Handout to seminar 2
ECON 4330

February 25, 2013

Problem 4

We solve for consumption and find

$$C_t = (1+g)^t C_0$$

where $C_0 = (r-g)Y/r$ and $g = (1+r)/(1+\rho) - 1$. Therefore:

1. $\rho = r$ gives $g = 0$, so consumption is constant
2. $\rho > r$ gives $g < 0$, so consumption is forever falling.
3. $\rho < r$ gives $g > 0$, so consumption is forever increasing

What happens to net foreign assets? One can derive

$$\frac{B_{t+1}}{Y} = \frac{1}{r}\sum_{s=0}^{t}(1+r)^s\left(r-(r-g)(1+g)^{t-s}\right)$$

but this is hard to interpret. Let us simulate the solution. First take the case with $\rho < r$. Figure 1 plots how consumption and net foreign assets evolve over time. As is clear, consumption and assets both go to infinity as $T \to \infty$.

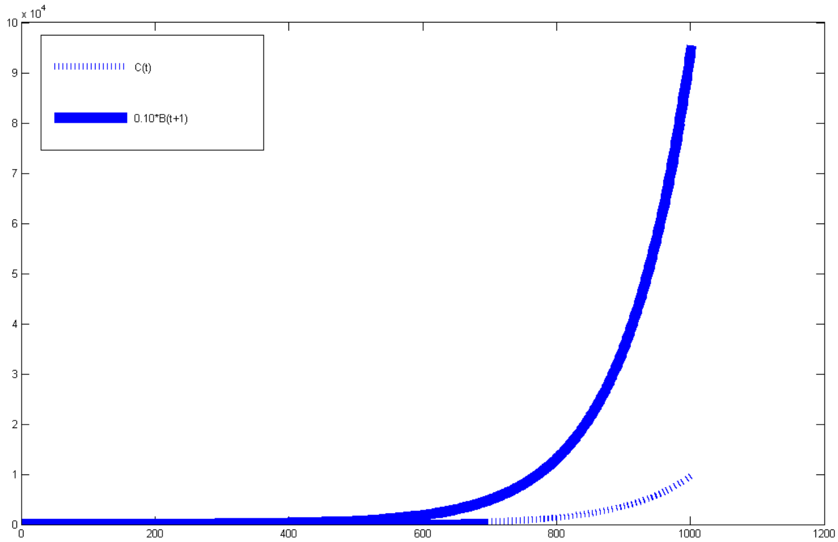

Figure 1: Consumption and NFA when $\rho < r$.

But how can assets go to infinity? Have we forgotten to impose the no Ponzi-game condition? Not at all. Figure 2 plots the logs of consumption and NFA. We see that net foreign assets converge to a path with constant growth rate. Furthermore, we know that this growth rate must be smaller than r, since the solution for C_0 was derived from an intertemporal BC that incorporated the no Ponzi-game condition.

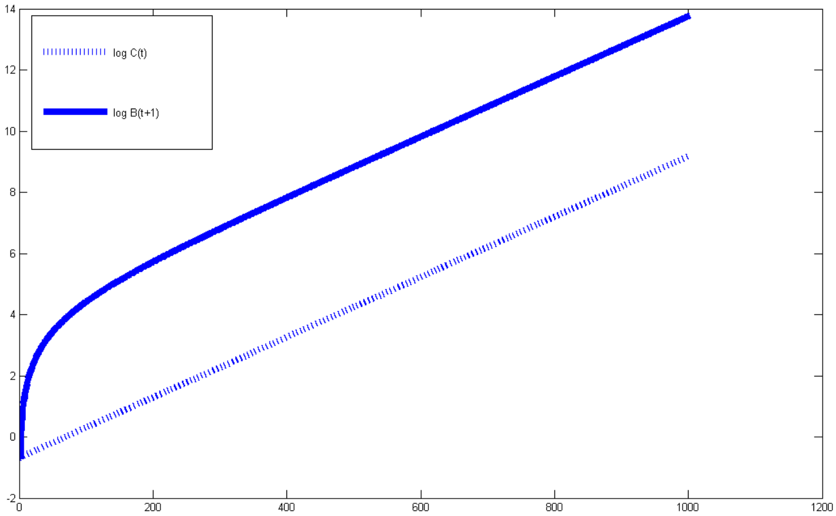

Figure 2: Logs of consumption and NFA when $\rho < r$.

What if $\rho > r$? Once more, I was a bit quick/unclear at the seminar. This case is somewhat different from the $\rho < r$-case, since consumption has a lower limit at zero. Figure 3 plots the outcome for consumption and net foreign assets.

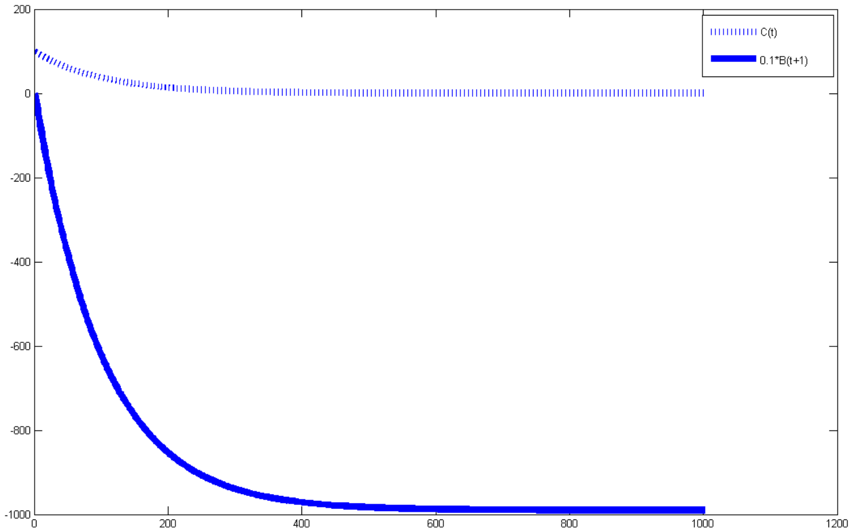

Figure 3: Consumption and NFA when $\rho > r$.

Consumption will decline geometrically, and in the limit converge to zero. Net foreign assets continue to increase as long as consumption is larger than private disposable income ($Y + rB_t$), but when consumption is close to zero, we see that B_{t+1} converge as well. This happens at the point where $B_t = -Y/r$, so that the country's income is just sufficient to pay the interests on its debt. Hence assets do *not* go to infinity, but the country ends up with zero consumption in the limit.

Going from partial to general equilibrium will only help us get rid of the $B_{t+1} \to \infty$ possibility. When $\rho^* < \rho$, we get a situation where Home behaves as in the $\rho > r$-case, i.e. its consumption level will converge to zero, while Foreign will be the creditor who (in the limit) recieves the entire GDP of Home every period.

Problem 5

Answer to question 2 is based on using

$$\mathbf{E_t}[u'(Y_{t+1})] \geq u'(\mathbf{E_t} Y_{t+1})$$

BUT: Risk aversion is not sufficient to guarantee this. Need to assume precautionary savings (i.e., $u''' > 0$). Sorry about that.