## ECON4330: Questions for seminar 1

## Part A: The current account in two-period equilibrium models

## From the exam spring 2008 (weight 50 per cent then)

We are looking at an open economy that exists for two periods. Output in each period $Y_{1}$ and $Y_{2}$ respectively, is given exogenously. A representative consumer maximizes expected utility

$$
\begin{equation*}
U=u\left(C_{1}\right)+\beta u\left(C_{2}\right) \tag{1}
\end{equation*}
$$

where $C_{1}$ and $C_{2}$ are consumption in the two periods and $\beta$ is a subjective discount factor, $0<\beta<1$. The country can borrow and lend in world markets at a given real interest rate, r. There is no initial foreign debt. Hence, the budget constraint can be written

$$
\begin{equation*}
C_{1}+\frac{C_{2}}{1+r}=Y_{1}+\frac{Y_{2}}{1+r} \tag{2}
\end{equation*}
$$

1. Derive the first order condition for optimal consumption and interpret it.
2. Discuss briefly the effect of an increase in the interest rate on consumption and the current account in the first period.
3. Suppose the rest of the world has the same preferences as the home country, but different output levels, $Y_{1}^{*}$ and $Y_{2}^{*}$. Explain with a graph how the world interest rate will be determined.
4. Assume $Y_{2}=Y_{2}^{*}$. Explain how the first period endowments $Y_{1}$ and $Y_{1}^{*}$ then determine which country that will have a current account surplus in the first period.
5. We now extend the model to include a production side. Each country has access to the same technology and the technology does not change. The production function is $Y=F(N, K)$ where $N$ and $K$ are respectively the inputs of labor and capital. The production function is homogeneous of degree one and has standard neoclassical properties. The labor input is the same in both countries and both periods, equal to $\bar{N}$. Each country has inherited a capital stock from the past, $K_{1}$ and $K_{1}^{*}$ respectively, that can be used in production in period 1. The capital stock can be augmented by investment in period 1 , which then adds to the input of capital in period $2, K_{2}$ and $K_{2}^{*}$. At the end of period 2 the remaining capital stock is consumed. The budget constraint of the home country can then be written

$$
\begin{equation*}
C_{1}+I_{1}+\frac{C_{2}+I_{2}}{1+r}=Y_{1}+\frac{Y_{2}}{1+r} \tag{2}
\end{equation*}
$$

where $I_{1}=K_{2}-K_{1}$ and $I_{2}=-K_{2}$. Explain how the home country's investment demand in period 1 is determined and briefly what the inclusion of capital means for the relationship between the world interest rate and the home country's current account in the first period.
6. Compare the capital stocks of the two countries in the second period. Suppose the home country has inherited more capital than the foreign country ( $K_{1}>K_{1}^{*}$ ). What does this imply for the current accounts in the two periods?
7. Finally, suppose international borrowing and lending had not been possible. From the point of view of wage earners in the home country, would this be an advantage or a disadvantage?

## Part B: Effect of productivity shocks in a small open economy

We look at a small open economy. The country can borrow and lend at a given world interest rate $r$. There is a representative consumer with utility function

$$
\begin{equation*}
U_{t}=\sum_{s=t}^{\infty} \beta^{s-t} u\left(C_{s}\right) \tag{1}
\end{equation*}
$$

where $0<\beta<1$ is a subjective discount factor and $C_{S}$ is consumption in period $s$. The production function is:

$$
\begin{equation*}
Y_{t}=A_{t} F\left(K_{t}\right), \quad t=1,2, \ldots \tag{2}
\end{equation*}
$$

where $Y_{t}, A_{t,}$ and $K_{t}$ are respectively output, productivity and the capital stock in period $t . F$ is increasing and concave. The capital stock evolves according to

$$
\begin{equation*}
K_{t}=K_{t-1}+I_{t-1}, \quad t=1,2, \ldots \tag{3}
\end{equation*}
$$

where $I_{t}$ is investment in period $t$. The country's foreign assets, $B_{t}$, evolve according to

$$
\begin{equation*}
B_{t+1}-B_{t}=C A_{t}=r B_{t}+Y_{t}-C_{t}-I_{t}, \quad t=1,2, \ldots \ldots \tag{4}
\end{equation*}
$$

where $C A_{t}$ is the current account surplus. The budget constraint is given by

$$
\begin{equation*}
\lim _{T \rightarrow \infty}\left(\frac{1}{1+r}\right)^{T} B_{t+T+1}=0 \tag{5}
\end{equation*}
$$

The initial values of the capital stock and the foreign debt are given.

1. Explain the meaning of the budget constraint (5).
2. Derive the first-order conditions for consumers and producers.
3. Suppose that productivity is constant and that by coincidence $\beta(1+r)=1$. What can you then say about the time paths of consumption, investment and the current account.
4. Compare the effects on consumption, investment and the current account of
a. An unexpected temporary increase in productivity in period $t+1$.
b. A temporary increase in productivity in period $t+1$ that becomes known at the beginning of period $t$.
c. An unexpected permanent increase in productivity in period $t+1$.

Sketch the different time-paths in graphs.

