A two-period current account model

Lecture 1, ECON 4330

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Intro

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- **The traditional macro approach:** Less focus on explicit optimization in micro, more focus on macro behavioral equations that seem to have empirical support. Nominal rigidities and unemployment problems. **Rødseth**

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- We will not do exactly that – but almost
- You will learn how to use fairly simple microfounded open-economy models to discuss various issues. After that, using more advanced models (including business cycle models) will be within the range of what you can learn yourselves.

Outline

1. Simple two-period model: What drives CA surpluses/deficits?
2. Two-period world equilibrium: What determines the world interest rate?
3. Simple two-period model with investment: Invest home or abroad?
4. Two-period world model with investment: The role of productivity differences

The current account

The first model we look at is very simple, and it provides us with a nice benchmark for how to think about modeling open economies. Main variable in focus will be the *current account*. In general, the CA is defined as:

$$\text{CA} = \text{Trade account} + \text{Primary income account} + \text{Secondary income account}$$

Trade account: Exports minus imports (trade balance)
Primary income account: Payments for use of labor and financial resources
Secondary income account: Foreign aid, remittances, etc.

The current account II

We want a model that can help us explain the main determinants for the current account. But first let us look at with something you know from before.

A two-period consumption model

Imagine an agent that lives for two periods. Income in period t is exogenous (Y_t), and the agent can borrow/lend at an exogenous interest rate r. The agent faces the following optimization problem:

$$\begin{aligned} \max_{C_1, C_2} \quad & \{u(C_1) + \beta u(C_2)\} \\ s.t. & \\ C_1 + B_2 &= Y_1 + (1+r)B_1 \\ C_2 &= Y_2 + (1+r)B_2 \end{aligned}$$

where B_t are assets carried over from period t to $t+1$. Assume $B_1 = 0$ (no assets to begin with) and substitute out B_2 to get the intertemporal budget constraint:

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

A two-period consumption model II

We then solve

$$\max_{C_1, C_2} \quad \{u(C_1) + \beta u(C_2)\}$$

$$s.t.$$

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r}$$

Letting λ be the Lagrange multiplier, the first-order conditions are:

$$u'(C_1) = \lambda$$

$$\beta(1+r)u'(C_2) = \lambda$$

A two-period consumption model III

Combine the two to find the *consumption Euler equation*:

$$u'(C_1) = \beta(1+r)u'(C_2) \tag{1}$$

This equation, together with the intertemporal budget constraint

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} \tag{2}$$

define the solution to the agent's optimization problem (the optimal values of C_1 and C_2). Model solved.

Open economy model?

Now let us translate this model into one for a small open economy. If we assume that:

- Behavior of the country can be modeled using a representative agent
- It is an endowment economy (income fixed)
- The country has access to a risk-free international credit market
- The country is 'small': it takes the world interest rate r as given

then the two-period model we just solved can also be used for a small open economy!

Open economy model? II

This means that if we consider a two-period model for a small open endowment economy with a representative agent (and no uncertainty), we just have to specify income (Y_1, Y_2) and a world interest rate r (as well as the utility function). The consumption path is then determined by equations (1) and (2) (the Euler equation together with the budget constraint).

CA in our simple model

But what about the current account? Recall:

$$\text{CA} = \text{Trade account} + \text{Primary income account} + \text{Secondary income account}$$

In period t the trade surplus is $Y_t - C_t$, while the primary income account is $r_t B_t$. The current account is therefore

$$CA_t = Y_t - C_t + r_t B_t$$

for $t = 1, 2$. Remember that we've assumed $r_1 = r_2 = r$. Inserting for $Y - C + rB$ using the budget constraints, we see that the current account measures the growth in net foreign assets:

$$CA_t = B_{t+1} - B_t$$

CA in our simple model II

For a given interest rate r and income Y_1 and Y_2 and $B_1 = 0$, we know that C_1 and C_2 are defined by the two equations

$$u'(C_1) = \beta(1+r)u'(C_2)$$
$$C_2 = (1+r)(Y_1 - C_1) + Y_2$$

More compactly:

$$u'(C_1) = \beta(1+r)u'((1+r)(Y_1 - C_1) + Y_2)$$

This equation implicitly defines $C_1(r, Y_1, Y_2)$, giving us the current account:

$$CA_1(r, Y_1, Y_2) = Y_1 - C_1(r, Y_1, Y_2)$$

What about CA_2? Since we don't save for the afterlife, $CA_2 = -CA_1$, so we focus mostly at CA_1.

Example

Assume we have a CES utility function:

$$u(C) = \frac{1}{1 - 1/\sigma} C^{1-1/\sigma}$$

The first derivative is $u'(C) = C^{-1/\sigma}$, making the Euler equation:

$$\beta \left(\frac{C_2}{C_1} \right)^{-1/\sigma} = \frac{1}{1+r}$$

Example II

Combine this with the budget constraint, you can show that that (homework)

$$C_1 = \frac{(1+r)Y_1 + Y_2}{2 + r + \{[\beta(1+r)]^\sigma - 1\}}$$

which makes the current account (assuming $B_1 = 0$)

$$CA_1 = Y_1 - C_1 = \frac{Y_1 - Y_2 + \{[\beta(1+r)]^\sigma - 1\}Y_1}{2 + r + \{[\beta(1+r)]^\sigma - 1\}}$$

Implications:

- There are two motives for saving in period 1 (having $CA_1 > 0$): Consumption smoothing (if $Y_1 > Y_2$), or if the rate of return is high ($\beta(1+r) > 1$). A low value of σ will make the substitution effect smaller.
- The current account as share of GDP (CA_1/Y_1) is independent of the absolute level of income (but not the growth rate)

Example III

What if $\beta(1+r)=1$? From the Euler equation that implies $C_1 = C_2 = C^*$ (complete consumption smoothing). Consumption is given by (from the BC)

$$C^* = \frac{(1+r)Y_1 + Y_2}{2+r}$$

while the *current account* is

$$CA_1 = Y_1 - C^* = \frac{Y_1 - Y_2}{2+r}$$

Implications:

- The main determinant of CA is the difference between present and future income
- CA_1/Y_1 depends only on the growth rate $(Y_2 - Y_1)/Y_1$, not the absolute level of income

The autarky real interest rate

A concept that will make it easier to think about whether an economy will run a current account surplus or deficit is the *autarky real interest rate*.

- Definition: The equilibrium interest rate under the counterfactual where the country has no access to international credit (autarky).
- Since autarky implies $C_1 = Y_1$ and $C_2 = Y_2$, we have r^A defined by the Euler equation evaluated at these consumption levels:

$$\frac{\beta u'(Y_2)}{u'(Y_1)} = \frac{1}{1+r^A}$$

- Alternatively, we can use the implicitly defined 'current account function', since at the autarky interest rate, $CA_1 = 0$:

$$CA_1(r^A, Y_1, Y_2) = 0$$

- If $r^A > r$, then future consumption is *cheaper* under autarky in our country compared to the rest of the world. We will import current consumption and export future consumption – i.e. run a current account deficit in period 1.

Lessons so far

From this (very simple) model, we've learned that

- The absolute level of income is not likely to be an important determinant of CA. At least not CA/Y.
- Countries that expect high income growth should tend to have CA deficits (and vice versa)
- Patient countries ($\beta(1+r) > 1$) should tend to have CA surpluses
- Comparing the world interest rate with the *autarky* real interest rate gives an indication of the sign of CA_1

Note

- We will most of the time assume that a higher interest rate reduces consumption, leading to an increse in the current account
- Questions for yourself: What are the income and substion effects of r on C_1? Is the net effect unambiguously positive or negative? Review section 1.3.2 on your own.

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General equilibrium

So far we have assumed that r is exogenously determined by 'the outside world'. Let us instead assume that we have two countries (Home and Foreign) that trade goods. How will the equilibrium interest rate be determined?

General equilibrium II

Let Home refer to the country we looked at in the small open economy case. We use the Euler equation together with the budget constraint to find optimal consumption, and therfore also the current account for any given r (conditional on income in the two periods):

$$CA_1(r; Y_1, Y_2) = Y_1 - C_1(r; Y_1, Y_2)$$

Assume now that the other country we loook at (Foreign) face a similar utility maximization problem, leading to its current account being given by a function:

$$CA_1^*(r; Y_1^*, Y_2^*) = Y_1^* - C_1^*(r; Y_1^*, Y_2^*)$$

General equilibrium IV

So we know what current account levels Home and Foreign will choose for a given world interest rate. The equilibrium world interest rate is the level of r that clears the world market. The market clearing condition is:

$$CA_1(r) + CA_1^*(r) = 0$$

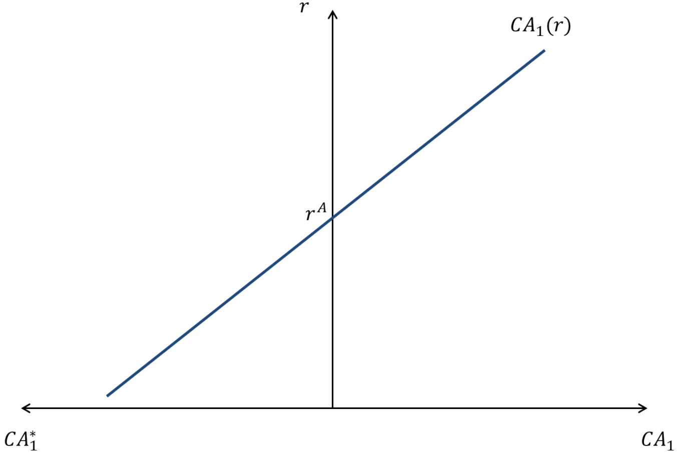
$\Rightarrow$ A surplus in one country must be met by a deficit somewhere else.

General equilibrium V

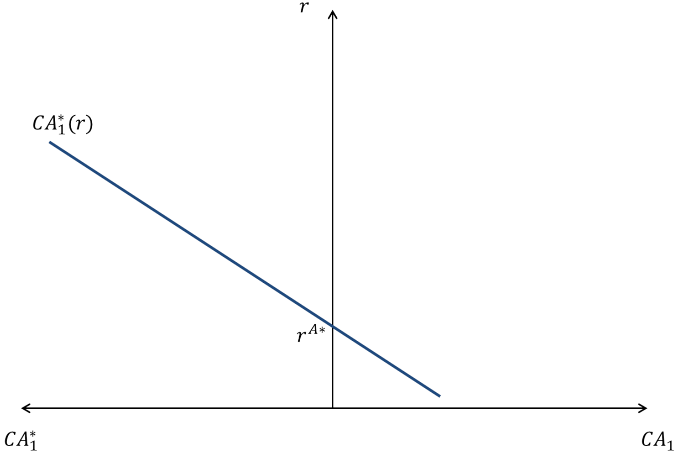
Let us analyze the equilibrium graphically. Assume that both current account functions are linear in r. Remember that we defined the *autarky real interest rate*. For the two countries, the autarky rate defines what interest rate that would lead to a zero current account:

$$CA_1(r^A) = 0$$
$$CA_1^*(r^{A*}) = 0$$

General equilibrium VI



General equilibrium VII

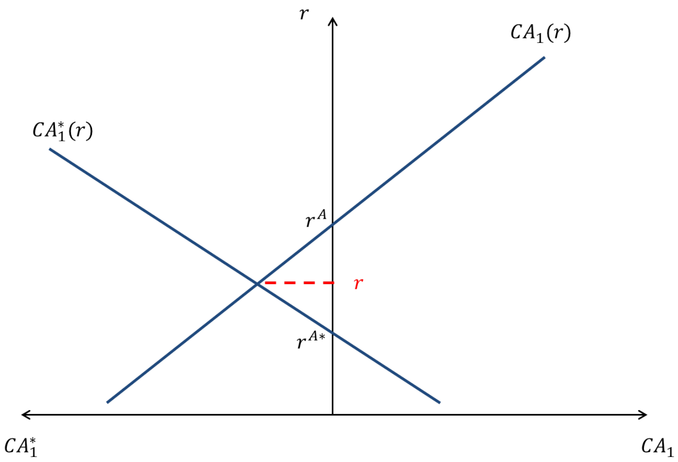


General equilibrium VIII

- Conclusion in the small open economy model: $r^A > r$ should lead to $CA_1 < 0$ since the internal price of future consumption is lower than the world market price.
- Conclusion in the world equilibrium model: Comparing r^A and r^{A*} will help us determine the sign of CA_1.

General equilibrium IX

$r^A > r^{A*} \Rightarrow CA_1 < 0 \& CA_1* > 0$



General equilibrium X

What variables determine r? Analytical example: Return to the CES-case from before. Assume identical utility functions and same discount factor. The current account functions are:

$$CA_1 = \frac{Y_1 - Y_2 + \{[\beta(1+r)]^\sigma - 1\}Y_1}{2 + r + \{[\beta(1+r)]^\sigma - 1\}}$$
$$CA_1^* = \frac{Y_1^* - Y_2^* + \{[\beta(1+r)]^\sigma - 1\}Y_1^*}{2 + r + \{[\beta(1+r)]^\sigma - 1\}}$$

Equlibrium condition is that

$$CA_1 = -CA_1^*$$

After inserting for CA_1 and CA_1^*, this can be solved to yield

$$1 + r = \frac{1}{\beta}\left(\frac{Y_2 + Y_2^*}{Y_1 + Y_1^*}\right)^{1/\sigma}$$

General equilibrium XI

How do we interpret the last equation? First introduce the *discount rate* ρ, defined as

$$\beta = \frac{1}{1+\rho}$$

Then let us take a log approximation of each side of the equilibrium condition. Since $\log(1+x) \approx x$ when x is small:

$$\begin{aligned} \log(1+r) &\approx r \\ \log 1/\beta = \log(1+\rho) &\approx \rho \\ \log\left(\frac{Y_2 + Y_2^*}{Y_1 + Y_1^*}\right)^{1/\sigma} = \frac{1}{\sigma}\log(1+g) &\approx \frac{1}{\sigma} g \end{aligned}$$

where $g = (Y_2 + Y_2^*)/(Y_1 + Y_1^*) - 1$ is the global growth rate. Hence, the equlibrium condition is approximately

$$r = \rho + \frac{1}{\sigma} g$$

General equilibrium XII

$$r = \rho + \frac{1}{\sigma} g$$

The world interest rate is thus a function of the discount rate ('patience'), the global growth rate, and the elasticity of substitution (σ). The elasticity is usually found to be quite small (less than one). Hence a high global growth rate will push up the world interest rate.

General equilibrium XIII

How do differences in size matter? Note that

$$g = \alpha g^{home} + (1-\alpha) g^{foreign}$$

where $\alpha = Y_1/(Y_1 + Y_2)$ is Home's share of world output in period 1. So if Home is the largest country ($\alpha > 0.5$), it will matter most for r what g^{home} is.

General equilibrium XIV

Not discussed: Who gains in general equilibrium from changes in output? The possibility of **immiserizing growth**. Higher output may harm the country's welfare. The possibility is due to the two effects that are caused by higher output.

- Higher output gives a direct positive impact on income
- But it also gives a change in the real interest rate

General equilibrium XV

In the previous graph, Home is a net borrower ($CA_1 < 0$). If Y_1 increases, CA_1 shifts to the right. If Y_2 increases, it shifts to the left. Hence a larger Y_1 will reduce r, while a higher Y_2 will increase r.

- A higher Y_1 will therefore give two benefits to Home: More income plus a lower interest rate. Foreign looses since it receives less for its lending.
- But a higher Y_2 will have one positive (higher income) and one negative (higher interest rate) effect for Home. The last may dominate.

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Investment

Let us now add investment to the model. First we return to the the small open economy case where we take the world interest rate r as given.

Investment II

Instead of assuming an endowment economy with fixed production, we'll have a production function:

$$Y_t = A_t F(K_t) \tag{3}$$

where A_t is productivity and K_t is the capital stock available at the beginning of time t. Assume $F' > 0$, $F'' < 0$ and $F(0) = 0$. With no depreciation, the law of motion is:

$$K_{t+1} = K_t + I_t \tag{4}$$

The model has only two periods; $t = 1$ and $t = 2$. K_1 is therefore given by past history, while it is obvious that the country wants $K_3 = 0$. Implies that $I_2 = -K_2$.

Investment III

What are the budget constraints? Without capital the period-constraints were

$$\begin{aligned} C_1 + B_2 &= Y_1 \\ C_2 &= Y_2 + (1+r)B_2 \end{aligned}$$

With capital we get

$$\begin{aligned} C_1 + I_1 + B_2 &= Y_1 \\ C_2 + I_2 &= Y_2 + (1+r)B_2 \end{aligned}$$

as well as the law of motion $K_2 = K_1 + I_1$ and the reasonable assumption $K_3 = 0$. Combined together we get the present-value constraint:

$$C_1 + \frac{C_2}{1+r} = Y_1 + \frac{Y_2}{1+r} + \frac{K_1 - r(K_2 - K_1)}{1+r} \tag{5}$$

Investment IV

What is the new optimization problem? The problem is:

$$\max_{C_1,K_2} u(C_1) + \beta u\left([1+r](Y_1 - C_1) + Y_2 + K_1 - r(K_2 - K_1)\right)$$

subject to $Y_2 = A_2 F(K_2)$ and given $Y_1 = A_1 F(K_1)$.

- Notice how consumption and investment decisions are separated
- Optimal K_2 is given by the value that maximizes $Y_2 - r(K_2 - K_1)$. It is independent of the utility function
- Optimal consumption is, for given levels of capital, determined in the same way as in the model without investment (i.e. using the Euler equation and budget constraint)

First-order conditions will confirm this intuition.

Investment V

The first-order conditions are:

$$A_2 F'(K_2) = r$$
$$u'(C_1) = \beta(1+r)u'\left([1+r](A_1 F(K_1) - C_1) + A_2 F(K_2) + K_1 - r(K_2 - K_1)\right)$$

Interpretation of first condition? Invest at home versus abroad. This implicitly defines $K_2(r, A_2)$. Rate of investment, $I(r, A_2, K_1) = K_2(r, A_2) - K_1$, will depend negatively on r and K_1, and positively on A_2. The Euler equation defines $C_1(r, A_1, A_2, K_1)$.

Investment VI

The solution is best illustrated by drawing indifference curves and the *production possibility frontier* (PPF). The PPF is defined by the intertemporal budget constraint under autarky:

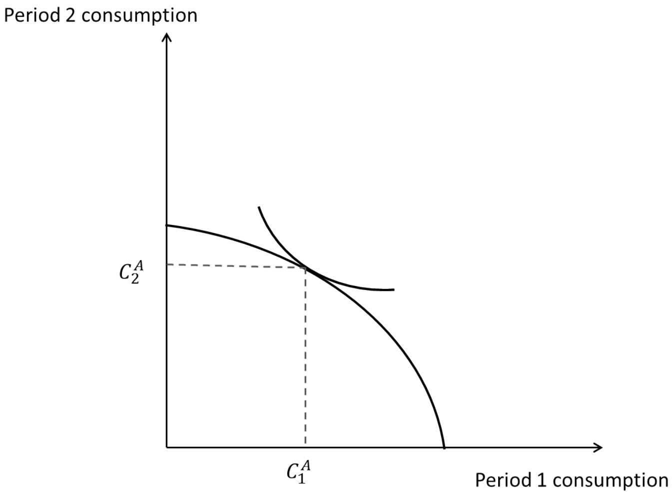
$$C_2 = F(K_1 + F(K_1) - C_1) + K_1 + Y_1 - C_1$$

which gives

$$\frac{dC_2}{dC_1} = -(1 + F'(K_2))$$

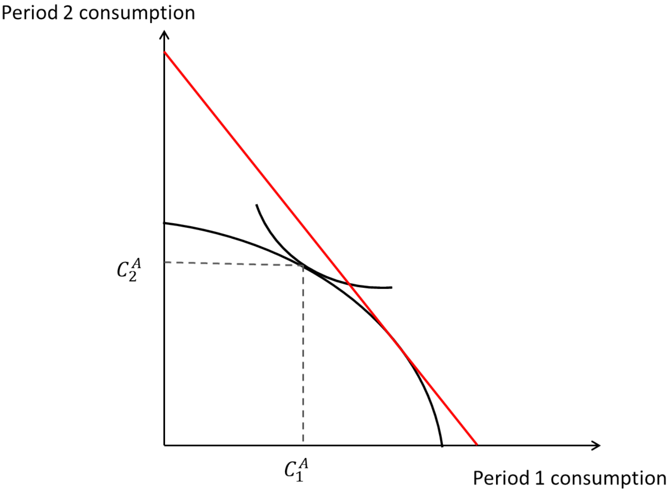
Investment VII

If autarky: Slope of the indifference curve must equal the slope of the PPF which is $-(1 + F'(K_2))$.



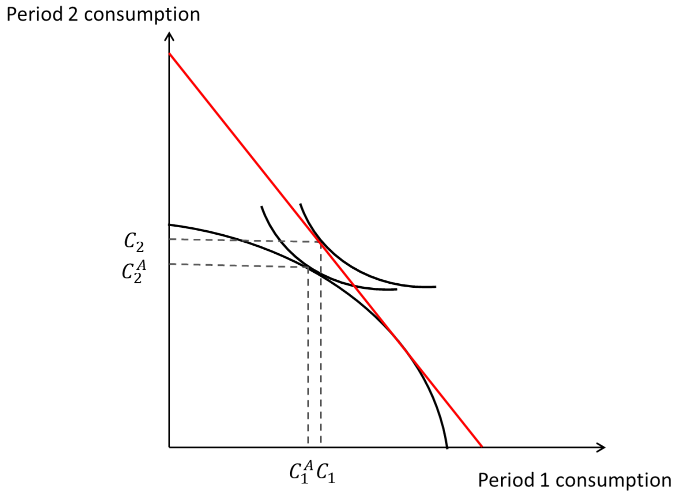
Investment VIII

With trade: Can borrow or lend in order to change the rate of investment – gives a BC with slope $-(1+r)$.



Investment IX

When going from autarky to trade, we see that in this figure we start with $F'(K_2) < r$. Home will do better if it invests less in capital, and rather lend to the outside world.



Current account and investment

What about the current account? For a given period t, it is (as before) defined as the trade account plus primary income account:

$$CA_t = Y_t - C_t - I_t + r_t B_t$$

For period 1:

$$CA_1 = A_1 F(K_1) - C_1 - (K_2 - K_1)$$

while we still have $CA_2 = -CA_1$ since $B_3 = 0$.

Current account and investment II

Relation to investment and saving?

$$\begin{aligned} \text{Saving} &= \text{Net investment in real capital} + \text{Net investment in financial assets} \\ &= \text{Investment in real capital at home} + \text{CA} \end{aligned}$$

If I denotes real investment and S is saving:

$$CA_t = S_t - I_t$$

A current account surplus means that you have a net investment abroad

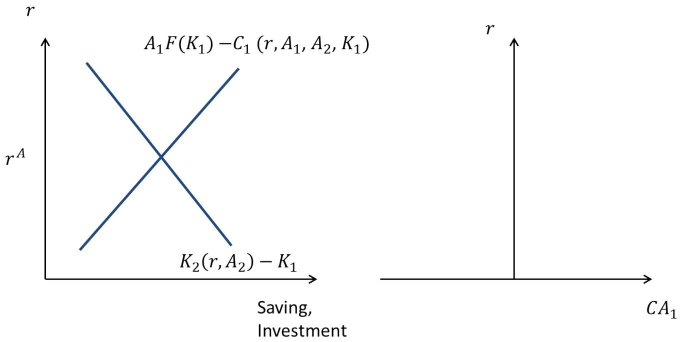
Current account and investment III

Since optimum is defined by $C_1(r, A_1, A_2, K_1)$ and $K_2(r, A_2)$, our new current account function is

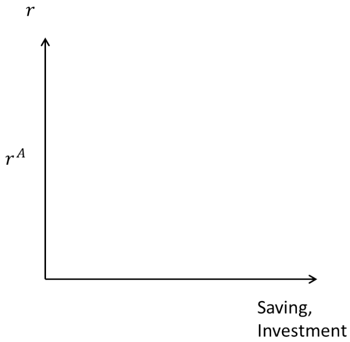
$$CA_1(r, A_1, A_2, K_1) = A_1 F(K_1) - C_1(r, A_1, A_2, K_1) - (K_2(r, A_2) - K_1)$$

We can thus either analyze the CA function alone or the savings and investment functions jointly (as is done in OR).

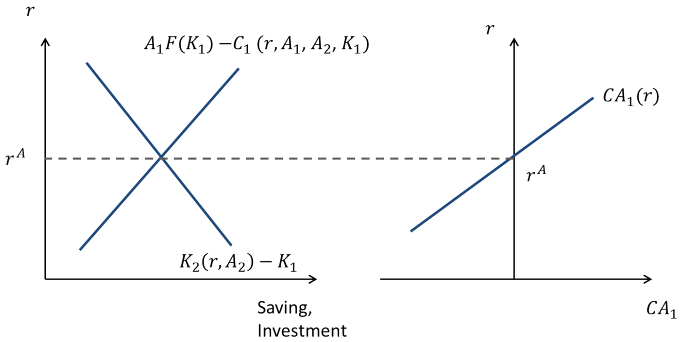
Current account and investment IV



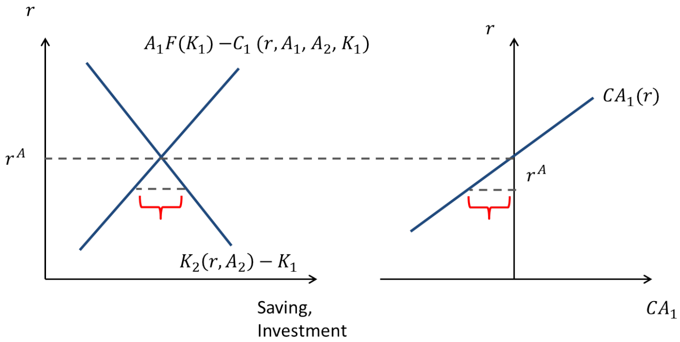
Current account and investment V



Current account and investment VI



Current account and investment VII



Current account and investment VIII

What are the effects of shifts in the exogenous variables?

Current account and investment IX

First let us increase A_2. Higher productivity in period 2 will detoriorate the current account ($\frac{dCA_1}{dA_2} < 0$) because:

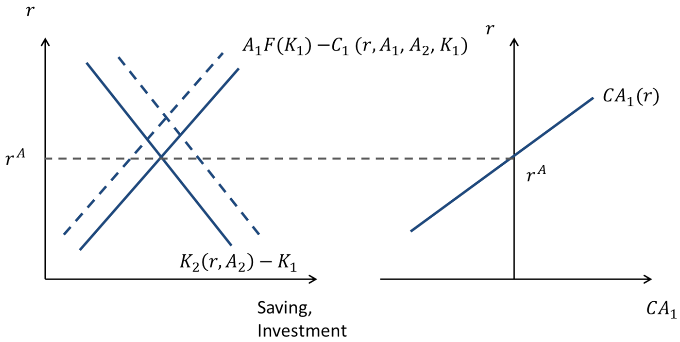
- For a given K_2, this works like an increase in Y_2 in the endowment economy. Increases consumption in both periods, and thus reduces the CA
- In addition, the optimal value of K_2 increases. This increases period 1 investment, leading to a lower CA

Note: The larger K_2 has no impact on *income*, $A_2 F(K_2) - rK_2$, because of the Envelope theorem. Reason:

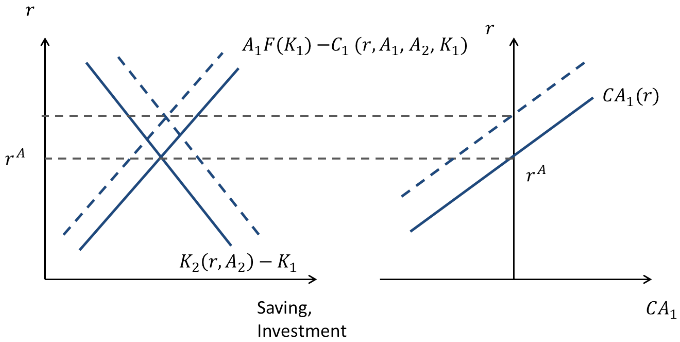
$$
\begin{aligned}
\frac{d}{dA_2}\left[A_2 F(K_2(r, A_2) - rK_2(r, A_2))\right] &= F(K_2(r, A_2)) + \left[A_2 F'(K_2) - r\right] \frac{dK_2}{dA2} \\
&= F(K_2(r, A_2))
\end{aligned}
$$

since the FOC for optimal investment holds.

Current account and investment X



Current account and investment XI



Current account and investment XII

An increase in period 1 productivity will improve the CA-balance ($\frac{dCA_1}{dA_1} > 0$)

- Works like an increase in Y_1 in the endowment economy. Consumption smoothing leads you to save some of the extra income abroad.

A larger inital capital stock (K_1) will also improve the CA ($\frac{dCA_1}{dK_1} > 0$)

- A larger K_1 reduces the need for investment (and optimal K_2 is unchanged). Improves the CA.
- It also increases the trade account through higher production (for unchanged level of period 1 consumption). Improves the CA.
- However, we also have an effect on consumption. C_1 goes up, causing CA to drop. But it can be shown to be smaller than the two positive effects (due to consumption smoothing), so net effect is positive.

Current account and investment XIII

Finally: What is the effect of a higher world interest rate? Homework! (Remember: It is homework to look at the effects in the endowment economy, too).

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General equilibrium with investment

As in the endowment case, assume that there are two countries, Home and Foreign. They will share utility and production functions, but may differ in levels of capital and productivity. Home's behavior is summarized by $CA_1(r, A_1, A_2, K_1)$. Foreign's by $CA_1^*(r, A_1^*, A_2^*, K_1^*)$. We find the equilibrium world interest rate by imposing market clearing

$$CA_1 + CA_1^* = 0$$

The CA-functions are more complex, but other than that much of the intuition from the endowment economy remains valid.

General equilibrium with investment II

The functions $K_2(r, A_2)$ and $K_2^*(r, A_2^*)$ are implicitly defined by first-order conditions which together imply:

$$A_2 F'(K_2) = A_2^* F'(K_2^*) = r$$

$\Rightarrow$ Efficiency in production.

General equilibrium with investment III

The functions $C_1(r, A_1, A_2, K_1)$ and $C_1^*(r, A_1^*, A_2^*, K_1^*)$ are implicitly defined by first-order conditions which together imply:

$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{\beta^* u'(C_2^*)}{u'(C_1^*)} = \frac{1}{1+r}$$

$\Rightarrow$ Efficiency in distribution

General equilibrium with investment IV

From the last two implications we find:

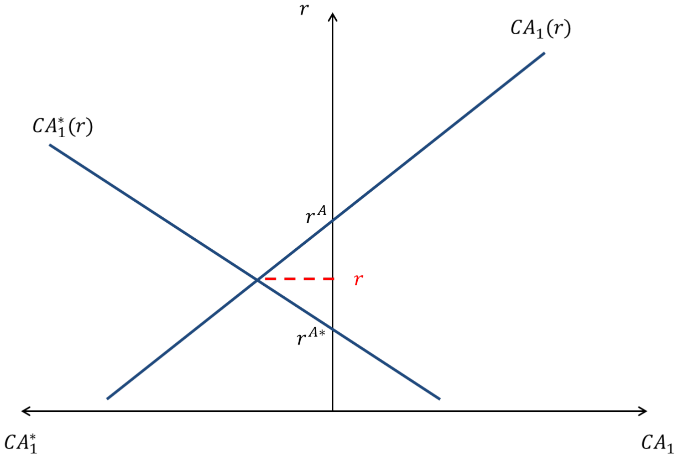
$$\frac{\beta u'(C_2)}{u'(C_1)} = \frac{\beta^* u'(C_2^*)}{u'(C_1^*)} = \frac{1}{1 + A_2 F'(K_2)} = \frac{1}{1 + A_2^* F'(K_2^*)}$$

$\Rightarrow$ Overall efficiency. $MRS = MRT$, so no gains from moving output between the periods.

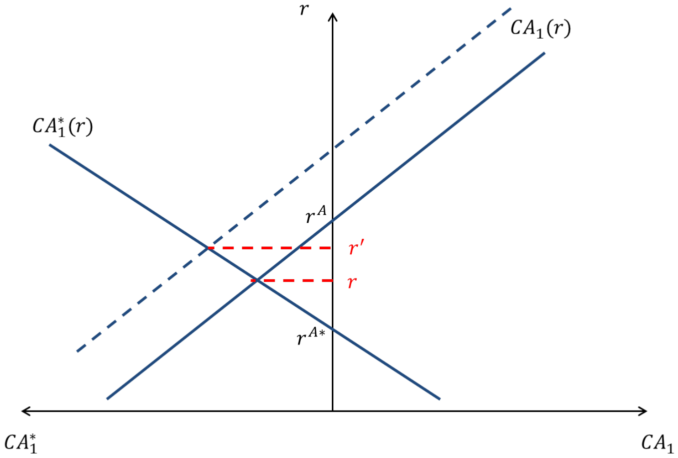
GE effects

Assume that A_2 increases. What are the GE-effects? From the partial equilibrium analysis we saw that the CA-function shifted to the left. Reason was two-fold: Higher future output increases current consumption, while higher productivity increases period 1 investment.

GE effects II



GE effects III



GE effects IV

In GE, this shift to the left will put upward pressure on the equilibrium interest rate.

- A higher interest rate will reduce consumption (unless it has a sufficiently large CA surplus causing the income effect to be very large and positive).
- It also reduceds the rate of investment

In total this reduces the fall in *CA* compared with the constant-interest rate case.

GE effects V

What is the net effect on the rate of investment? Intuitively, one would think that a larger A_2 increases investment, even though the interest rate effect may dampen the increase. But it is theoretically possible that the rate of investment falls! This is similar to the immiserizing growth point made earlier. See Application in section 1.3.3.3.

GE effects VI

What about Foreign? All the effects come through the increase in r. Foreign gains if it initially was a net lender (since it will earn more from what it is selling). It looses if it was initially a borrower.

Summing up

As we see, even though this is a rather simple two-periods, two-countries model, it is possible to get rather complicated effects. Could have added labor, but I will not go through section 1.5, but you should have a look at it.