Partial solution to ECON4330, seminar 1, exercise B, spring 2010
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1 Explain the meaning of the budget constraint

See Lecture notes to second lecture, slides 21 - 23 for derivation and explanation of the budget constraint.

2 Derive the first order conditions

In general, the way to approach these problems is to reduce the system to as few equations and endogenous variables as possible, and then optimize. (You could also use Lagrangian methods here)

The candidates are physical capital K, investment I and foreign assets B (not debt as it says in the exercise; $-B$ is debt). I choose to eliminate I, but any of these would be fine.

I get the expression of consumption from (E.4) (I'll use E. to refer to equation numbers in the exercise)

$$C_t = (1+r)B_t + Y_t - I_t - B_{t+1} \tag{1}$$

I can then insert for Y from (E.2) and for I from (E.3) to get

$$C_t = (1+r)B_t + A_tF(K_t) + K_t - K_{t+1} - B_{t+1} \tag{2}$$

This is straigtforward to interpret: your "income" is your foreign assets B (plus the interest you gain this period), your production income Y and your capital stock K. After you deduct next period's investment, you consume what's left.

Insert for C (now indexed by "periods forward from now", s) in (E.1) to get

$$U_t = \sum_{s=t}^{\infty} \beta^{s-t} u((1+r)B_s + A_sF(K_s) + K_s - K_{s+1} - B_{s+1}) \tag{3}$$

If this looks complex, we can instead (at least for our own explanation) write it as

$$\begin{aligned} U_t = {} & u((1+r)B_t + A_tF(K_t) + K_t - K_{t+1} - B_{t+1}) \\ & + \beta u((1+r)B_{t+1} + A_{t+1}F(K_{t+1}) + K_{t+1} - K_{t+2} - B_{t+2}) \\ & + \beta^2 u((1+r)B_{t+2} + A_{t+2}F(K_{t+2}) + K_{t+2} - K_{t+3} - B_{t+3}) \\ & + \beta^3 u((1+r)B_{t+3} + A_{t+3}F(K_{t+3}) + K_{t+3} - K_{t+4} - B_{t+4}) \\ & + ... \end{aligned}$$

We then pick the series of next period capital, K_{s+1} and next period assets, B_{s+1}, from now until infinity (that is, for s from t to ∞):

$$\max_{\{K_{s+1},B_{s+1}\}_{s=t}^{\infty}} \sum_{s=t}^{\infty} \beta^{s-t} u((1+r)B_s + A_s F(K_s) + K_s - K_{s+1} - B_{s+1}) \tag{4}$$

and take the first order conditions, for B_{s+1}:

$$-\beta^{s-1}\frac{\partial u}{\partial C_{t+s-1}} + \beta^s \frac{\partial u}{\partial C_{t+s}}(1+r) = 0 \tag{5}$$

$$\beta(1+r) = \frac{\frac{\partial u}{\partial C_{t+s-1}}}{\frac{\partial u}{\partial C_{t+s}}} \tag{6}$$

$$\beta(1+r) = \frac{\frac{\partial u}{\partial C_{t+s}}}{\frac{\partial u}{\partial C_{t+s+1}}} \tag{7}$$

$$(1+r) = \frac{\frac{\partial u}{\partial C_{t+s}}}{\beta\frac{\partial u}{\partial C_{t+s+1}}} \tag{8}$$

Equivalently, for K_{s+1}:

$$-\beta^{s-1}\frac{\partial u}{\partial C_{t+s-1}} + \beta^s \frac{\partial u}{\partial C_{t+s}}(1 + A_{t+s}\frac{\partial F}{\partial K_{t+s}}) = 0 \tag{9}$$

$$\beta(1 + A_{t+s}\frac{\partial F}{\partial K_{t+s}}) = \frac{\frac{\partial u}{\partial C_{t+s-1}}}{\frac{\partial u}{\partial C_{t+s}}} \tag{10}$$

$$\beta(1 + A_{t+s+1}\frac{\partial F}{\partial K_{t+s+1}}) = \frac{\frac{\partial u}{\partial C_{t+s}}}{\frac{\partial u}{\partial C_{t+s+1}}} \tag{11}$$

These can then be combined to get the familiar

$$A_s\frac{\partial F}{\partial K_s} = r \tag{12}$$

Equation (8) says that the marginal utility of consumption across periods must equal the market return across periods. Equation (12) says that the return to financial saving should be equal to the market return across periods. As we have assumed that the market return is given from the international capital market, the decisions can be solved independently of each other.

3 Suppose that...

If $\beta(1+r) = 1$, we get from Equation (7) that $\frac{\partial u}{\partial C_{t+s}} = \frac{\partial u}{\partial C_{t+s+1}}$. This means that consumption must be constant. From Equation (12) we see that capital must be constant as well. What about foreign assets? From the two conditions above, we could imagine a situation where we borrow, say, \$100 000 each year. That would keep consumption constant. But due to the borrowing/saving constraint (E.5), this is not allowed. Hence, any B path has to be constant for C and K to be constant, and we have a current account deficit/surplus of 0.

4 Time paths

I will do this "variable-by-variable". I will use as a benchmark a situation where $\beta(1+r) = 1$, so the path of all variables is constant when there is no shock.

Notation: $AF(K)$ becomes $A^H F(K)$ for the duration of the shock. I denote the corresponding higher value of Y as Y^H. Note that the size of $Y^H - Y$ varies with scenario (see below).

Consumption:

Situation a:

- In period t we know nothing. Hence, no change. Also, no extra capital in period $t+1$ because we do not know beforehand that things will be different.
- In period $t+1$ we realize we got richer. We want to spread this utility gain across all periods, and because $\beta(1+r) = 1$, we want to do so perfectly smoothly. Hence, we consume the return to the shock, or $r(Y^H - Y)$ more in all periods starting with $t+1$.

Situation b:

- In period t we realize we can gain from the increased productivity by increasing next period's capital stock. Hence, the overall income gain from the shock is higher in this scenario.
- In period t we realize we will get richer next period, and we still want to spread this utility gain across each period. We hence start immediately, and consume a bit less than $r(Y^H - Y)$ more in all periods starting with t

Situation c:

- In period $t+1$ we realize the shock. Since we know it's permanent there will be some capital adjustment and the increase will be higher in period $t+2$ and onwards than in $t+1$. We smooth this (because $\beta(1+r) = 1$ we want our consumption path to be perfectly smooth) and hence consume a bit more than $Y^H_{t+1} - Y$ and a bit less than $Y^H_{t+2} - Y$ forever.

Investment: (it makes sense to do investment before current account). Remember that investment is just the change in K from one period to the next.

Situation a:

- No change. The shock is temporary and we do not have time to adjust our capital before we learn of the shock.

Situation b:

- Higher capital level (positive investment) in period t in anticipation of next period's shock. See equation (12), the RHS is the same but A on the LHS has gone up, for the equality to hold $\frac{\partial F}{\partial K}$ must go down, meaning K should go up. But in period $t+2$ productivity will be back to normal, hence capital stock should go back to normal, and negative investment in period $t+1$ equals the positive investment made in period t.

Situation c:

- As soon as we learn of the productivity increase, we will adjust to it, following equation (12). Hence, we will have the same positive investment as in situation b but no corresponding negative investment.

Current account:
Assuming no initial debt, the expression for net assets B is $Y_t - C_t - I_t$.
Situation a:

- There is never any change in investment.
- In period t we know nothing. No change.
- In period $t+1$ and onwards consumption increases. However, the entire income gain came in period $t+1$. It follows that the income gain is saved, and we get an increase in B. This is equal (from the point of view of the country) to storing the entire income gain in the bank (the bank being the international capital market), only consuming the interest of this wealth forever.
- Conclusion: Positive current account in $t+1$, zero other times

Situation b:

- In period t we increase both consumption and investment without having gotten the income gain yet. This means borrowing against the international market. B decreases.
- In period $t+1$ we get the income gain. We also do the disinvestment. Both of these help "paying off" the debt from period 1. As in situation a, the remaining balance is put "in the bank" and consumed forever. There is an increase in the current account.
- Conclusion: Negative current account in period t, positive in period $t+1$

Situation c:

- In period t nothing happens.
- In period $t+1$ we get the income increase, and consume it (and even a bit more, as we know there will be one more income increase next period, as we increase investment). We also invest. Both of these means a lowering of B.
- In period $t+2$ and onwards we perpetually consume a bit less than we produce, because the "initial splurge" in period $t+1$ is repaid. In addition, the investment is never reversed, and the investment was a movement of wealth from the foreign to the domestic capital market. This means that B is lower forever.
- Conclusion: Negative current account in period $t+1$, positive in $t+2$ (but not as large as the previous one in absolute value)