{r}) - z_t \\
r_{t+2} - \bar{r} &= \epsilon(r_{t+1} - \bar{r}) - z_{t+1} = \epsilon^2(r_t - \bar{r}) - \epsilon z_t - z_{t+1} \\
r_{t+3} - \bar{r} &= \epsilon(r_{t+2} - \bar{r}) - z_{t+2} = \epsilon^3(r_t - \bar{r}) - \epsilon^2 z_t - \epsilon z_{t+1} - z_{t+2}
\end{aligned}$$

By continuing

$$r_{t+s+1} - \bar{r} = \epsilon^{s+1}(r_t - \bar{r}) - \sum_{i=0}^{s} \epsilon^{s-i} z_{t+i}$$

Proof, continued

$$r_{t+s+1} - \bar{r} = \epsilon^{s} \left[\epsilon(r_t - \bar{r}) - \sum_{i=0}^{s} \epsilon^{-i} z_{t+i} \right]$$

First term goes to infinity with s, second term has to go to zero, which requires:

$$r_t - \bar{r} = \frac{1}{\epsilon} \sum_{i=0}^{\infty} \epsilon^{-i} z_{t+i}$$

Bring back expectations, and you have (8).

Solution for the remaining variables

Real variables and expected inflation

With r_t given, ρ_t follows from (5).

y_t is found by inserting for r_t and ρ_t in IS-curve (1)

$\mathbf{E}_t \pi_{t+1}$ is found from Phillips-curve (3)

The nominal exchange rate

$$e_t = r_t + p_t - p_{*,t}$$

p_t predetermined, $p_{*,t}$ exogenous.
Cost-push shock $u_{\pi,t}$ raises p_t and, hence, e_t one for one, while r_t is unaffected.

Effects of transitory shocks

Assume

Shocks are independently distributed from period to period.

CB has no advance information on cost push or other shocks.

Then the solution (8) for r_t is reduced to

$$r_t - \bar{r} = \frac{1}{\epsilon} z_t = \frac{1}{\epsilon}[\frac{-\phi\beta}{1+\phi\beta\alpha_1} u_{y,t} + (\rho_{*,t} - \bar{\rho}) - u_{e,t}]$$

All later terms in sum drop out
Supply shock drops out, since it will be unexpected

$$r_t - \bar{r} = -\frac{\phi\beta}{1+\phi\beta(\alpha_1+\alpha_2)}u_{y,t} + \frac{1+\phi\beta\alpha_1}{1+\phi\beta(\alpha_1+\alpha_2)}[(\rho_{*,t}-\bar{\rho}) - u_{e,t}] \quad (9)$$

$$\rho_t - \bar{\rho} = \frac{\phi\beta}{1+\phi\beta(\alpha_1+\alpha_2)}u_{y,t} + \frac{\phi\beta\alpha_2}{1+\phi\beta(\alpha_1+\alpha_2)}[(\rho_{*,t}-\bar{\rho}) - u_{e,t}] \quad (10)$$

$$y_t - \bar{y} = \frac{1}{1+\phi\beta(\alpha_1+\alpha_2)}u_{y,t} + \frac{\alpha_2}{1+\phi\beta(\alpha_1+\alpha_2)}[(\rho_{*,t}-\bar{\rho}) - u_{e,t}] \quad (11)$$

Summary: Effect of shock confined to period t

Positive demand shock $u_{y,t} > 0$

- Appreciates the exchange rate (real and nominal)
- Raises the real interest rate
- Raises output
- Raises expected inflation

A more vigorous anti-inflation policy (higher ϕ)

- reinforces the effect on the real interest rate and the real exchange rate
- dampens the effect on output and inflation

Effect of transitory shocks confined to period t

Positive shock in the foreign interest rate $\rho_* - \bar{\rho} > 0$

- Depreciates the exchange rate (real and nominal)
- Raises the real interest rate, but by less than the foreign real interest rate
- Raises output
- Raises expected inflation

A more vigorous anti-inflation policy (higher ϕ)

- dampens the effect on output and inflation
- reinforce the effect on the real interest rate
- dampens the effect on the realexchange rate

An expected one-time cost-push shock

- Appreciates the exchange rate (real and nominal)
- Raises the real interest rate
- Lowers output
- Raises expected inflation

A more vigorous anti-inflation policy (higher ϕ)

- Increases the effect on the real interest rate, the exchange rate *and output*
- Reduces the effect on expected inflation

Milton Friedman's warning

- Long and variable lags
- Observation lag
- Decision lag
- Impact lag