ECON4330: Questions for seminar 2, week 10

1 Overlapping generations

1. Consumers live for two periods. The utility function of a representative consumer who is young in period t is

$$U = \ln(c_t^Y) + \beta \ln(c_{t+1}^O) \tag{1}$$

where c_t^Y and c_{t+1}^O are his levels of consumption when young and old respectively. The consumer supplies one unit of labor when young and 0.5 units of labor when old. The wage per unit of labor in period t is w_t. The consumer pays lump-sum taxes τ_t^Y when young and τ_{t+1}^O when old. The consumer starts life without any financial assets and leaves no bequests. He can borrow and lend at a given real interest rate r.

Write down the life-time budget constraint of the consumer in present values terms and solve for his consumption levels in the two periods.

2. Compare the effects on the savings a young consumer, s_t^Y, of a tax increase when he is young and an expected tax increase when he is old.

3. The consumers live in a small country with free access to international capital markets. The size of the population and the level of productivity is constant. Explain why in this case the capital stock and the wage rate will both be constant and unaffected by the taxes the consumers pay. Suppose that taxes are constant ($\tau_t^Y = \tau^Y$ and $\tau_t^O = \tau^O$)and that the government budget is balanced. What will be the level of the country's current account balance?

In the sequel the size of a generation is normalized to 1.

4. The net foreign assets of the country at the end of period t can be expressed as

$$B_{t+1} = s_t^Y - D_{t+1} - K \tag{2}$$

where D_{t+1} is the government debt at the end of period t and K is the constant level of the capital stock. Suppose that at the beginning of period t the government decides to introduce a tax reform which moves some of the tax burden from young to old ($\Delta\tau_t^Y = -\Delta\tau_t^O$). How does this affect:

   (a) The saving of the old in period t

   (b) The saving of the young in period t

(c) National saving in period t

(d) The foreign debt at the end of period t

5. We now want to compare alternative stationary states where the current account and the government budget is balanced and where taxes and government expenditures are constant (at levels τ^0, τ^Y and G respectively). Balancing of the government budget requires that

$$\tau^Y + \tau^0 = G + rD \tag{3}$$

where D is the stationary level of the government debt. What does a higher level of government debt, D, mean for the level of net foreign assets, B, when higher interest payments on the government debt are financed by higher taxes on the young. Explain in words what happens.

2 Determination of the world interest rate.

In this exercise we want to compare the steady-state interest rates in the life-cycle model (OR Ch 3) and the infinite horizon model (OR Ch 2). In both cases we assume that all countries have the same logarithmic preferences with subjective discount rates β and the same growth rates of labor productivity g and population n. Countries also have the same size and the same productivity levels and the same Cobb-Douglas production function with capital's share equal to α. In the life-cycle model we assume that individuals work only when young. In the infinite horizon model we assume that all generations have equal weight in the utility function independent of their size.

It can be shown that the two models lead to the following formulas for the steady-state interest rates:

The life-cycle model:

$$\bar{r} = \frac{\alpha(1+\beta)(1+n)(1+g)}{\beta(1-\alpha)} \tag{1}$$

The infinite horizon model:

$$\bar{r} = \frac{1+g-\beta}{\beta} \tag{2}$$

Compare the two solutions for the interest rates. Try to give intuitive reasons for why they differ. (You may also want to convince yourself that you are able to the expressions).

3 Explaining current account imbalances

Discuss the relative merits of the life-cycle model and the infinite horizon model in explaining current account imbalances.