ECON4330 Seminar 5

A Equilibrium in the foreign exchange market

Consider a world with two currencies, kroner and dollars.The table below shows the balance sheets. Government is the consolidated government and central bank sector. Taken together the two governments are net borrowers. The last line is equal to the line immediately above. Implicitly this equality defines the net assets (or wealth) of the four sectors measured in their respective currencies. (If this helps, you may think of the initial price levels as being equal to one).

Consider a period which is too short for new savings to add significantly to the stock of wealth. The only thing the agents can do then is to change one currency for another within the constraint that

$$B_j + EF_j = B_j^0 + EF_j^0 \tag{1}$$

for all sectors j.

The demands for dollars by the domestic and foreign private sectors are given as:

$$F_p = [\phi + \xi(i_* + \mu_e - i)]W_p/E = fW_p/E \tag{2}$$

$$F_{p*} = [\phi_* + \xi(i_* + \mu_e - i)]W_{p*} = f_*W_{p*} \tag{3}$$

Here, $0 \leq \phi < \phi_* \leq 1$ and $\xi > 0$ are constants.

1. Discuss how the total private demand for dollar denominated assets depends on the level of the exchange rate.

Assets	Home		Foreign		Total
	Govern.	Private	Govern.	Private	
Kroner	B_g	B_p	B_{g*}	B_{p*}	0
Dollars	F_g	F_p	F_*	F_{p*}	0
Sum	$B_g + EF_g$	$B_p + EF_p$	$B_* + EF_*$	$B_* + EF_{g*}$	0
Sum	W_g	W_p	EW_{g*}	EW_{p*}	0

Solution

The demand for dollars can be written

$$\begin{aligned} F_p &= f\frac{W_p}{E} = f[\frac{B_p^0}{E} + F_p^0] \\ F_{p_*} &= f_{p*}W_{p_*} = f_{p*}[\frac{B_{p_*}^0}{E} + F_{p_*}^0] \end{aligned}$$

The dollar demand effects of a depreciation $dE > 0$ can be decomposed into a portfolio composition effect (*i.e.* given constant f) and an expectation effect (f changes because μ_e changes)

Portfolio effect:

- When the exchange rate increases the distribution and the value of wealth changes. As a result, investors rebalances their investments to keep the distributions constant.
- If initially the investors hold positive amounts in both currencies, then the demand for dollars will go down (just take the derivative of F_p wrt E, holding f constant)
  - intuition
    - domestic investors are wealthier $dW_p = F_p^0 dE > 0$, since their dollar assets are now worth more Kroner. Given no rebalancing $F_p = F_p^0$, the share of wealth invested in foreign assets would go up. However, since the optimal share f is constant, they want to sell dollars and buy NOK after the depreciation.
    - if domestic private sector have *positive* net wealth but either (a) negative NOK assets or (b) negative \$ assets then the effect is reversed
    - if (a) then $f > 1$ and the \$ increase in wealth lead them to want even more \$ (by borrowing in NOK)
    - if (b) then $f < 0$ and there is an increase in \$ debt and thus a reduction in wealth. If no rebalancing the dollar debt goes up and the wealth goes down, implying that debt-to-wealth goes up. To keep debt-to-wealth constant, they borrow less in \$ (hence increased demand for dollar).
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    - The symmetric argument can be made about foreign investors

Expectation effect: What if something happens to μ_e?

- assuming positive net wealth in both private sectors $W_j > 0$, then for $j = p, p_*$

– If expectations are regressive $\mu_e' < 0$ the dollar risk premium goes down, reducing the demand for dollar

$$\frac{\partial F_j}{\partial f}\frac{\partial f}{\partial E} = \frac{W_j}{E}\xi\mu_e' < 0$$

– if expectations are extrapolative $\mu_e' > 0$ the dollar risk premium goes up, increasing the demand for dollar

$$\frac{\partial F_j}{\partial f}\frac{\partial f}{\partial E} = \frac{W_j}{E}\xi\mu_e' > 0$$

In general the sign of $d(F_p + F_{p^*})/dE$ is ambiguous. However, under the assumption that private sectors home and abroad hold positive amounts in both currencies, and that depreciation expectations are regressive, then $d(F_p + F_{p^*})/dE < 0$. It is standard to assume this sign, i.e. assume that the demand for dollars goes down when the price of dollars goes up.

2. Assume that the exchange rate is floating freely. a) Write down the equilibrium condition for the foreign exchange market. b) Explain how you can use this to solve for the exchange rate as a function of exogenous and predetermined variables only. (You are free to do the calculations,but this is not asked for).

Solution

Equilibrium $F_g = -F_p - F_{p_*} - F_*$. Exogenous variables are F_g and F_{g^*}, i, i^* and B_j^0, F_j^0 for $j = p, p_*$. Insert for demand equations F_p and F_{p*}, and solve for E. We get a simple closed form as long as there's no expectation effect. Note that if the dollar market is in equilibrium, then the Kroner market will also be in equilibrium.

3. Suppose the domestic central bank sells dollars. a) What effect does this have on the exchange rate? b) Which of the entries in the balance sheet would change and in what direction?

Solution

Differentiation (possibly implicit) of the above equation wrt F_g. Or, assuming a upwards sloping supply curve $-(F_p + F_{p_*} + F_{g*})$, we can simply shift the vertical line representing the domestic Gov't demand for foreign currency to the left. the effect is an appreciation $dE < 0$. Effects

- Private sector demand for dollar goes up $dF_j > 0$ $j = p, p_*$
- Private sector demand for Kroner goes down $dB_j < 0$ $j = p, p_*$ corresponding to the reduced supply of Kroner $dB_g < 0$, coming from the fact that the Central bank sell dollars and buys Kroner.

- CB sells dollars buys Kroner, the private sectors buy dollars and sell Kroner. The exchange rate appreciates such that the private sector is willing to absorb the dollar sold by the domestic CB.
- Of course, if the foreign Central bank intervenes and buys the dollars the domestic CB sells, the exchange rate doesn't move and private sectors allocations are unchanged.

4. Imagine that the private holdings of foreign currency instead of F_p^0 and F_{p*}^0 had been $F_p^0 + \Delta$ and $F_{p*}^0 - \Delta$ in the initial balance sheet. What would this have meant for the exchange rate now?

Solution

In this thought experiment we redistribute dollars from foreign to domestic private sector. Intuitively, this will have effect as long as there is home bias. And we do have home bias since $\phi_* > \phi \Rightarrow f_{p*} > f_p$.

- The foreign investors reduce their demand for dollars by $-f_{p*}\Delta$
- The domestic investors increase their demand for dollars by $f_p\Delta$
- Effect on Total private sector demand is given by $\Delta(f_p - f_{p*}) < 0$. Hence, supply of dollars to the CB goes up and the exchange rate appreciates, such that with the new initial distribution of dollars, the demand for dollars is unchanged.

5. Suppose both governments have balanced budgets. The home country has a current account surplus equal to S per period. The foreign country has a corresponding deficit. You may assume that all payments on the current account are made in dollars. How many dollars does the home central bank have to buy per period if it is to avoid a gradual appreciation? Would the figure be the same if the surpluses and deficits were in the government sectors, while the private sectors were in balance?

Solution

- Now savings (i.e. current account surplus/deficits) add to the stock of wealth. In effect, the fact that the home surplus corresponds to the foreign deficit, and all payments are made in dollars, this corresponds to question 4, with S replacing Δ. The CB must increase Foreign reserves with $-S(f_p - f_{p*}) > 0$.
- If the home Gov't runs a surplus of S and the foreign Gov't a deficit of S, the shifts in the curves would leave the exchange rate unchanged, and no need for the CB to intervene. The shifts would be equal to S. Think of a situation in which home Gov't lends to foreign Gov't.

B Mean-variance model of portfolio choice

An investor with financial wealth W is considering how to divide her investments between assets denominated in domestic and in foreign currency. Her preferences between risk and return are described by:

$$\mathcal{E}(\pi) - \frac{1}{2} R var(\pi) \tag{4}$$

where π is the real rate of return and R is the degree of relative risk aversion and $\mathcal{E}$ is for expectation. Let

$f = EF/PW$ = share of foreign currency in portfolio

i, i_* = domestic and foreign interests rate

e = rate of depreciation

p = inflation rate

The Variables e and p are stochastic with

Expectations μ_e and μ_p

Variances σ_{ee}, σ_{pp}

Covariance σ_{ep}

1. a) Calculate the expectation and variance of the return on a portfolio with share of foreign currency f. b) Use the result to show that the optimal share is
$$f = \frac{\sigma_{ep}}{\sigma_{ee}} - \frac{i - i_* - \mu_e}{R\sigma_{ee}} \tag{5}$$
c) Interpret this equation.

Solution

The return, expectation and variance on a portfolio is given by

$$\begin{aligned}
\pi &= (1-f) \underbrace{(i-\rho)}_{\text{real return domestic}} + f \underbrace{(i_* + e - \rho)}_{\text{real return foreign}} \\
&= (1-f) \underbrace{i}_{\text{domestic nominal return}} + f \underbrace{(i_* + e)}_{\text{foreign nominal return}} - \rho \\
E(\pi) &= (1-f)i + f(i_* + \mu_e) - \mu_p \\
var(\pi) &= f^2 \sigma_{ee} + \sigma_{\rho\rho} - 2f\sigma_{e\rho}
\end{aligned}$$

the derivative of expected return and variance is

$$\begin{aligned}\frac{dE(\pi)}{df} &= i_* + \mu_e - i = -r \\ \frac{dvar(\pi)}{df} &= 2f\sigma_{ee} - 2\sigma_{ep}\end{aligned}$$

where $r = i - i_* - \mu_e$ is the risk premium on domestic asset. The FOC is

$$\begin{aligned}\frac{dE(\pi)}{df} - \frac{1}{2}R\frac{dvar(\pi)}{df} &= 0 \\ -r - R(f\sigma_{ee} - \sigma_{ep}) &= 0 \\ f &= \frac{\sigma_{ep}}{\sigma_{ee}} - \frac{r}{R\sigma_{ee}}\end{aligned}$$

interpretation. Trade off between mean return and variance. It is useful to discuss the two terms separately

- *Minimum variance portfolio*, i.e. the portfolio that minimizes risk.: $\frac{\sigma_{ep}}{\sigma_{ee}}$ As long as there is a positive covariance between depreciation and inflation (what we expect), you want to invest in foreign assets to hedge against inflation risk (the foreign asset yields high nominal return when inflation is high). But the more exchange rate risk σ_{ee}, the less strong is this hedging motive

- *Speculative portfolio.* If there is a positive risk premium on domestic asset $r > 0$, you move away from foreign assets. Move towards the asset that yields highest expected return. The response to differences in expected returns is stronger the lower risk aversion and exchange rate risk.

- Overall, if *exchange rate risk goes up* the effect on foreign assets depends on the whether you have a positive or negative asset position

$$\frac{df}{d\sigma_{ee}} = -\frac{1}{\sigma_{ee}^2}\left[\sigma_{ep} - \frac{r}{R}\right]$$

  if positive $f > 0$ you invest less when σ_{ee} goes up, and if you borrow $f < 0$, you borrow less, i.e. you reduce exposure to exchange rate risk.

2. Suppose you are advising an investor who knows no maths or stats. How would you explain to him that, everything else equal, he should invest more in foreign currency the higher is the covariance, σ_{ep}?

Solution

High (low) inflation reduces (increases) the real return on your portfolio. If the covariance between depreciation and inflation is high, the foreign asset yields high (low) nominal return precisely when inflation is high (low). Hence if you invest in foreign assets, you are less exposed to inflation risk.

3. Actual portfolios seem to have a "home bias". Explain what is meant by this expression. How can home bias be explained?

In case you need it: If x and y are two stochastic variables, then

$$Var(ax + by) = a^2 Var(x) + b^2 Var(y) + 2abCov(xy)$$

Solution

Home bias: domestic investors invest a larger share of their wealth in domestic currency than do foreigners. When would this be the case? From the perspective of a foreign investor, optimal share invested in Kroner is

$$\underbrace{1 - f_*}_{b_*} = \frac{-\sigma_{ep_*}}{\sigma_{ee}} + \frac{r}{R\sigma_{ee}}$$

Home bias if

$$\begin{aligned} 1 - f &> b_* \\ 1 - \frac{\sigma_{ep}}{\sigma_{ee}} + \frac{r}{R\sigma_{ee}} &> \frac{-\sigma_{ep_*}}{\sigma_{ee}} + \frac{r}{R\sigma_{ee}} \\ 1 - \frac{\sigma_{ep}}{\sigma_{ee}} + \frac{\sigma_{ep_*}}{\sigma_{ee}} &> 0 \\ \sigma_{ee} - \sigma_{ep} + \sigma_{ep_*} &> 0 \end{aligned}$$

If

- Covariance between depreciation and inflation is zero there's a strong home bias, because there's no hedging motive! from a risk minimizing perspective, investment in foreign assets is then a pure increase in risk.
- The exchange rate risk goes up σ_{ee} investors move towards their own currency, creating home bias.

Suppose relative PPP holds: $e = p - p_*$ (i.e. constant real exchange rate)

$$\begin{aligned} var(e) &= var(p) + var(p_*) - 2cov(p, p_*) \\ cov(e, p) &= cov(p - p_*, p) = var(p) - cov(p, p_*) \\ cov(e, p_*) &= cov(p - p_*, p_*) = cov(p, p_*) - var(p_*) \\ \sigma_{ee} - \sigma_{ep} + \sigma_{ep_*} &= \sigma_{pp} + \sigma_{p_*p_*} - 2\sigma_{p.p_*} - \sigma_{pp} + \sigma_{p.p_*} - \sigma_{p_*p_*} + \sigma_{p.p_*} = 0 \end{aligned}$$

deviations from relative PPP creates home bias.