

Handout to Seminar 1

ECON 4330

Tord Krogh

February 4, 2013

Notes to Part 2, question 4

Question: Now assume (as in question 5 of the first problem) that output is a function of the capital stock, $Y_t = A_t F(K_t)$. The utility function is specified as $U_t = \sum_{s=t}^{\infty} \beta^{s-t} u(C_s)$. Use the period s budget constraint to insert for C_s in the utility function.

First-order conditions with respect to K_{t+1} and B_{t+1} can be shown to be

$$\begin{aligned}u'(C_t) &= \beta(1 + A_{t+1}F'(K_{t+1}))u'(C_{t+1}) \\u'(C_t) &= \beta(1 + r)u'(C_{t+1})\end{aligned}$$

Combine the two to get $A_{t+1}F'(K_{t+1}) = r$.

Question (b)

Suppose productivity is constant $A_s = A_t$ for all $s \geq t$ and that, by coincidence, $\beta(1 + r) = 1$. Describe the time-profiles of consumption, investment and the current account (you can assume that initial net foreign assets, B_t , are zero).

Since $\beta(1 + r) = 1$, consumption is constant. Let K^* be the capital stock that makes $F'(K) = r$. Assume $K_t < K^*$. Clearly, investment in period t is positive and equal to $K^* - K_t$. For all future periods it is zero. What about the current account? In period t it will be

$$CA_t = AF(K_t) - C^* - (K^* - K_t)$$

What happens in period $t + 1$? From now on output, consumption and investment are constant, so CA must also return immediately to its long-run value. For period $t + 1$ we have

$$CA_{t+1} = AF(K^*) - C^* + rB_{t+1}$$

where $B_{t+1} = CA_t$. The long-run value of CA must be zero, in which case $AF(K^*) - C^* = rCA_t$. Why not positive or negative? Since if it is positive then we will pay down on our debt, and therefore get an ever increasing CA , which cannot be an optimum, since we would rather consume more. If $CA < 0$, we would borrow more and more to pay the interests (neither an equilibrium).

Question (c)

Sketch the effects on consumption, investment and the current account from

- An unexpected temporary increase in productivity in period $t + 1$ (that only lasts one period)
 - **ANSWER:** Variables will jump in period $t + 1$. Consumption is still constant, and will now jump to a higher level where some of the increase in lifetime wealth is consumed every period. Investment does not react since productivity is back to its old level tomorrow. Current account is positive in period $t + 1$ and then zero.
- A temporary increase in productivity in $t+1$ (that only lasts one period) that becomes known at the beginning of period t
 - **ANSWER:** Same effects as above, but now investment also responds. Investment is positive in period t and negative in $t + 1$ (to make the capital stock go back to its previous level). Will (as long as the investment response is larger than the increase in savings) drive the CA to be negative in period t and then positive in $t + 1$. Zero afterwards.
- An unexpected permanent increase in productivity
 - **ANSWER:** Investment adjusts immediately in period $t + 1$, and returns to zero afterwards. Consumption jumps up to a higher level and stays there. Current account is negative for one period, and then zero.