Dynamics of Small Open Economies

Econ 4330 Open Economy Macroeconomics Spring 2010

Third lecture

Asbjørn Rødseth February 5 2010

- Small, open economy
- Infinite horizon
- Can borrow and lend abroad
- Can invest in real capital
- Given international real interest rate r constant
- A representative consumer
- Labor supply fixed

The budget equation for period s-1:

$$B_{s+1} - B_s = Y_s + rB_s - (C_s + G_s + I_s), \quad s = t, t+1, t+2, \ldots. \qquad (1)$$

- B_s is net foreign assets at end of period $s-1$

The present value budget constraint:

$$\sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} (C_s + I_s + G_s) \leq (1+r)B_t + \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} Y_s \qquad (2)$$

or, equivalently:

$$LIM = \lim_{T\to\infty} \left(\frac{1}{1+r}\right)^{T} B_{t+T+1} \geq 0 \qquad (3)$$

- PV of expenditures should not exceed initial wealth + PV of future output
- Foreign debt should grow slower than the interest rate

- With an infinite horizon debt can be rolled over forever as long as some of the interest is paid from present income.
- Utility maximization requires that the present value budget constraint is satisfied with equality
- The PV budget constraint presupposes that growth is not forever higher than the interest rate

The debt limit

$$-(1+r)B_t \leq \sum_{s=t}^{\infty} \left(\frac{1}{1+r}\right)^{s-t} [Y_s - (C_s + I_s + G_s)] \qquad (4)$$

- Future *trade* surpluses must be sufficiently large
- How large trade surpluses are achievable?

- Default risk may give rise to a lower debt limit
- Debt limits are on individual borrowers, not on nations

Debt to GDP	Interest rate	Growth rate	Required trade surplus to GDP
1	0.04	0.02	0.02
2	0.04	0.02	0.04
2	0.04	0.03	0.02
2	0.03	0.01	0.04

The model

Utililty function

$$U_t = u(C_t) + \beta u(C_{t+1}) + \beta^2 u(C_{t+2}) + \beta^3 u(C_{t+3}) + \cdots = \sum_{s=t}^{\infty} \beta^{s-t}\, u(C_s) \quad (5)$$

Production functions:

$$Y_t = A_t F(K_t), \qquad t = 1,2, \ldots \qquad (6)$$

Accumulation equations

$$K_t = K_{t-1} + I_{t-1}, \qquad t = 1,2, \ldots . \quad (7)$$

$$CA_t = B_{t+1} - B_t = rB_t + Y_t - C_t - I_t - G_t, \qquad t = 1,2, \ldots \ldots \qquad (8)$$

Optimization

Max $U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$ (9)

with respect to B_{s+1} and K_{s+1}, $s = t, t+1, \ldots$

given

$$C_s = (1+r)B_s - B_{s+1} + A_s F(K_s) - (K_{s+1} - K_s) - G_s \qquad s = t, t+1, \ldots. \quad (10)$$

and given K_t, B_t, *LIM=0.*

First order condition for B_{s+1}:

$$\frac{\partial U_t}{\partial B_{s+1}} = \beta^{s-t} u'(C_s) \frac{\partial C_s}{\partial B_{s+1}} + \beta^{s+1-t} u'(C_{s+1}) \frac{\partial C_{s+1}}{\partial B_{s+1}}$$
$$= \beta^{s-t} u'(C_s)(-1) + \beta^{s+1-t} u'(C_{s+1})(1+r) = 0$$

Hence, the consumption Euler equation

$$u'(C_s) = \beta(1+r)u'(C_{s+1}) \qquad (11)$$

First order condition for K_{s+1}:

$$\frac{\partial U_t}{\partial K_{s+1}} = \beta^{s-t} u'(C_s) \frac{\partial C_s}{\partial K_{s+1}} + \beta^{s+1-t} u'(C_{s+1}) \frac{\partial C_{s+1}}{\partial K_{s+1}}$$
$$= \beta^{s-t} u'(C_s)(-1) + \beta^{s+1-t} u'(C_{s+1})(A_{s+1}F'(K_{s+1}) + 1) = 0$$

or

$$\beta u'(C_{s+1})(A_{s+1}F'(K_{s+1}) + 1) = u'(C_s)$$

Or after taking account of the Euler equation

$$A_{s+1}F'(K_{s+1}) + 1 = \frac{u'(C_s)}{\beta u'(C_{s+1})} = 1 + r$$

and surprise!

$$A_{s+1}F'(K_{s+1}) = r \qquad (12)$$

$$\frac{\beta u'(C_{s+1})}{u'(C_s)} = \frac{1}{1+r} \quad (11)$$

The marginal rate of substitution between two subsequent periods should equal the discount rate

$$A_{s+1}F'(K_{s+1}) = r \quad (12)$$

On the margin the returns from investing in real capital at home and in financial assets abroad should be the same

Consumption and investment decisions can be separated

Time paths of C_s and K_s can be found from 1.o.conditions, initial conditions and PV budget constraint. Current account and foreign debt follows from accounting rel.

CES example again: Solving for C_t

$$u(C) = \frac{1}{1-\frac{1}{\sigma}} C^{1-\frac{1}{\sigma}} \qquad (13)$$

The Euler equation reduces to

$$C_{s+1} = [\beta(1+r)]^{\sigma} C_s = (1+v)C_s$$

where $v = [\beta(1+r)]^{\sigma} - 1$ is the growth rate of consumption.

Hence, $C_s = (1+v)^{s-t} C_t$,

Consumption grows if $\beta(1+r) > 1$

Is $v < r$? Yes, always when $\beta < 1$ and $\sigma \leq 1$ (and maybe even when $\sigma > 1$).

The present value of consumption is

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s = \sum_{s=t}^{\infty}\left(\frac{1+v}{1+r}\right)^{s-t} C_t = \frac{1}{1-\frac{1+v}{1+r}} C_t = \frac{1+r}{r-v} C_t \quad (14)$$

(Use formula for sum of infinite geometric series, $r > v$).

Recall the present value budget constraint

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} C_s = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} (Y_s - I_s - G_s) = W_t$$

W_t = total wealth (measured at the beginning of period t

Replacing the lhs by$(1+r)/(r-v)\ C_t$

from (14), we find that

$$C_t = \frac{r - v}{1 + r} W_t \qquad (15)$$

$v = 0$ Consume the permanent income from your total wealth.

$v > 0$ Consume less than your permanent income if you want a rising consumption path

Characterizing the solution for the current account

Define the *"permanent" value* $\tilde{X}_t$ of a variable X_s

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\tilde{X}_t = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$$

Using the formula for the sum of an infinite geometric series:

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} = \frac{1}{1-\frac{1}{1+r}} = \frac{1+r}{r}$$

Hence,

$$\frac{1+r}{r}\tilde{X}_t = \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s \quad \Leftrightarrow \quad \tilde{X}_t = \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}X_s$$

W_t can then be rewritten

$$W_t = (1+r)B_t + \sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}(Y_s - \mathrm{I_s} - G_s) = (1+r)B_t + \frac{1+r}{r}(\tilde{Y}_t - \tilde{I}_t - \tilde{G}_t)$$

Hence, the solution for C_t can be rewritten

$$C_t = \frac{r-v}{1+r}W_t = rB_t + \tilde{Y}_t - \tilde{I}_t - \tilde{G}_t - \frac{v}{1+r}W_t \qquad (16)$$

By definition

$$CA_t = rB_t + Y_t - C_t - I_t - G_t \qquad (17)$$

After inserting for C_t from (16)

$$CA_t = Y_t - \tilde{Y}_t - (I_t - \tilde{I}_t) - (G_t - \tilde{G}_t) + \frac{v}{1+r}W_t \qquad (18)$$

- Deviations between actual and permanent values of *Y, I and G.*
- Total wealth times growth factor (impatience versus interest rate)

A Stochastic Current Account Model

- Future levels of output, investment and government spending are stochastic

- Only financial asset is riskless bond which pays a constant interest rate *r*

- Rational expectations: Agent’s expectations are equal to the mathematical conditional expectations based on the economic model and all available information about current and past value of economic variables

- Current values of all exogenous variables are known by all decision makers before decisions are made

Want to look more closely at effect of income shocks

Optimization

Utility function

$$U_t = \mathbf{E}_{\mathrm{t}}\left\{\sum_{s=t}^{\infty} \beta^{s-t}\, u(C_s)\right\} \qquad (19)$$

Same budget equation and constraints, same initial conditions, same procedure.

First order condition with respect to B_{s+1} (compare slide 6):

$$\mathbf{E}_{\mathrm{t}}[\beta^{s-t}u'(C_s)(-1)+\beta^{s+1-t}u'(C_{s+1})(1+r)] = 0$$

or

$$\mathbf{E}_{\mathrm{t}}[u'(C_s)] = \mathbf{E}_{\mathrm{t}}[\beta(1+r)u'(C_{s+1})] \quad s = t, t+1, \ldots. \quad (20)$$

For *s=t* this specializes to

$$u'(C_t) = \mathbf{E}_{\mathrm{t}}[\beta(1+r)u'(C_{t+1})] \qquad (21)$$

First order condition with respect to K_{s+1} (compare slide 8):

$$\mathbf{E}_{\mathrm{t}}[\beta^{s-t}u'(C_s)(-1)+\beta^{s+1-t}u'(C_{s+1})(A_{s+1}F'(K_{s+1})+1)]=0$$

For *s*=*t* this specializes to

$$\mathbf{E}_{\mathrm{t}}\{\beta u'(C_{t+1})(A_{t+1}F'(K_{t+1})+1)\}=u'(C_t)$$

$$\mathbf{E}_{\mathrm{t}}\left\{\frac{\beta u'(C_{t+1})}{u'(C_t)}A_{t+1}F'(K_{t+1})\right\}+\mathbf{E}_{\mathrm{t}}\left\{\frac{\beta u'(C_{t+1})}{u'(C_t)}\right\}=1$$

Or, after inserting from the consumption Euler equation

$$\mathbf{E}_{\mathrm{t}}\left\{\frac{\beta(1+r)u'(C_{t+1})}{u'(C_t)}A_{t+1}F'(K_{t+1})\right\}=r \qquad (22)$$

The linear-quadratic example

Exogenous endowments (Y_t), no investment.

No trend growth in consumption: $\beta(1+r) = 1$

Quadratic utility function

$$u(C) = C - \frac{a_0}{2}C^2, \quad a_0 > 0 \qquad (23)$$

Euler equation $\mathbf{E}_{\mathrm{t}}[u'(C_s)] = \mathbf{E}_{\mathrm{t}}[\beta(1+r)u'(C_{s+1})]$

$$\mathbf{E}_{\mathrm{t}}[1 - a_0 C_s] = \mathbf{E}_{\mathrm{t}}[\beta(1+r)(1 - \mathrm{a}_0\mathrm{C}_{\mathrm{s+1}})]$$

$$1 - a_0\mathbf{E}_{\mathrm{t}}C_s = 1 - a_0\mathbf{E}_{\mathrm{t}}\mathrm{C}_{\mathrm{s+1}}$$

$$\mathbf{E}_{\mathrm{t}}C_{s+1} = \mathbf{E}_{\mathrm{t}}C_s \quad s = t, t+1, \ldots .(24)$$

For *s=1* we get Robert Hall's random walk result:

$$\mathbf{E}_{\mathrm{t}}C_{t+1} = C_t \quad (25)$$

Taking expectations on both sides of the budget constraint, we find

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathbf{E}_{\mathrm{t}}C_s=(1+r)B_t+\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathbf{E}_{\mathrm{t}}(Y_s-G_s)=W_t$$

Since $\mathbf{E}_{\mathrm{t}}C_{s+1}=C_t$ for all *s>t*, the lhs is (compare (14))

$$\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathbf{E}_{\mathrm{t}}C_s=\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}C_t=C_t\frac{1+r}{r}$$

Hence (compare (15))

$$C_t=\frac{r}{1+r}W_t=rB_t+\frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t}\mathbf{E}_{\mathrm{t}}(Y_s-G_s)\qquad(26)$$

Certainty equivalence: Act as if the expected values were certain to be realized.

Constraints and first-order conditions are linear in all the stochastic variables.

Necessary: Quadratic utility function (dubious) and non-stochastic *r*.

Response-impulse relations for output shocks

C_t is determined by $\mathbf{E}_t Y_s$, s=t+1,t+2,….How, are these expectations formed?

Example: Consumers believe income follows the stochastic process

$$Y_{s+1} - \bar{Y} = \rho(Y_s - \bar{Y}) + \varepsilon_{s+1} \qquad (27)$$

where $0 \leq \rho \leq 1$, $\mathbf{E}_t \varepsilon_s = 0$ for s=t+1, t+2,… ,and $\varepsilon_t$is serially uncorrelated.

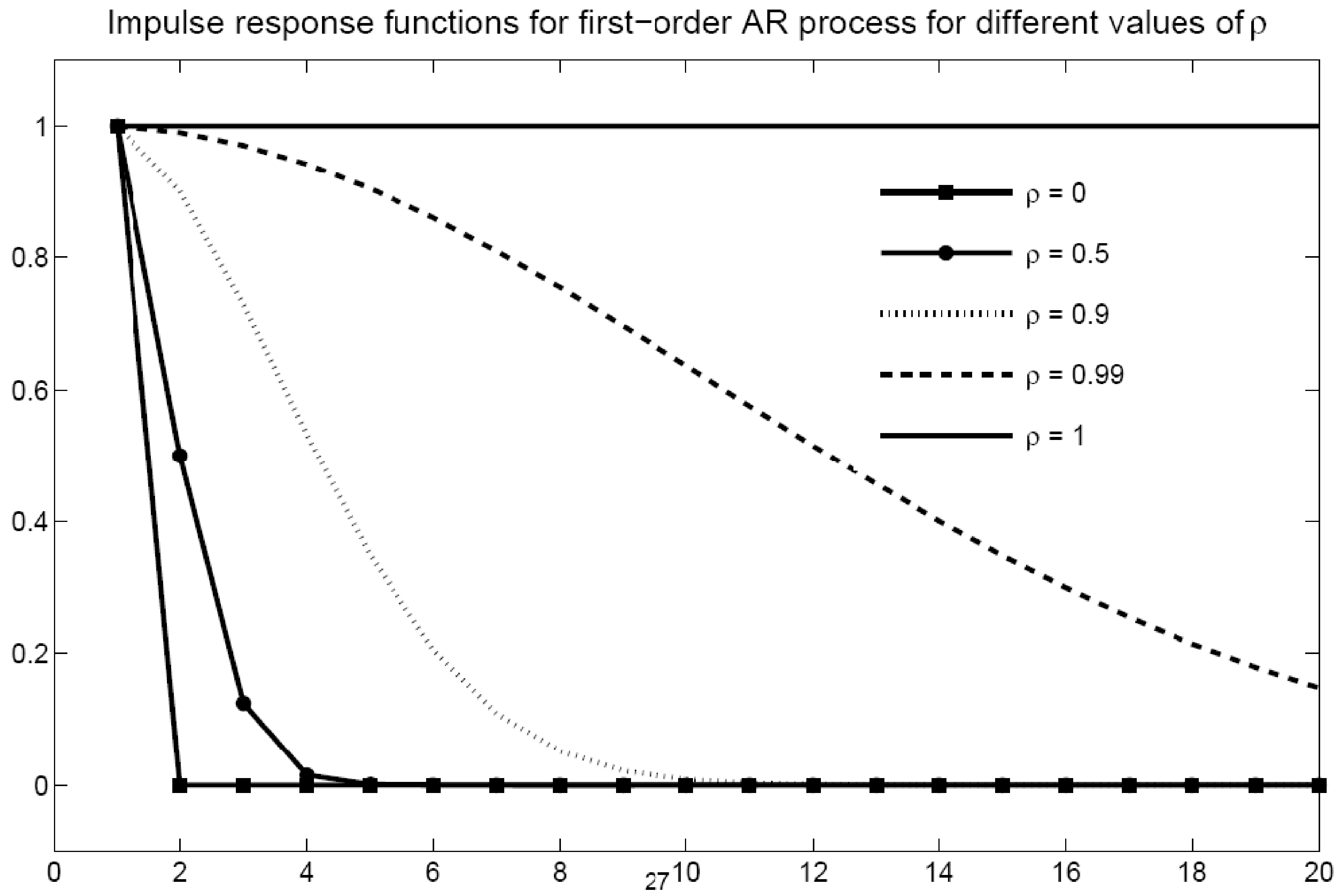
ρ is the coefficient of autocoregression.

ρ measures the degree of persistence of the process

$\rho = 0$ Y_s varies randomly around $\bar{Y}$. No serial correlation.

$0 < \rho < 1$ $Y_s$returns gradually towards $\bar{Y}$ after a shock. Positive serial corr.

$\rho = 1$ Y_s random walk, no tendency to return to $\bar{Y}$, $Y_{s+1} - Y_s = \varepsilon_{s+1}$


Impulse response functions for first−order AR process for different values of ρ
ρ = 0
ρ = 0.5
ρ = 0.9
ρ = 0.99
ρ = 1
1
0.8
0.6
0.4
0.2
0
0
2
4
6
8
10
12
14
16
18
20

27

By successive insertions in (27) we find (details on slide 25)

$$Y_s - \bar{Y} = \rho^{s-t}(Y_t - \bar{Y}) + \sum_{i=t+1}^{s} \rho^{i-t}\varepsilon_i \qquad (28)$$

Take expectations on both sides of (28):

$$\mathbf{E}_{\mathrm{t}}[Y_s - \bar{Y}] = \rho^{s-t}(Y_t - \bar{Y}) \qquad (29)$$

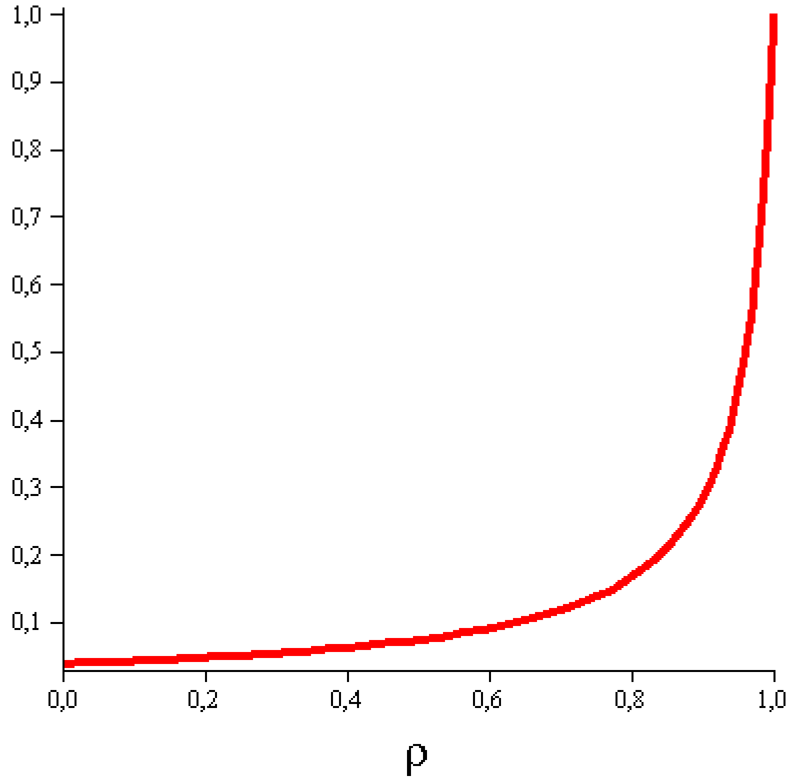
Insert the expectations from (29) in the consumption function (26) and you find (details on slide 26)

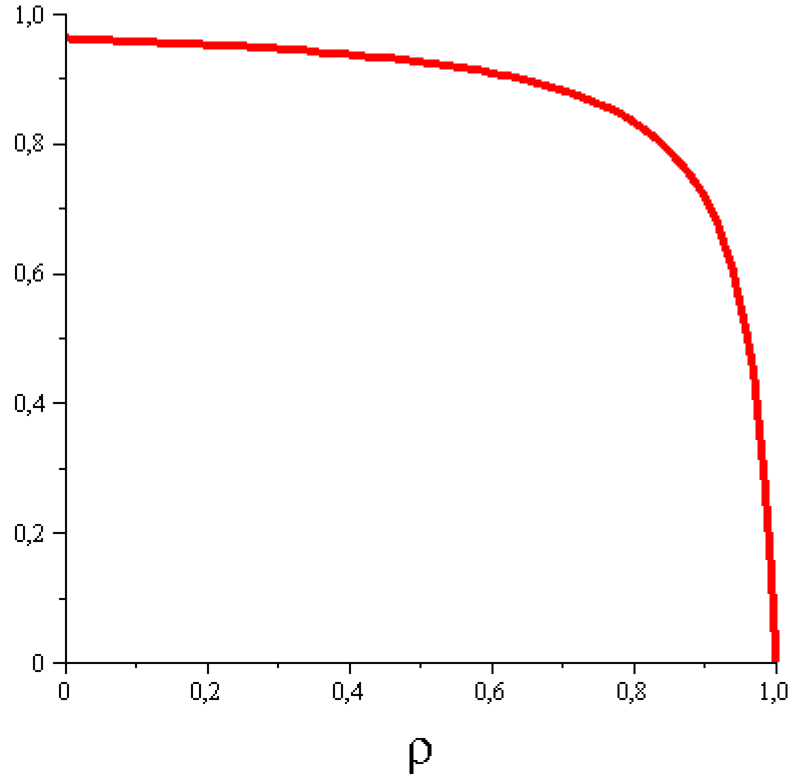
$$C_t = rB_t + \bar{Y} + \frac{r}{1+r-\rho}(Y_t - \bar{Y}) \qquad (30)$$

By definition $CA_t = rB_t + Y_t - C_t$. After inserting for C_t:

$$CA_t = \frac{1-\rho}{1+r-\rho}(Y_t - \bar{Y}) \qquad (31)$$

CA does not depend on B_t.





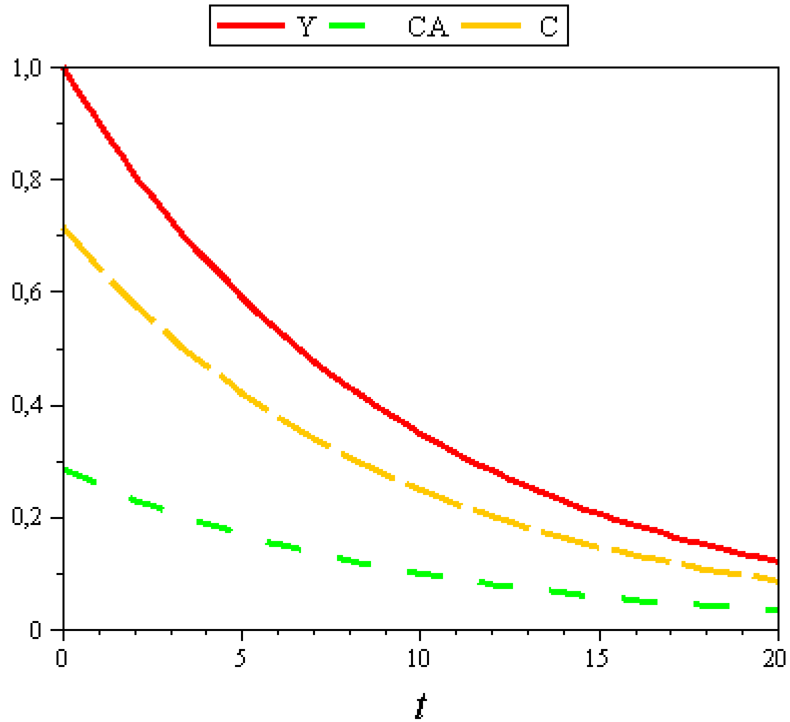
$r = 0.04$

Effect of Y_t on C_t. (MPC)

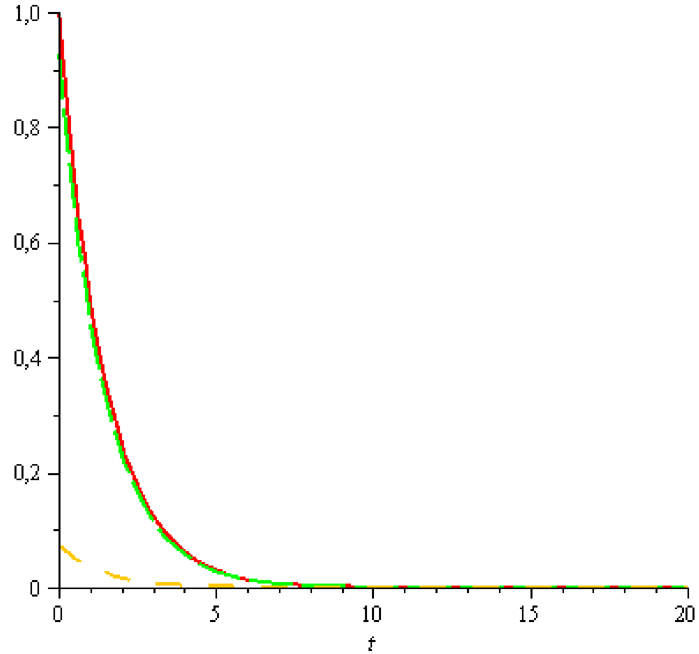
$$\frac{r}{1+r-\rho}$$

Effect of Y_t on CA_t

$$\frac{1-\rho}{1+r-\rho}$$



Implluse response ρ = 0.9, r = 0.04



Impulse response ρ = 0.5, r = 0.04

Derivation of (28)

Start from (27) with s=t:

$$Y_{t+1} - \bar{Y} = \rho(Y_t - \bar{Y}) + \varepsilon_{t+1}$$

Move forward 1 period:

$$Y_{t+2} - \bar{Y} = \rho(Y_{t+1} - \bar{Y}) + \varepsilon_{t+2}$$

Insert for $Y_{t+1} - \bar{Y}$:

$$Y_{t+2} - \bar{Y} = \rho^2(Y_t - \bar{Y}) + \rho\varepsilon_{t+1} + \varepsilon_{t+2}$$

Move forward 1 period:

$$Y_{t+3} - \bar{Y} = \rho(Y_{t+2} - \bar{Y}) + \varepsilon_{t+3}$$

Insert:

$$Y_{t+3} - \bar{Y} = \rho^3(Y_t - \bar{Y}) + \rho^2\varepsilon_{t+1} + \rho\varepsilon_{t+2} + \varepsilon_{t+3}$$

Continue until *s* and you end up with (28).

Derivation of (30)

Start with the consumption function

$$C_t = rB_t + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \mathbf{E}_{\mathrm{t}}Y_s$$

Add and subtract $\bar{Y}$.

$$C_t = rB_t + \bar{Y} + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} (\mathbf{E}_{\mathrm{t}}Y_s - \bar{Y})$$

Insert for the expectations from (29)

$$C_t = rB_t + \bar{Y} + \frac{r}{1+r}\sum_{s=t}^{\infty}\left(\frac{1}{1+r}\right)^{s-t} \rho^{s-t}(Y_t - \bar{Y})$$

Use the formula for the sum of an infinite geometric series

$$C_t = rB_t + \bar{Y} + \frac{r}{1+r-\rho}(Y_t - \bar{Y})$$