Inflation targeting in the MFT model

Lecture 12, ECON 4330

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Today:

- Talk about inflation targeting
- Display a different representation of the same MFT-model

This representation might seem unfamiliar, but we are still in the same "world" as last week.

We are moving from this:

$$Y = C(Y_p, W_p, \rho, \rho_*) + I(\rho, \rho_*) + G + X(R, Y, Y_*)$$

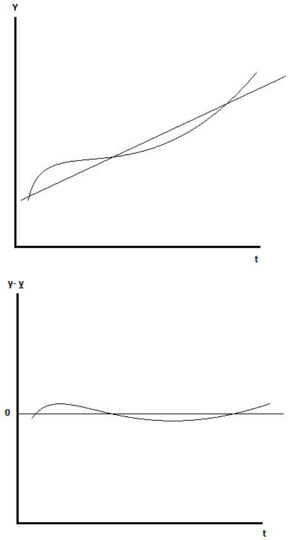
To this:

$$y - \bar{y} = -\alpha_\rho(\rho - \bar{\rho}) + \alpha_r(r - \bar{r}) + \alpha_* u_{y*} + u_d$$

This transformation consists of three parts:

1. From absolute levels to deviations from norm
2. From levels to logs
3. From general to parameterized model

From absolute levels to deviations from norm.



From levels to logs

Throughout this lecture we will use a model defined in terms of logarithms and relative changes of the variables.

$$x = LnX$$

$$\dot{x} = \frac{dx}{dt} = \frac{\dot{X}}{X}$$

$\bar{x}$ is the long run equilibrium of the variable

In terms of output:
Log transformation: $y = Ln(Y)$
Equilibrium value : $\overline{y}$

The representation we use is one in terms of percentage points from equilibrium or long run average:

$$y - \overline{y}$$

Parameterization

This is the step where we move from general functions to functions with parameters which can be solved explicitly.

$$y - \bar{y} = -\alpha_\rho(\rho - \bar{\rho}) + \alpha_r(r - \bar{r}) + \alpha_* u_{y*} + u_d$$

Here the parameters are α_ρ, α_r and α_*. They explain the relationship between the variables of the model; the deviations from long run equilibrium of y, ρ, r and y_* in addition to the domestic demand shock u_d.

Environment

- Home and foreign goods
- Average producer prices change only gradually
- CB targets inflation in prices on home goods
- The nominal interest rate is the only instrument used
- CB takes inflation rate expected by private agents as given
- UIP

Strategy

- Neglect informational problems. Shocks observed immediately.
- Full credibility, then how to build credibility.
- Temporary disturbances, then permanent changes
- Strict inflation targeting, then flexible

Mechanism

$$\left.\begin{matrix}\dot{p}_e \\ i\end{matrix}\right\} \longrightarrow \rho \underset{FX}{\longrightarrow} r \underset{IS}{\longrightarrow} y \underset{Philips}{\longrightarrow} \dot{p}$$

$$\left.\begin{matrix}\text{Expected inflation} \\ \text{Policy interest rate}\end{matrix}\right\}$$

$$\left.\begin{matrix}\rightarrow \text{Real interest rate} \\ \rightarrow \text{Real exchange rate}\end{matrix}\right\}$$

$\rightarrow$ Output

$\rightarrow$ Inflation

Real rates, definitions

Real interest rate:

$$\rho = i - \dot{p}_e \tag{1}$$

Real exchange rate:

$$r = e + p_* - p \tag{2}$$

i = nominal interest rate, p= price of home goods, e = nominal exchange rate, p_*= price of foreign goods (in foreign currency)

Interest rate parity

UIP

$$i = i_* + \dot{e}_e \tag{3}$$

Expect. consistent with $r = p_* + e - p$:

$$\dot{p}_e = \dot{e}_e + \dot{p}_* - \dot{r}_e$$

Subtract this from (3)

$$i - \dot{p}_e = i_* - \dot{p}_* + \dot{r}_e$$

RIP

$$\rho = \rho_* + \dot{r}_e \tag{4}$$

= Real Interest rate Parity

FX-equation

Real exchange rate expected to return to normal:

$$\dot{r}_e = -\epsilon(r - \bar{r}) + u_r, \qquad \epsilon > 0 \tag{5}$$

Insert in RIP:

$$\rho = \rho_* - \epsilon(r - \bar{r}) + u_r$$

FX-equilibrium

$$r = \bar{r} - \frac{1}{\epsilon}(\rho - \rho_* - u_r) \tag{6}$$

u_r =*exchange rate shock (risk premium shock)*

IS-equation

$$y - \bar{y} = -\alpha_\rho(\rho - \bar{\rho}) + \alpha_r(r - \bar{r}) + \alpha_* u_{y*} + u_d \qquad (7)$$

y= output, u_{y*}=foreign demand disturbance, u_d = domestic demand disturbance

Phillips-curve

Price setting:

$$\dot{p} = \dot{p}_e + \gamma(y - \bar{y}) + u_s, \qquad \gamma > 0. \tag{8}$$

Full credibility of target:

$$\dot{p}_e = \bar{\pi}$$

u_s= *cost-push shock*, $\bar{\pi}$= *inflation target*

Note that we have a cost shock that only influence prices. It would be realistic also to include it in the IS-equation because increasing costs may reduce output, but as we shall see it will influence GDP through the governments response.

To sum up our model we have three main equations:

IS:

$$y - \bar{y} = -\alpha_\rho(\rho - \bar{\rho}) + \alpha_r(r - \bar{r}) + \alpha_* u_{y*} + u_d$$

FX:

$$r = \bar{r} - \frac{1}{\epsilon}(\rho - \rho_* - u_r)$$

Phillips:

$$\dot{p} = \dot{p}_e + \gamma(y - \bar{y}) + u_s$$

The first two represent short term equilibrium in the gods- and foreign exchange-markets, while the third describe how the physical markets affect inflation.

What we do when we try to control inflation is in a sense to manage the third equation with the two first ones as constraints. There are two main ways to analyze this in our model:

1. Direct policy analysis on an ISFX curve
2. Generate a monetary policy (MP-) curve

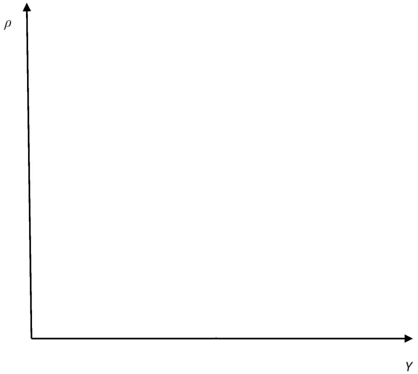
Aggregate demand The *ISFX*-curve:

Shows effect of ρ on aggregate demand taking account of effect through r
Insertion of FX in IS yields $ISFX$:

$$y - \bar{y} = -(\alpha_\rho + \alpha_r/\epsilon)(\rho - \bar{\rho}) + (\alpha_r/\epsilon)(\rho_* - \bar{\rho} + u_r) + \alpha_* u_* + u_d \tag{9}$$

- slope negative, sum of direct interest rate effect (α_ρ) and exchange rate effect α_r/ϵ
- small effect through exchange rate when expected return is fast
- Assumption: Normal real interest rate the same in both countries

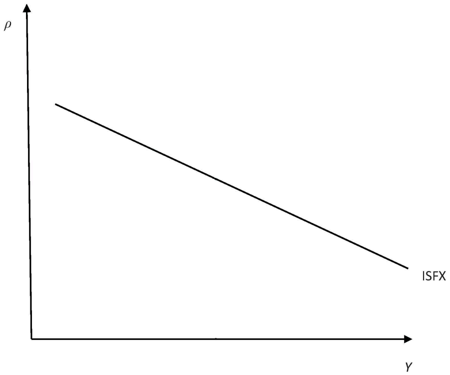
As before we want to represent the equilibrium in a diagram with the interest rate (now the real interest rate) and GDP (now output gap).



The derivative of the output gap in the ISFX equation with respect to ρ is:

$$\frac{\partial(y - \overline{y})}{\partial \rho} = -(\alpha_\rho + \frac{\alpha_r}{\epsilon}) < 0$$

We now know the slope and can start the policy analysis.



Inflation

Inflation follows from

$$\dot{p} = \bar{\pi} + \gamma(y - \bar{y}) + u_s$$

$$\left.\begin{matrix} \dot{p}_e \\ i \end{matrix}\right\} \longrightarrow \rho \underset{FX}{\longrightarrow} r \underset{IS}{\longrightarrow} y \underset{Philips}{\longrightarrow} \dot{p}$$

Here we see that we have the effect we postulated earlier in the lecture.

The three channels

There are three main channels from the policy of the central bank to actual inflation:

1. The interest rate channel: Increase in nominal interest rate raises real interest rate, lowers output gap, reduces inflation.
2. The exchange rate channel: Increase in nominal interest rate increases the demand for domestic currency leading to appreciation (real and nominal), lowering the output gap and reducing inflation. (In addition direct effect on consumer prices of foreign goods becoming cheaper).
3. The expectations channel: If central bank actions or communication reduces $\dot{p}_e$, this reduces inflation directly..

Setting policy 1

Hitting target requires:

$$y - \bar{y} = -\gamma^{-1} u_s$$

Intersection with *ISFX* gives the required real interest rate

$$\rho - \bar{\rho} = \frac{\alpha_r}{\alpha_r + \epsilon\alpha_\rho}(\rho_* - \bar{\rho} + u_r) + \frac{\epsilon}{\alpha_r + \epsilon\alpha_\rho}\left[\alpha_* u_{y*} + u_d + \gamma^{-1} u_s\right]$$

This is the real interest rate response. Note that:

- A demand shock would be completely neutralized: $y - \overline{y} = 0$
- Cost-push shocks would cause changes in the output gap even if they don't influence it directly.

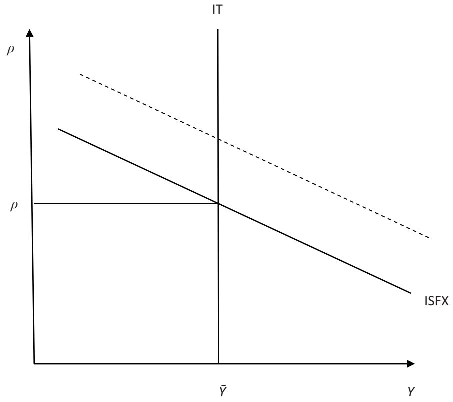


Figure : Effect of a demand shock

A positive demand shock will shift the ISFX-curve upwards forcing the central bank to increase the interest rate to cool the economy and avoid inflation.

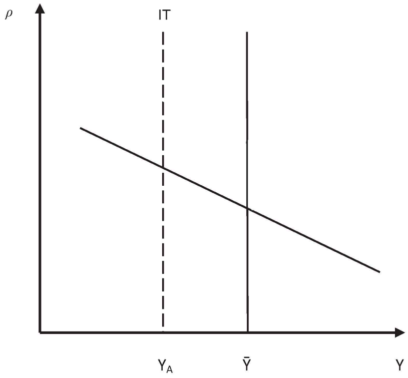


Figure : Effect of a cost-push shock

The central-bank will increase the interest rate such as to push the economy bellow it's long level equilibrium for the duration of the shock to counteract the inflation pressure.

Setting policy 1

$$\rho - \bar{\rho} = \frac{\alpha_r}{\alpha_r + \epsilon\alpha_\rho}(\rho_* - \bar{\rho} + u_r) + \frac{\epsilon}{\alpha_r + \epsilon\alpha_\rho}\left[\alpha_* u_{y*} + u_d + \gamma^{-1} u_s\right]$$

ρ responds

- positively to ρ_*, u_r u_d, u_{y*}, and u_s.
- less than one for one to $\rho*$ and u_s
- more to ρ_* and u_r when economy more open (α_r high)
- more to u_d and u_s when economy less open

Setting policy 2 The *MP*-curve

- Insert for $(y - \overline{y})$ from the phillips-curve with $\dot{p}_e = \bar{\pi}$
- $y = \bar{y} - \gamma^{-1} u_s$
- we have the MP-curve:

$$y - \bar{y} = -\alpha_\rho(\rho - \bar{\rho}) + \alpha_r(r - \bar{r}) + \alpha_* u_{y*} + u_d - \gamma^{-1} u_s = 0$$

or rewrite as:

$$r - \bar{r} = \frac{1}{\alpha_r}\left[\alpha_\rho(\rho - \bar{\rho}) - \alpha_* u_{y*} - u_d - \gamma^{-1} u_s\right]$$

Analyze in MP-FX diagram with the FX-curve:

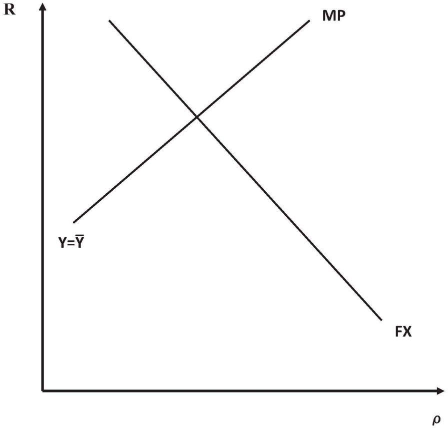
$$r = \bar{r} - \frac{1}{\epsilon}(\rho - \rho_* - u_r)$$

We find the slope of the MP-curve as:

$$\frac{\partial(r-\bar{r})}{\partial\rho} = \frac{\alpha_\rho}{\alpha_r} > 0$$

and the slope of the FX-curve is:

$$\frac{\partial(r-\bar{r})}{\partial\rho} = -\frac{1}{\epsilon} < 0$$


R
MP
Y=Ȳ
FX
ρ

Let's look at how we analyze the effects of a demand shock in this setting.

- The effects are of course the same, but now we see the effect on the exchange rate.
- Lets start with a positive demand shock

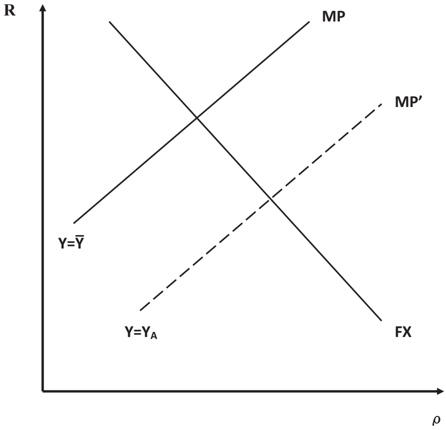


Figure : Effect of a demand shock

We can also look at the time paths of different variables in response to a single shock (with persistence).

Effect of a demand shock, strict target

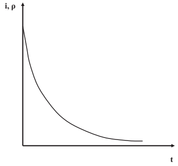


Figure : Interest rates

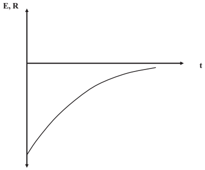


Figure : Exchange rates

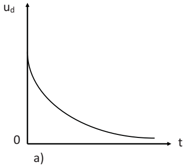


Figure : demand shock

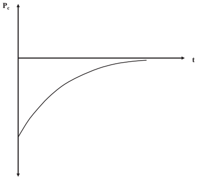


Figure : Consumer prices

Consistent expectations?

Solution for real exchange rate (Insert solution for ρ in FX-equation):

$$r - \bar{r} = \frac{1}{\alpha_r + \epsilon\alpha_\rho}\left[(\rho_* - \bar{\rho}) + \alpha_\rho u_r - \alpha_* u_{y*} - u_d + \gamma^{-1} u_s\right] \tag{10}$$

Time derivatives:

$$\dot{r} = \frac{1}{\alpha_r + \epsilon\alpha_\rho}\left[\dot{\rho}_* + \alpha_\rho \dot{u}_r - \alpha_* \dot{u}_{y*} - \dot{u}_d + \gamma^{-1} \dot{u}_s\right]$$

Suppose

$$\dot{u}_i = -\epsilon u_i$$

for all disturbances i ($\dot{\rho}_* = -\epsilon(\rho_* - \bar{\rho}$). Insertion in equation for $\dot{r}$ gives

$$\dot{r} = \frac{-\epsilon}{\alpha_r + \epsilon\alpha_\rho}\left[(\rho_* - \bar{\rho}) + \alpha_\rho u_r - \alpha_* u_{y*} - u_d + \gamma^{-1} u_s\right] = -\epsilon(r - \bar{r})$$

Need for more refined expectations?

- Full consistency only if disturbances die out with same speed as exchange rate is expected to return to normal.
- differentiate ϵ according to source and duration of shock?

Short duration of shocks (high ϵ) may mean:

- less impact of shocks on exchange rate
- less need for CB to react to disturbances from FX market (ρ to ρ_* and u_r)
- stronger interest response needed to counter disturbances from the goods market (u_d and u_s)

Expected nominal depreciation

Implied expected depreciation rate:

$$\dot{e}_e = \dot{r} + \dot{p}_e - \dot{p}_* = -\epsilon(r - \bar{r}) + \bar{\pi} - \dot{p}_* + u_r$$

- Compensates one for one for inflation abroad

Conflicting goals

- Output vs inflation
  - Demand side shocks - No conflict
  - Cost push shocks - Output effect enhanced by CB's response
  - Flexible inflation targeting
- Consumer vs producer prices
  - Targeting consumer prices instead

Consumer prices

Price index:

$$P_c = P^{1-\phi}(EP_*)^{\phi}$$

Consumer price inflation:

$$\dot{p_c} = (1-\phi)\dot{p} + \phi(\dot{e} + \dot{p}_*), \qquad 0 < \phi < 1 \tag{11}$$

$\dot{p}$ = domestic component, $(\dot{e} + \dot{p}_*)$ = imported or foreign component.

Consumer prices

After a positive demand shock consumer prices:

- First make a downward jump as the nominal exchange rate appreciates
- Then grow faster than target as the nominal exchange rate depreciates
- Average inflation over the whole period equal to target
- Same conclusion for other shocks when strict inflation target

Maybe we should be content with targeting producer prices?

Flexible inflation targeting

Loss function

$$L = \frac{1}{2}(\dot{p} - \bar{\pi})^2 + \frac{\lambda}{2}(y - \bar{y})^2, \qquad \lambda > 0 \tag{12}$$

Phillips-curve

$$\dot{p} = \bar{\pi} + \gamma(y - \bar{y}) + u_s, \qquad \gamma > 0.$$

$$L = \frac{1}{2}(\gamma(y - \bar{y}) + u_s)^2 + \frac{\lambda}{2}(y - \bar{y})^2$$

First-order condition

$$dL/dy = \gamma[\gamma(y - \bar{y}) + u_s] + \lambda y - \bar{y}) = 0 \tag{13}$$

$$\dot{p} - \bar{\pi} = -(\lambda/\gamma)(y - \bar{y}) \tag{14}$$

Output choice

$$\dot{p} - \bar{\pi} = -(\lambda/\gamma)\,(y - \bar{y}) \tag{15}$$

$$y - \bar{y} = -\frac{\gamma}{\gamma^2 + \lambda} u_2 \tag{16}$$

$$\dot{p} - \bar{\pi} = \frac{\lambda}{\gamma^2 + \lambda} u_2 \tag{17}$$

- Demand side disturbances: $y = \bar{y}$ as before
- Cost push
  - A fraction of the shock allowed to pass through
  - Reduction in output less than before $(\gamma/(\gamma^2 + \lambda) < 1/\gamma)$
  - Inflation and output gaps should have opposite signs
  - Lasting effect on price level

Credibility

In this lecture we have assumed all along that the central banks target is credible. That is $\dot{p_e} = \pi$. If this was not the case we could have all sorts of trouble:

- If agents believe that inflation will go over the inflation target the central bank must take extra measures to dampen the economy.
- If agents believe inflation will be below the target, then the CB must try to increase inflation.
- The target may be more credible the better the CB does.
- Covered in chapter 3.3 of the inflation note

Today

- Introduced inflation explicitly
- Derived the MFT model in terms of log variables and parameterized it
- Analyzed todays most relevant policy option: inflation target with interest rate tool

Next time

- Move to a long run economy
- Short summary and questions