Exchange rate determination under inflation targeting

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Abstract

This lecture presents a highly stylized model of exchange rate determination under inflation targeting. Essentially it is the Dornbush overshooting model except that the money supply target is replaced by an inflation target.

1 Introduction

Previous lectures have provided two benchmark models of exchange rate determination with rational expectations:

1. The simple monetary model with fully flexible prices
2. The Dornbusch overshooting model with short-run nominal rigidity in prices of home goods

Common to both are that money supply is exogenous. However, hardly any central bank practices money supply targeting today. Countries with floating exchange rates instead usually practice some kind of inflation targeting. The present lecture provides a model for how exchange rates are determined under inflation targeting. It is a parallel to the Dornbusch model, assuming the same structure of the economy, but a different monetary policy.

Modeling inflation targeting regimes immediately raises some problems. Inflation targeting is not a uniquely defined policy, but a broad category. One can distinguish between *strict inflation targeting*, when inflation is the only policy goal, and *flexible inflation targeting*, which allows the central bank to pursue other goal (e.g output stability) as long as inflation is reasonably stable around a target. Sometimes inflation targeting is seen just as a broad framework for conducting monetary policy in a way that maximizes welfare. It is then combined with the underlying belief that stable inflation is important for welfare.

When a country targets the exchange rate or the money supply, it is usually in the belief that this will contribute to price stability. A strict inflation target is one step closer to the ultimate policy goals. However, it is also further from the instruments of monetary policy. Central bank policy has an immediate impact on the exchange rate and the supply of (central bank) money, and these variables can be monitored continuously. Hence the central bank can be asked to keep either the exchange rate or the money supply in a narrow band around the target continuously. In contrast central bank policy affects inflation with long lags and inflation itself is only observed with a lag. Thus, there is a question about at which time horizon the inflation should be expected to be back at the target after a shock. To keep the inflation rate on target continuously is impossible.

In the literature about exchange rates under inflation targeting there is no canonical model that is used as a benchmark in the same way as the Dornbusch model. There are two main approaches. In one central banks are modeled as optimizing agents endowed with a preference function. Central bank utility is then usually assumed to depend negatively on the deviations of inflation and output from their target levels. The other approach starts by assuming a central bank reaction function that describes how the bank's interest rate responds to the state of the economy. Often the reaction functions is the *Taylor rule*, where the interest rate is a linear function of the inflation gap (inflation minus its target level) and the output gap (the deviation of output from its full equilibrium level). Below we follow the latter approach, because it yields the simples model.

As mentioned above the lag structure and the time horizon are important with inflation targeting. Complicated lag structures are easier to represent in discrete than in continuous time. Hence, most models of inflation targeting are in discrete time. Because complicated lag structures are combined with forward-looking expectations, the models often do not have closed form solutions and simulation techniques are frequently used. In the model below the lag structure is deliberately made simple to ensure that the model has a closed form solution. This enhances the pedagogical value, but conceals some of the inherent difficulties and dilemmas of inflation targeting.

2 The model

As mentioned, the basic structure is the same as in the Dornbusch model. There are home and foreign goods. Prices on home goods change only gradually (short run nominal price rigidity). Capital mobility is close to perfect, which means that, except for a stochastic disturbance, interest rate parity holds. The only important difference from the Dornbusch model is the monetary policy. We focus on a case where the public believes that the inflation target will be met over time. In other words, we assume that the policy has full credibility.

The model consists of four equations:

- An IS-curve
- An interest parity condition
- A Phillips-curve
- An interest setting equation

The focus is on real variables, real exchange rates, real interest rates and output and, of course, on the inflation rate. The nominal exchange rate and nominal interest rate will be discussed later. The key variables are (in logs, except for the real interest rate [1].):

- π_{t+1} rate of inflation for home goods from period t to $t+1$,
- y_t output
- r_t the real exchange rate (price of foreign relative to home goods)
- ρ_t the domestic real interest rate from period t to $t+1$, (the home currency nominal interest rate minus the expected increase in prices of home goods)
- $\rho_{*,t}$ the corresponding foreign real interest rate

Values of these variables in a stationary state are: $\bar{\pi}$, $\bar{y}$, $\bar{r}$, $\bar{\rho}$, $\bar{\rho}_*$. $\bar{y}$ is equilibrium output (full capacity output), $\bar{\pi}$ is target inflation. Due to capital mobility: $\bar{\rho} = \bar{\rho}_*$. It is practical to write all equations in terms of deviations from the stationary state.

The aggregate demand equation can be written

$$y_t = \alpha_0 - \alpha_1 \rho_t + \alpha_2 r_t + u_{y,t}$$

where $u_{y,t}$ is a stochastic demand shock and $\alpha_1 > 0$, $\alpha_2 > 0$. In long run equilibrium

$$\bar{y} = \alpha_0 - \alpha_1 \bar{\rho} + \alpha_2 \bar{r}$$

Taking the difference between the two equations above yields:

$$y_t - \bar{y} = -\alpha_1(\rho_t - \bar{\rho}) + \alpha_2(r_t - \bar{r}) + u_{y,t} \tag{1}$$

which shall be our IS-equation. Note that $\bar{r}$, is determined by $\bar{y}$ and $\bar{\rho}$.

The second equation of the model is the *real interest rate parity* condition:

$$\mathbf{E}_t r_{t+1} - \bar{r} = (r_t - \bar{r}) + (\rho_t - \bar{\rho}) - (\rho_{*,t} - \bar{\rho}) + u_{e,t} \tag{2}$$

where $u_{e,t}$ is a stochastic risk premium shock. This means that capital mobility is virtually perfect, the only deviation being a stochastic risk premium shock centered on zero. Alternatively the shock can be interpreted as random deviations from model-consistent expectations[2].

In order to see how the real interest rate parity condition follows from the more usual nominal interest rate parity condition, we can start with the definition of the real exchange rate

$$r_t = e_t + p_{*,t} - p_t$$

[1]If Θ is the level of the real interest rate, then ρ must be defined as $\rho = \ln(1+\Theta)$ for the relations to hold exactly. For reasonable levels of Θ, $\rho \approx \Theta$

[2]The first interpretation is standard in the literature, but somewhat problematic. If there is a risk premium, theory tells us that it will depend on the exchange rate. This is usually neglected.

This means that

$$\begin{aligned}
\mathbf{E}_t r_{t+1} - r_t &= \mathbf{E}_t(e_{t+1} + p_{*,t+1} - p_{t+1}) - (e_t + p_{*,t} - p_t) \\
&= (\mathbf{E}_t e_{t+1} - e_t) + (\mathbf{E}_t p_{*,t+1} - p_{*,t}) - (\mathbf{E}_t p_{t+1} - p_t) \\
&= i_t - i_{*,t} + u_{e,t} + (\mathbf{E}_t p_{*,t+1} - p_{*,t}) - (\mathbf{E}_t p_{t+1} - p_t) \\
&= \rho - \rho_* + u_{e,t}
\end{aligned}$$

which is the same as the real interest parity condition (2) (just add and subtract $\bar{r}$ and $\bar{\rho}$ In the third line we inserted from the nominal interest rate parity condition and in the fourth line we used the definitions of the two real interest rates. The only extra assumption in addition to nominal interest rate parity is that prices at home and abroad on the same goods are the same when measured in the same currency.

The third equation of the model is the *Phillips curve*:

$$\pi_{t+1} = \bar{\pi} + \beta(y_t - \bar{y}) + u_{\pi,t+1} \tag{3}$$

where $u_{\pi,t}$ is a cost-push shock and $\beta > 0$. This, can be seen as a standard expectations-augmented Phillips-curve. Note, however, that expected inflation here is set equal to the inflation target. Implicitly we have then assumed that this is credible.

There may be a slight inconsistency here, because later on we assume model-consistent expectations, and in the short run these may deviate from $\bar{\pi}$. One defense is that since prices change only gradually, it may be rational for price setters to emphasize more long-run expectations of inflation. In the literature one can find models with more sophisticated equations for price setting. Equation (3) is a simple way of creating a model that produces conclusion that resemble those obtained in more realistic models.

The last equation of the model is the one for *interest rate setting*:

$$\rho_t = \bar{\rho} + \phi(\mathbf{E}_t \pi_{t+1} - \bar{\pi}) \tag{4}$$

Here, $\phi > 0$, which means that the real interest rate is increased when inflation exceeds target. This has some similarity to the Taylor-rule, as one can see by rewriting it in terms of nominal interest rates:

$$i = \rho_t + \mathbf{E}_t \pi_{t+1} = \bar{\rho} + \bar{\pi} + (1 + \phi)(\mathbf{E}_t \pi_{t+1} - \bar{\pi})$$

However, there are also important differences. One conspicuous difference is that we have left out the output gap. This is of limited importance given the simple structure that we have assumed for the economy. Another deviation is that our central bank targets the price increase on home goods (i.e. *producer price inflation* rather than *consumer price inflation*, which is more common). This simplifies the solution. Some consequences of these two deviations will be commented on below. A more fundamental difference is that while in the original Taylor rule the interest rate depends on the *actual past inflation*, here the interest rate depends on *expected future inflation.* This is in accord with the fact that most inflation-targeting central banks claim to be forward-looking.

When reaction functions for central banks have been estimated empirically, they are usually much more complex than (4), containing more variables and different lags and leads. However, the singular focus on the inflation rate in (4) means that we focus on a common element in the policies of all inflation targeters.

Equations (1) - (4) determine the time paths of the four endogenous variables ρ, r, y and π given the values of the exogenous shocks $u_{y,t}$, $u_{\pi,t}$, $u_{e,t}$ and $\rho_{*,t}-\bar{\rho}_*$. The latter can be seen as just another stochastic shock. We assume that all shocks have unconditional expectations that are equal to zero and that they are stationary (meaning that they have a constant variance and do not act like random walks). We shall also make the somewhat heroic assumption that contemporary shocks are observed by the central bank.

3 Solving the model

As a first step we can relate the interest rate setting of the central bank to the current state of the economy and the expectations about future shocks instead of to the expected rate of inflation. Take expectations of the Phillips-curve (3):

$$\mathbf{E}_t\pi_{t+1} = \bar{\pi} + \beta(y_t - \bar{y}) + \mathbf{E}_t u_{\pi,t+1}$$

Insert this in the interest rule (4) and then insert from the IS-curve (1):

$$\begin{aligned}
\rho_t - \bar{\rho} &= \phi(\mathbf{E}_t\pi_{t+1} - \bar{\pi}) \\
&= \phi\beta(y_t - \bar{y}) + \phi\mathbf{E}_t u_{\pi,t+1} \\
&= \phi\beta(-\alpha_1(\rho_t - \bar{\rho}) + \alpha_2(r_t - \bar{r}) + u_{y,t}) + \phi\mathbf{E}_t u_{\pi,t+1}
\end{aligned}$$

Since the current real interest rate affects expected inflation, ρ_t appears on both sides of the equation above. By solving it for ρ_t, we get:

$$\rho_t - \bar{\rho} = \frac{\phi\beta\alpha_2}{1+\phi\beta\alpha_1}(r_t - \bar{r}) + \frac{1}{1+\phi\beta\alpha_1}(\phi\beta u_{y,t} + \phi\mathbf{E}_t u_{\pi,t+1}) \tag{5}$$

This shows that the central bank will raise the real interest rate in response to:

- A depreciated real exchange rate
- A positive demand shock
- A positive expected cost push shock

The next step in solving the model is to derive a differential equation for the real exchange rate. This we get simply by inserting for real interest rate from (5) in real interest parity condition (2):

$$\begin{aligned}
\mathbf{E}_t r_{t+1} - \bar{r} &= (r_t - \bar{r}) + (\rho_t - \bar{\rho}) - (\rho_{*,t} - \bar{\rho}) + u_{e,t} \\
&= (r_t - \bar{r}) + \frac{\phi\beta\alpha_2}{1+\phi\beta\alpha_1}(r_t - \bar{r}) \\
&\quad + \frac{1}{1+\phi\beta\alpha_1}(\phi\beta u_{y,t} + \phi\mathbf{E}_t u_{\pi,t+1}) - (\rho_{*,t} - \bar{\rho}) + u_{e,t}
\end{aligned}$$

or, with a simplification of notation,

$$\mathbf{E}_t r_{t+1} - \bar{r} = \epsilon(r_t - \bar{r}) - z_t \tag{6}$$

where

$$\epsilon = \frac{1 + \phi\beta(\alpha_1 + \alpha_2)}{1 + \phi\beta\alpha_1} > 1$$

and

$$z_t = -\frac{1}{1 + \phi\beta\alpha_1}(\phi\beta u_{y,t} + \phi\mathbf{E}_t u_{\pi,t+1}) + (\rho_{*,t} - \bar{\rho}) - u_{e,t}$$

Since $\epsilon > 1$, equation (6) is unstable. If r_t deviates from equilibrium, it will be expected to be further away from equilibrium in the next period. The mechanism behind the instability is this

Depreciated real exchange rate

→ Demand pressure, inflationary pressure

→ High real interest rate

→ Further real depreciation

As in similar cases we have encountered before, there is a single saddle path that leads to the steady state. Confidence in the central banks ability to meet the inflation target in the long run means that expectations focus on this unique solution. This is the same principle as is used for selecting the solution in the Dornbusch model. The saddle path solution is

$$r_t - \bar{r} = \frac{1}{\epsilon}\sum_{i=0}^{\infty} \epsilon^{-i}\mathbf{E}_t z_{t+i} \tag{7}$$

Proof

From forwarding the equation for the real exchange rate (6):

$$\mathbf{E}_t r_{t+s+1} - \bar{r} = \epsilon(\mathbf{E}_t r_{t+s} - \bar{r}) - \mathbf{E}_t z_{t+s}, \qquad s = 0, 1, 2, ... \tag{8}$$

Simplify the notation by temporarily leaving out the expectations operator (but bearing in mind that it is a differential equation in expectations we are dealing with):

$$r_{t+s+1} - \bar{r} = \epsilon(r_{t+s} - \bar{r}) - z_{t+s},$$

Iterate from $s = 0$:

$$\begin{aligned}
r_{t+1} - \bar{r} &= \epsilon(r_t - \bar{r}) - z_t \\
r_{t+2} - \bar{r} &= \epsilon(r_{t+1} - \bar{r}) - z_{t+1} = \epsilon^2(r_t - \bar{r}) - \epsilon z_t - z_{t+1} \\
r_{t+3} - \bar{r} &= \epsilon(r_{t+2} - \bar{r}) - z_{t+2} = \epsilon^3(r_t - \bar{r}) - \epsilon^2 z_t - \epsilon z_{t+1} - z_{t+2}
\end{aligned}$$

By continuing

$$r_{t+s+1} - \bar{r} = \epsilon^{s+1}(r_t - \bar{r}) - \sum_{i=0}^{s} \epsilon^{s-i} z_{t+i}$$

Rewrite this by factoring out ϵ^s on the right hand side:

$$r_{t+s+1} - \bar{r} = \epsilon^s \left[\epsilon(r_t - \bar{r}) - \sum_{i=0}^{s} \epsilon^{-i} z_{t+i} \right]$$

The first term on the right hand side goes to infinity with s. The only possibility for approaching a stationary state is that the second term goes to zero, which requires:

$$r_t - \bar{r} = \frac{1}{\epsilon} \sum_{i=0}^{\infty} \epsilon^{-i} z_{t+i}$$

Bring back expectations, and you have (7).
Proof completed

Taking a closer look at equation (7), we observe that the solution for r_t is equal to $\bar{r}$ plus a weighted sum of the expected values of the present and all future shocks. The weights decline towards zero as we go further into the future. The real exchange rate depends only on the future and the present, not on the past in any way. Since the nominal exchange rate can jump at any point in time, the real exchange rate is also free to jump to the level that is consistent with long run equilibrium. Note that if there are no shocks, the model predicts that the real exchange rate will jump to its long run equilibrium value $\bar{r}$ and stay there. New information about the future affects the real exchange immediately since there can be no expected jumps in the exchange rate.

Bearing in mind the definition of z_t, equation (7) shows that

- Positive (present or expected) demand shocks lead to real appreciation
- Positive foreign interest rate shocks lead to real depreciation
- Positive *expected* cost push shocks leads to real appreciation
- Positive exchange rate shocks leads to real appreciation

Qualitatively these effects are the same as we would get in the Dornbusch model.

Remember though that in the Dornbusch model money demand shocks have real effects in the short run. Here they play no role, since the central bank supplies whatever quantity of money is demanded at the interest rate it sets. Instability in money demand have often turned out to be a problem when money supply targeting has been tried in modern economies.

With the solution for r_t in hand, the solutions for the remaining variables are easy to find.

- With r_t given, ρ_t follows from (5).
- y_t can be found by inserting for r_t and ρ_t in the IS-curve (1)
- $\mathbf{E}_t \pi_{t+1}$ is found from the Phillips-curve (3)

One implication is that in the end this period's interest rate depends on the whole future of expected shocks, even though the central bank only looks at expected inflation one period ahead. The reason is that thorough the real exchange rate all future takes part in determining aggregate demand, and, hence, inflationary pressures today.

Given r_t, the nominal exchange rate follows from the definitional relation

$$e_t = r_t + p_t - p_{*,t} \tag{9}$$

since p_t is predetermined (except for the exogenous cost-push shock) and $p_{*,t}$ is exogenous. Hence, most types of shocks have the same effect on the real and the nominal exchange rate. The exception is a shock to the international price of foreign goods. This is compensated by an opposite movement in the nominal exchange rate, and, hence, has no effect on the real exchange rate.

Unlike the real exchange rate, the nominal exchange rate depends on the past since it depends on p_t. It is not a pure forward-looking variable. An unexpected cost-push shock $u_{\pi,t}$ raises p_t and, hence, e_t one for one, while r_t is unaffected. This has a lasting effect on the levels of p and e. Since the shock is discovered too late, the central bank is unable to dampen the effect on p_t. Since its policy is strictly forward-looking, it does nothing to reverse the price increase later[3].

Temporary shocks in period $t-1$ will also have some effect on p_t. The monetary policy we have assumed is not sufficient to nullify the shocks (see the example in the next section). This means these shocks too will have lasting effects on the nominal price level and the nominal exchange rate. As the Dorbusch model shows, this is different with money supply targeting. Then temporary shocks never have lasting effects on the nominal price level.

The (actual) rate of depreciation can be written as

$$\begin{aligned} e_t - e_{t-1} &= r_t - r_{t-1} + \pi_t - \pi_{*,t} \\ &= \bar{\pi} - \bar{\pi} + (\pi_t - \bar{\pi}) - (\pi_{*,t} - \bar{\pi}) + (r_t - r_{t-1}) \end{aligned}$$

This decomposes the rate of depreciation in four:

- $\bar{\pi} - \bar{\pi}$, the difference between the inflation targets at home and abroad
- $\pi_t - \bar{\pi}$, the deviation from the inflation target at home
- $\pi_{*,t} - \bar{\pi}$, the deviation from the inflation target abroad
- $r_t - r_{t-1}$, the change in the real exchange rate

[3]Matters are more complicated if the target variable is consumer price inflation, not just inflation in home goods. Consumer price inflation can be influenced through the exchange rate channel even after it is too late to influence the prices of home goods. The central bank may still choose to be purely forward looking in its behavior, not caring about the present inflation rate. However, if it does care, it may raise the interest rate in period t in order to produce an appreciation that reduces the overall inflation rate from period $t-1$ to period t. There may then be a conflict between stabilizing inflation in periods t and $t+1$ and between stabilizing inflation and output.

The first component is the underlying deterministic trend that there will always be in the exchange rate when to countries have different inflation targets (and both targets are fully credible). The second and third components are the deviations from the inflation targets that happen in spite of the central bank's efforts. These deviations have lasting effects of the nominal price level and the nominal exchange rate. The last component is the jump in the real exchange rate that is required to keep it on track to long-run equilibrium.

4 Effects of independent transitory shocks

It is useful to look at the simplest possible example of transitory shocks. This we get by assuming that shocks are distributed independently from period to period and that the central bank has no advance information on any shock (including no information on cost push shocks before they happen). Then the solution (7) for r_t is reduced to

$$r_t - \bar{r} = \frac{1}{\epsilon} z_t = \frac{1}{\epsilon}\left[\frac{-\phi\beta}{1+\phi\beta\alpha_1} u_{y,t} + (\rho_{*,t} - \bar{\rho}) - u_{e,t}\right]$$

All later terms in the sum in (7) drops out and supply shocks drop out completely, since the central bank do not learn about them until it is too late to react.

When we also solve for the other variables as described above and insert for ϵ, we can summarize the solution as

$$r_t - \bar{r} = -\frac{\phi\beta}{1+\phi\beta(\alpha_1+\alpha_2)} u_{y,t} + \frac{1+\phi\beta\alpha_1}{1+\phi\beta(\alpha_1+\alpha_2)} [(\rho_{*,t} - \bar{\rho}) - u_{e,t}] \quad (10)$$

$$\rho_t - \bar{\rho} = \frac{\phi\beta}{1+\phi\beta(\alpha_1+\alpha_2)} u_{y,t} + \frac{\phi\beta\alpha_2}{1+\phi\beta(\alpha_1+\alpha_2)} [(\rho_{*,t} - \bar{\rho}) - u_{e,t}] \quad (11)$$

$$y_t - \bar{y} = \frac{1}{1+\phi\beta(\alpha_1+\alpha_2)} u_{y,t} + \frac{\alpha_2}{1+\phi\beta(\alpha_1+\alpha_2)} [(\rho_{*,t} - \bar{\rho}) - u_{e,t}] \quad (12)$$

$$\pi^e_{t+1} - \bar{\pi} = \frac{\beta}{1+\phi\beta(\alpha_1+\alpha_2)} u_{y,t} + \frac{\beta\alpha_2}{1+\phi\beta(\alpha_1+\alpha_2)} [(\rho_{*,t} - \bar{\rho}) - u_{e,t}] \quad (13)$$

where $\pi^e_{t+1} = \mathbf{E}_t \pi_{t+1}$.

We can then summarize the effects of a positive demand shock $u_{y,t} > 0$. It

- appreciates the exchange rate (real and nominal)
- raises the real interest rate
- raises output
- raises expected inflation

In case of a demand shock a more vigorous anti-inflation policy (higher ϕ) would

- reinforce the effect on the real interest rate and the real exchange rate

- dampen the effect on output and inflation

A positive shock to the foreign interest rate $\rho_* - \bar{\rho} > 0$

- depreciates the exchange rate (real and nominal)
- raises the real interest rate, but by less than the increase in foreign real interest rate
- raises output
- raises expected inflation

In case of an international interest rate shock a more vigorous anti-inflation policy (higher ϕ) would

- reinforce the effect on the real interest rate
- dampen the effect on the real exchange rate
- dampen the effect on output and inflation

As can be seen from equations (10)-(13), the effects of a negative risk premium shock and a positive foreign interest rate shock are the same.

Note that when shock emanate on the demand side of the economy or in the foreign exchange market, there is no conflict between stabilizing inflation and stabilizing output. Because both types of shocks affect inflation through the output gap and the Phillips-curve, inflation is stabilized by stabilizing the output gap[4].

However, conflicts between output stability and inflation stability arises if there are expected cost-push shocks. An expected one-time cost-push shock

- appreciates the exchange rate (real and nominal)
- raises the real interest rate
- lowers output
- raises expected inflation

A more vigorous anti-inflation policy (higher ϕ) in this case

- increases the effect on the real interest rate, the exchange rate *and output*
- reduces the effect on expected inflation

If we look beyond the present model, conflicts between inflation stabilization and output stabilization may arise also for other reasons:

[4]For shocks from the foreign exchange market this gets more complicated if the inflation target includes the prices on foreign goods. Whether or not a more vigorous policy for stabilizing inflation dampens the output response may then depend on the values of the parameters of the model.

- If the inflation target is consumer prices, and the central bank is concerned with not just inflation to the next period, but also inflation to this period.
- Because of different time lags in the effects of monetary policy on inflation and output.
- If the inflation target does not have full credibility, e.g. if price- and wage-setters expect inflation to be far above the target, while at the same there is a negative demand shock ("stagflation").

That some disturbances create conflicts between output stability and inflation stability, while others do not, gives an argument for a more flexible type of inflation targeting where the response of the central bank can be tailored to the shocks.

5 Permanent shocks

The analysis of the last section can be extended by assuming that shocks are autocorrelated, meaning that shocks in one period contain information about the level of the shocks in the following periods. However, the assumption of stationarity means that sooner or later the effect of a given shock is expected to die out. Given that the shocks are of the same size, more persistent shocks will have a stronger immediate impact.

If we also want to analyze the effect of permanent shocks, we can do that by looking at the effects of one-time changes in the stationary values of the variables. The key is then what happens to $\bar{r}$. From the derivation of the IS-curve we know that

$$\bar{r} = (\bar{y} - \alpha_0 + \alpha_1 \bar{\rho})/\alpha_2 \tag{14}$$

A permanent increase in capacity output $\bar{y}$, must in the long run lead to a real depreciation which is exactly large enough to make demand for home goods increase by in line with supply. A permanent decline in domestic demand has the same effect. If follows from the solution for the present real exchange rate in equation (7) that the immediate real exchange rate effect is the same as the permanent effect. This takes place through an immediate correction of the nominal exchange rate. There is no overshooting.

A permanent increase in the world interest rate $\bar{\rho} = \bar{\rho}_*$ also leads to a permanent appreciation of the real exchange rate according to (14). Again the immediate effect is equal to the permanent effect and it comes about through an immediate nominal appreciation.

6 A final comment

The model we have analyzed is highly simplified relative to the considerations that are necessary to make in actually pursuing an inflation target. Milton Friedman used to stress that macroeconomic policies work with long and variable lags. It is difficult to get the right result at the right time. An active policy to stabilize the economy can easily end up being destabilizing. This was one of the main reason that he advocated a mechanical rule, a constant growth rate for the money supply, instead of an active stabilization policy. His proposal has been discredited, but his warnings still deserve to be remembered.

First there is an observation lag. Statistics are collected after the fact, and are often substantially revised later on. Hence, policy makers do not have full knowledge of the present state of the economy. Second there is a decision lag. Time is needed to analyze the data and make decisions. The decision lag is usually shorter, though, for monetary policy than for fiscal policy. Then there are the lags from decision to impact. Whether they are variable, as Friedman claimed, or whether the problem is that our estimates of the lag lengths are poor, is hard to tell. Anyhow, before the full effect of a policy change has materialized the state of the economy can be different from what was expected.