

# Exercises to Seminar 3

## ECON 4330

February 21, 2013

### Debt overhang and buy-backs

Consider a two-period representative agent model. The country we look at starts out with an initial level of debt  $D$  (exogenously given) which must be repaid in period 2. The world interest rate is assumed to equal zero ( $r = 0$ ). For simplicity, we assume that the country is not able to borrow or lend anything extra from the world credit market.

Income in period 1 is exogenous and equal to  $Y_1$ , while income in period 2 is  $A_2F(K_2)$ , where  $A_2$  is the (stochastic) level of productivity and  $K_2$  is the capital stock. Assume complete depreciation. This means that  $K_2$  is equal to the level of investment in period 1.  $A_2$  can take values between  $A_L$  and  $A_U$  and has probability density function  $\pi(A_2)$  with  $E(A_2) = 1$ .

When period 2 arrives, the country may *default*. In that case we assume that the country incurs a default cost  $\eta A_2F(K_2)$ . Consequently, the country only repays the loan if the default cost exceeds  $D$ .

1. Let  $V(D, K_2)$  be the *market value* of  $D$  in period 1. Explain why

$$V(D, K_2) = \eta F(K_2) \int_{A_L}^{\frac{D}{\eta F(K_2)}} A \pi(A) dA + D \int_{\frac{D}{\eta F(K_2)}}^{A_U} \pi(A) dA$$

2. The value  $V$  depends on  $K_2$ , i.e., it depends on how much the country decides to invest. Assume that the country chooses  $K_2$  to maximize  $F(K_2) - K_2 - V(D, K_2)$ . Find the first-order condition for optimal  $K_2$ . Give it an interpretation.
3. The first-order condition implicitly defines optimal  $K_2$  as a function of  $D$ ,  $K(D)$ . We will take for granted that  $K' < 0$  (this is cumbersome to show). What does this mean in economic terms?

4. Use the preceding answer to give intuition for why  $V$  is a *concave* function in  $D$ , once you take into account how  $K_2$  depends on  $D$ .
5. Now we want to evaluate the possibility of debt buybacks. We imagine a scenario where a country observes that its debt is traded at low values in the credit market. Can it be a good idea to purchase its own debt at a low value in period 1, rather than waiting for maturity in period 2? Let  $Q$  be the face value of the debt the country decides to re-purchase. We assume the country maximizes its expected net income:

$$Y_1 - K_2 + F(K_2) - pQ - V(D - Q, K_2)$$

where  $p$  is the *market price* of the debt. If  $Q$  must be determined before optimal investment, we know that  $K_2$  is given by  $K(D - Q)$ . Further, if  $Q$  must be officially announced before it is bought and since  $p$  is the price of debt traded in a rational market, we know that the market price must equal

$$p = \frac{V(D - Q, K(D - Q))}{D - Q}$$

i.e. the price the government must pay for its debt is affected by how much it decides to buy.

- (a) Without doing any differentiation, discuss what effects a debt buy-back will have on the country's expected net income.

## Mean-variance model of the FX market

1. Derive the demand for foreign currency for a risk-averse investor.
2. Discuss how the demand for foreign currency depends on the investor's perception of exchange rate risk.
3. The exchange rate is floating. Derive the solution for the equilibrium exchange rate for given interest rates and given exchange rate expectations. (You only need to find the equation that defines the exchange rate. No need to solve it for  $E$ ).
4. What will be the impact of an increase in the home interest rate on
  - (a) home and foreign investors' demand for foreign currency given the initial exchange rate,
  - (b) the equilibrium exchange rate, and

- (c) the equilibrium foreign currency holdings of the two groups of investors?
5. In this question you are allowed to simplify by assuming that both the covariances between inflation and depreciation and the expected rate of depreciation are zero. Suppose the perceived level of exchange rate uncertainty increases. What effect will this then have on the exchange rate? How does the effect depend on the interest rate differential?