

# Exchange rate determination under inflation targeting

## Econ 4330 Lecture 9

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# Inflation targeting

Welfare the ultimate goal

Inflation closer to the ultimate goal than money supply or exchange rate

Inflation also further from the instruments of monetary policy

Strict inflation targeting - focus on inflation rate alone

Flexible inflation targeting - allowing for other goals as long as inflation is reasonably stable around a target

Not a uniquely defined policy, but a broad category / framework

# Modeling inflation target regimes

## Two main approaches

- 1 Central banks as optimizing agents
- 2 Central bank reaction functions (e.g. Taylor rule)

Lag structure important

Discrete time

Simulation models

Gross simplifications below

# The core of the model

IS-curve

Interest parity condition

Phillips-curve

Interest setting equation

Home and foreign goods

Interest rate parity except for stochastic disturbance

Focus on real exchange rates, real interest rates

Return to nominal rates later

Inflation target assumed to be credible

Key variables (in logs):

$\pi_{t+1}$  rate of inflation from period  $t$  to  $t + 1$

$y_t$  output

$r_t$  real exchange rate

$\rho_t$  real interest rate from period  $t$  to  $t + 1$

$\rho_{*,t}$  foreign real interest rate

Values of these variables in a stationary state are:  $\bar{\pi}$ ,  $\bar{y}$ ,  $\bar{r}$ ,  $\bar{\rho}$ ,  $\bar{\rho}_*$

$\bar{y}$  = capacity output,  $\bar{\pi}$  = inflation target

Due to capital mobility:  $\bar{\rho} = \bar{\rho}_*$

# IS-curve

Demand relation

$$y_t = \alpha_0 - \alpha_1 \rho_t + \alpha_2 r_t + u_{y,t}$$

$u_{y,t}$  = demand shock,  $\alpha_1 > 0$ ,  $\alpha_2 > 0$

Long run equilibrium

$$\bar{y} = \alpha_0 - \alpha_1 \bar{\rho} + \alpha_2 \bar{r}$$

Difference yields:

$$y_t - \bar{y} = -\alpha_1(\rho_t - \bar{\rho}) + \alpha_2(r_t - \bar{r}) + u_{y,t} \quad (1)$$

Note that  $\bar{r}$ , is determined by  $\bar{y}$  and  $\bar{\rho}$ .

## Real interest rate parity

$$\mathbf{E}_t r_{t+1} - \bar{r} = (r_t - \bar{r}) + (\rho_t - \bar{\rho}) - (\rho_{*,t} - \bar{\rho}) + u_{e,t} \quad (2)$$

$u_{e,t}$  = risk premium shock

Since  $r_t = e_t + \rho_{*,t} - \rho_t$ ,

$$\begin{aligned} \mathbf{E}_t r_{t+1} - r_t &= \mathbf{E}_t (e_{t+1} + \rho_{*,t+1} - \rho_{t+1}) - (e_t + \rho_{*,t} - \rho_t) \\ &= (\mathbf{E}_t e_{t+1} - e_t) + (\mathbf{E}_t \rho_{*,t+1} - \rho_{*,t}) - (\mathbf{E}_t \rho_{t+1} - \rho_t) \\ &= i_t - i_{*,t} + u_{e,t} + (\mathbf{E}_t \rho_{*,t+1} - \rho_{*,t}) - (\mathbf{E}_t \rho_{t+1} - \rho_t) \\ &= \rho_t - \rho_{*,t} + u_{e,t} \end{aligned}$$

## Phillips curve

$$\pi_{t+1} = \bar{\pi} + \beta(y_t - \bar{y}) + u_{\pi,t+1} \quad (3)$$

$u_{\pi,t}$  = cost-push shock,  $\beta > 0$

Expected inflation equal to the target  
Slightly inconsistent, but simplifies



## Interest rate setting

$$\rho_t = \bar{\rho} + \phi(\mathbf{E}_t\pi_{t+1} - \bar{\pi}) \quad (4)$$

$\phi > 0$  Real interest rate is increased when inflation exceeds target

$$i_t = \rho_t + \mathbf{E}_t\pi_{t+1} = \bar{\rho} + \bar{\pi} + (1 + \phi)(\mathbf{E}_t\pi_{t+1} - \bar{\pi})$$

Resembles Taylor-rule, but

- Forward looking interest rate setting

- Output gap left out (simplifies)

- Import prices left out (simplifies)

- Empirical reaction functions more complex

# The exogenous shocks

$$u_{y,t}, u_{\pi,t}, u_{e,t}, \rho_{*,t} - \bar{\rho}_* = u_{\rho_{*},t}$$

Assume:

Unconditional expectation of shocks is zero

Shocks are stationary (constant variance, not random walk)

Contemporary shocks are observed by central bank

## The ISFX-curve

IS: Equilibrium in goods market (1)

FX: Equilibrium in foreign exchange market (2)

$$y_t - \bar{y} = -\alpha_1(\rho_t - \bar{\rho}) + \alpha_2(r_t - \bar{r}) + u_{y,t}$$

Insert for  $r_t - \bar{r}$  from RIP:

$$r_t - \bar{r} = (\mathbf{E}_t r_{t+1} - \bar{r}) - (\rho_t - \bar{\rho}) + u_{\rho^*,t} - u_{e,t}$$

$$\begin{aligned} y_t - \bar{y} = & - (\alpha_1 + \alpha_2)(\rho_t - \bar{\rho}) \\ & + \alpha_2[(\mathbf{E}_t r_{t+1} - \bar{r}) + u_{\rho^*,t} - u_{e,t}] + u_{y,t} \end{aligned} \quad (5)$$

- ▶ Two channels for monetary policy: Interest rate  $\alpha_1$ , exchange rate  $\alpha_2$
- ▶ Expected future exchange rate taken as given

## The MPI-curve

Relationship between  $\rho$  and  $y$  that follows from Phillips-curve (3) and the monetary policy rule (4)

Take expectation of the Phillips-curve (3):

$$\mathbf{E}_t \pi_{t+1} - \bar{\pi} = \beta(y_t - \bar{y}) + \mathbf{E}_t u_{\pi,t+1}$$

Insert in monetary policy rule (4):

$$\rho_t - \bar{\rho} = \phi\beta(y_t - \bar{y}) + \phi\mathbf{E}_t u_{\pi,t+1} \quad (6)$$

## Equilibrium for given exchange rate expectations

Solution to (5) and (6)

$$y_t - \bar{y} = \mu \left[ u_{y,t} + \alpha_2 (\mathbf{E}_t r_{t+1} - \bar{r} + u_{\rho^*,t} - u_{e,t}) - (\alpha_1 + \alpha_2) \phi \mathbf{E}_t u_{\pi,t+1} \right] \quad (7)$$

Multiplier for real demand shock:

$$\mu = \frac{1}{1 + (\alpha_1 + \alpha_2) \phi \beta} < 1$$

- ▶ Positive real demand shock
- ▶ Higher output, higher inflation ( $\beta$ )
- ▶ CB raises real interest rate ( $\phi$ )
- ▶ This dampens the increase in output ( $\alpha_1 + \alpha_2$ )

## The real interest rate

Insert from (7) in MPI-curve (6):

$$\rho_t - \bar{\rho} = \mu\phi\beta [u_{y,t} + \alpha_2(\mathbf{E}_t r_{t+1} - \bar{r} + u_{\rho^*,t} - u_{e,t})] + \mu\phi\mathbf{E}_t u_{\pi,t+1} \quad (8)$$

Real interest rate goes up in response to

- ▷ Expected depreciation next period
- ▷ Increase in foreign interest rate this period
- ▷ A negative risk premium shock
- ▷ A positive costs push shock

## The real exchange rate

Insert the real interest rate from (8) in the real interest rate parity condition:

$$r_t - \bar{r} = \frac{1 + \alpha_1 \phi \beta}{1 + (\alpha_1 + \alpha_2) \phi \beta} (\mathbf{E}_t r_{t+1} - \bar{r} + u_{\rho^*, t} - u_{e, t}) - \frac{1}{1 + (\alpha_1 + \alpha_2) \phi \beta} (\phi \beta u_{y, t} - \phi \mathbf{E}_t u_{\pi, t+1}) \quad (9)$$

Expected exchange rate affects present less than 1 : 1

Dampened by the increase in the real interest rate

$$\frac{1 + \alpha_1 \phi \beta}{1 + (\alpha_1 + \alpha_2) \phi \beta} = 1 - \frac{1 + \alpha_2 \phi \beta}{1 + (\alpha_1 + \alpha_2) \phi \beta}$$

## Determining exchange rate expectations

Turn (9) around:

$$\begin{aligned}\mathbf{E}_t r_{t+1} - \bar{r} &= (r_t - \bar{r}) + \frac{\phi\beta\alpha_2}{1 + \phi\beta\alpha_1}(r_t - \bar{r}) \\ &+ \frac{1}{1 + \phi\beta\alpha_1}(\phi\beta u_{y,t} + \phi\mathbf{E}_t u_{\pi,t+1}) - u_{\rho^*,t} + u_{e,t}\end{aligned}$$

or

$$\mathbf{E}_t r_{t+1} - \bar{r} = \epsilon(r_t - \bar{r}) - z_t$$

where

$$\epsilon = \frac{1 + \phi\beta(\alpha_1 + \alpha_2)}{1 + \phi\beta\alpha_1} > 1$$

and

$$z_t = -\frac{1}{1 + \phi\beta\alpha_1}(\phi\beta u_{y,t} + \phi\mathbf{E}_t u_{\pi,t+1}) + u_{\rho^*,t} - u_{e,t}$$



$$\mathbf{E}_t r_{t+1} - \bar{r} = \epsilon(r_t - \bar{r}) - z_t \quad (10)$$

Depreciated real exchange rate

- ▷ Demand pressure, inflationary pressure
- ▷ High real interest rate
- ▷ Further real depreciation
- ▷ Instability

Confidence in CBs ability to reach inflation target leads to choice of saddle path leading to stationary state.

## Deriving the saddle path

From forwarding the equation for the real exchange rate (10):

$$\mathbf{E}_t r_{t+s+1} - \bar{r} = \epsilon(\mathbf{E}_t r_{t+s} - \bar{r}) - \mathbf{E}_t z_{t+s}, \quad s = 0, 1, 2, \dots \quad (11)$$

Solution:

$$r_t - \bar{r} = \frac{1}{\epsilon} \sum_{i=0}^{\infty} \epsilon^{-i} \mathbf{E}_t z_{t+i} \quad (12)$$

## Proof

Simplify the expressions by temporarily leaving out the expectations operator:

$$r_{t+s+1} - \bar{r} = \epsilon(r_{t+s} - \bar{r}) - z_{t+s},$$

Iterate from  $s = 0$ :

$$r_{t+1} - \bar{r} = \epsilon(r_t - \bar{r}) - z_t$$

$$r_{t+2} - \bar{r} = \epsilon(r_{t+1} - \bar{r}) - z_{t+1} = \epsilon^2(r_t - \bar{r}) - \epsilon z_t - z_{t+1}$$

$$r_{t+3} - \bar{r} = \epsilon(r_{t+2} - \bar{r}) - z_{t+2} = \epsilon^3(r_t - \bar{r}) - \epsilon^2 z_t - \epsilon z_{t+1} - z_{t+2}$$

By continuing

$$r_{t+s+1} - \bar{r} = \epsilon^{s+1}(r_t - \bar{r}) - \sum_{i=0}^s \epsilon^{s-i} z_{t+i}$$

## Proof, continued

$$r_{t+s+1} - \bar{r} = \epsilon^s \left[ \epsilon(r_t - \bar{r}) - \sum_{i=0}^s \epsilon^{-i} z_{t+i} \right]$$

First term goes to infinity with  $s$ , second term has to go to zero, which requires:

$$r_t - \bar{r} = \frac{1}{\epsilon} \sum_{i=0}^{\infty} \epsilon^{-i} z_{t+i}$$

Bring back expectations, and you have (12).

# Solution for the remaining variables

## Real variables and expected inflation

With  $r_t$  given,  $\rho_t$  follows from (5).

$y_t$  is found by inserting for  $r_t$  and  $\rho_t$  in IS-curve (1)

$\mathbf{E}_t\pi_{t+1}$  is found from Phillips-curve (3)

## The nominal exchange rate

$$e_t = r_t + p_t - p_{*,t}$$

$p_t$  predetermined,  $p_{*,t}$  exogenous.

Cost-push shock  $u_{\pi,t}$  raises  $p_t$  and, hence,  $e_t$  one for one, while  $r_t$  is unaffected.

# Effects of transitory shocks

## Assume

Shocks are independently distributed from period to period.

CB has no advance information on cost push or other shocks.

Then the solution (12) for  $r_t$  is reduced to

$$r_t - \bar{r} = \frac{1}{\epsilon} z_t = \frac{1}{\epsilon} \left[ \frac{-\phi\beta}{1 + \phi\beta\alpha_1} u_{y,t} + u_{\rho^*,t} - u_{e,t} \right]$$

All later terms in sum drop out

Supply shock drops out, since it will be unexpected

$$r_t - \bar{r} = -\frac{\phi\beta}{1 + \phi\beta(\alpha_1 + \alpha_2)} u_{y,t} + \frac{1 + \phi\beta\alpha_1}{1 + \phi\beta(\alpha_1 + \alpha_2)} [u_{\rho^*,t} - u_{e,t}] \quad (13)$$

$$\rho_t - \bar{\rho} = \frac{\phi\beta}{1 + \phi\beta(\alpha_1 + \alpha_2)} u_{y,t} + \frac{\phi\beta\alpha_2}{1 + \phi\beta(\alpha_1 + \alpha_2)} [u_{\rho^*,t} - u_{e,t}] \quad (14)$$

$$y_t - \bar{y} = \frac{1}{1 + \phi\beta(\alpha_1 + \alpha_2)} u_{y,t} + \frac{\alpha_2}{1 + \phi\beta(\alpha_1 + \alpha_2)} [u_{\rho^*,t} - u_{e,t}] \quad (15)$$

- ▷ Returns to full equilibrium next period
- ▷ Same multipliers as with given expectations

## Summary: Effect of shock confined to period $t$

Positive demand shock  $u_{y,t} > 0$

Appreciates the exchange rate (real and nominal)

Raises the real interest rate

Raises output

Raises expected inflation

A more vigorous anti-inflation policy (higher  $\phi$ )

reinforces the effect on the real interest rate and the real exchange rate

dampens the effect on output and inflation



## Effect of transitory shocks confined to period $t$

Positive shock in the foreign interest rate  $\rho_* - \bar{\rho} > 0$

Depreciates the exchange rate (real and nominal)

Raises the real interest rate, but by less than the foreign real interest rate

Raises output

Raises expected inflation

A more vigorous anti-inflation policy (higher  $\phi$ )

dampens the effect on output and inflation

reinforce the effect on the real interest rate

dampens the effect on the realexchange rate

## An expected one-time cost-push shock

Appreciates the exchange rate (real and nominal)

Raises the real interest rate

Lowers output

Raises expected inflation

## A more vigorous anti-inflation policy (higher $\phi$ )

Increases the effect on the real interest rate, the exchange rate *and output*

Reduces the effect on expected inflation

## Milton Friedman's warning

Long and variable lags

Observation lag

Decision lag

Impact lag