The monetary theory of exchange rate determination

Slides for Chapter 4 of Open Economy Macroeconomics

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Assumptions

Perfect mobility of goods; *purchasing power parity*, $P = EP_*$

Perfect mobility of capital; *interest rate parity*, $i = i_* + e_e$

Wage flexibility; output, Y, supply determined

Exogenous money supply, M

Model-consistent expectations, $e_e = \dot{E}/E$

The basic model

Money market equilibrium

$$\frac{M}{EP_*} = m(i, Y) \tag{1}$$

Foreign exchange market equilibrium

$$\frac{\dot{E}}{E} = i - i_* \tag{2}$$

Endogenous: E and i.
Exogenous: P_*, i_*, Y and M.

Solution

Solve (1) for i:

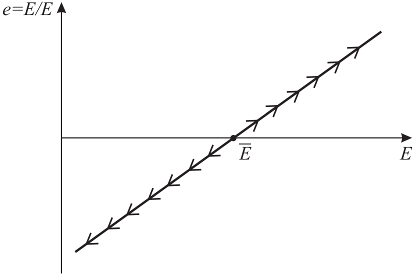
$$i = i\left(\frac{M}{EP_*}, Y\right) \qquad i_1 < 0, i_2 > 0$$

Insert in (2):

$$\frac{\dot{E}}{E} = i\left(\frac{M}{EP_*}, Y\right) - i_* \tag{3}$$

- One differential equation in $E(t)$
- Given an initial value for E, (3) determines the whole future path of E.
- E is free to jump at any time.
- How to determine the initial value of E?

Determining the initial value



$$\frac{\dot{E}}{E} = i\left(\frac{M}{EP_*}, Y\right) - i_*$$

$$\frac{d\dot{E}/E}{dE} = -i_1 \frac{M}{E^2 P_*} > 0$$

Equation unstable

Arbitrary starting point yields explosive path

Accelerating depreciation or appreciation

Choose the non-explosive path

Stationary point $\dot{E} = 0$

Solution: $i = i_*$

$E = \frac{M}{P_* m(i_*, Y)}$

Exchange rate proportional to quantity of money

Generalization

Paper money has zero intrinsic value

Its value depends entirely on beliefs

Usually there are several rational expectations paths for the value of money

Several self-fulfilling beliefs

If money supply is exogenous, there is at most one non-explosive rational expectations path for the value of money.

Assumption: Expectations coordinate on the non-explosive path

Confidence in the monetary system

The exogenous money supply acts as *nominal anchor*

A log-linear example

Money demand:

$$m - p = -\eta i + \kappa y \qquad (4)$$

Purchasing power parity

$$p = e + p_* \qquad (5)$$

Interest rate parity

$$\dot{e} = i - i_* \qquad (6)$$

$m = \ln M, p = \ln P$ etc

$\eta > 0,\ \kappa > 0$, constants

$\dot{e} = \dot{E}/E$ etc

Solution of the example

From money market equilibrium:

$$i = -(1/\eta)(m - p) + (\kappa/\eta)y$$

Insert this in $\dot{e} = i - i$:

$$\dot{e} = -(1/\eta)(m - e - p_*) + (\kappa/\eta)y \tag{7}$$

$$\dot{e} = (1/\eta)e - z \tag{8}$$

where

$$z = (1/\eta)(m - p_* - \kappa y) + i_*$$

First order linear differential equation

Sydsæter:

$$\dot{x} + a(t)x = b(t) \iff x = x(t_0)\mathrm{e}^{-\int_{t_0}^{t} a(\xi)d\xi} + \int_{t_0}^{t} b(\tau)\mathrm{e}^{-\int_{\tau}^{t} a(\xi)d\xi} d\tau$$

In our case $x = e$, $a(t) = -1/\eta$, $b(t) = -z(t)$ and

$$-\int_{\tau}^{t} a(\xi)d\xi = \frac{1}{\eta}(t - \tau)$$

Solution of the example, continued

$$\dot{e} = (1/\eta)e - z$$

has solution

$$e(t) = \left[e(t_0) - \int_{t_0}^{t} z(\tau)\mathrm{e}^{-(1/\eta)(\tau - t_0)} d\tau \right] \mathrm{e}^{(1/\eta)(t - t_0)}$$

Expression explodes unless

$$e(t_0) = \int_{t_0}^{\infty} z(\tau)\mathrm{e}^{-(1/\eta)(\tau - t_0)} d\tau$$

Choose this as the solution for all t:

$$e(t) = \int_{t}^{\infty} [(1/\eta)(m - p_* - \kappa y) + i_*]\mathrm{e}^{-(1/\eta)(\tau - t)} d\tau \qquad (9)$$

The exchange rate is determined by the whole future path of the money supply and the other exogenous variables.

More on the solution

$$e(t) = \int_t^{\infty} [(1/\eta)(m - p_* - \kappa y) + i_*] e^{-(1/\eta)(\tau - t)} d\tau$$

Note that

$$\int_t^{\infty} e^{-(1/\eta)(\tau - t)} d\tau = \eta$$

Hence, if the exogenous variables are constant

$$e(t) = m - p_* - \kappa y - \eta i_*$$

which is the log-linearized version the solution we found above.

Hyperinflation

Monetary model best when inflation very rapid

Hyperinflation: Inflation above 50 per cent per month

Possible causes:

1. Explosive growth in money supply
2. Expectations only

Cagan (1956)

- Studied 20th century European hyperinflations
- Caused by explosive growth in money supply
- Weak governments printing money to finance deficits

Hyper*de*flation?

Never observed

Nominal interest rate cannot be negative

No rational expectations path with $\dot{e} < -i_*$

(Perfect capital mobility)

With $i = 0$ money and bonds become equivalent

Cases where a nominal anchor is missing

1. Exogenous interest rate

$\dot{e} = i - i_*$. No way to pin down the *level* of the exchange rate.

2. "Inflation targeting"

$$i = i_* + (\bar{\pi} - \dot{p}_*) + \phi(\dot{p} - \bar{\pi})$$

$\bar{\pi}$ = inflation target, $\phi > 1$

$i_* + (\bar{\pi} - \dot{p}_*)$ = normal interest rate

$$\dot{e} = i - i_* = i_* + (\bar{\pi} - \dot{p}_*) + \phi(\dot{e} + \dot{p}_* - \bar{\pi}) - i_*$$

$$(1 - \phi)\dot{e} = (1 - \phi)(\bar{\pi} - \dot{p}_*)$$

$$\dot{e} = \bar{\pi} - \dot{p}_*$$

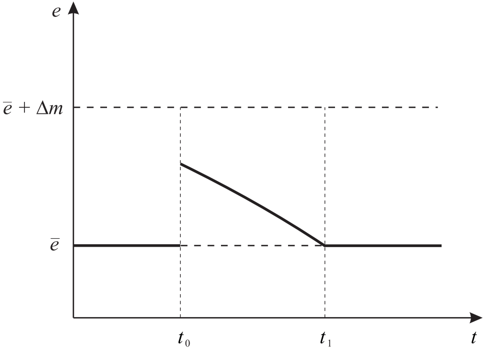
No way to pin down the *level* of the exchange rate.

Temporary and permanent changes and news

Four useful principles

- The exchange rate is forward looking
- Begin with the distant future and look backwards
- There are never expected jumps in the exchange rate
- The exchange rate jumps on news

An unexpected, temporary increase in the money supply

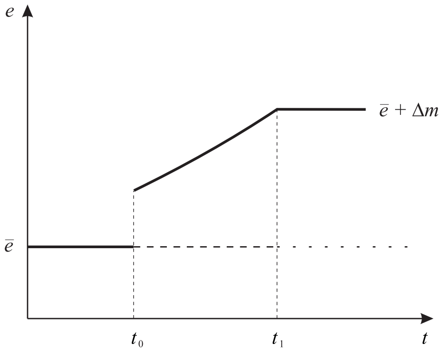


Money supply increased by Δm at t_0, reversed at t_1.

Regressive expectations
Interest rate down at t_0

$$e(t) = \int_t^{\infty} [\frac{1}{\eta}(m - p_* - \kappa y) + i_*] e^{-(1/\eta)(\tau - t)} d\tau$$

A preannounced permanent increase in the money supply



At t_0 it is announced that m will increase by Δm from t_1.

Initial depreciation followed by more.

Interest rate up at t_0

$$e(t) = \int_t^{\infty} [\frac{1}{\eta}(m - p_* - \kappa y) + i_*] e^{-(1/\eta)(\tau - t)} d\tau$$

Using the term structure to derive exchange rate expectations

Suppose

Perfect capital mobility.

Term structure is equal to expected future interest rates

Sine $\dot{e} = i - i_*$, if everything works out as expected,

$$e(t) - e_{t_0} = \int_{t_0}^{t} \dot{e} d\tau = \int_{t_0}^{t} (i - i_*) d\tau \tag{10}$$

The markets expectation at t_0 of $e(t)$ can be calculated from the term structure of interest rates