Seminar 4
ECON 4330

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1 Debt overhang and buy-backs

Consider a two-period representative agent model. The country we look at starts out with an initial level of debt D (exogenously given) which must be repaid in period 2. The world interest rate is assumed to equal zero ($r = 0$). For simplicity, we assume that the country is not able to borrow or lend anything extra from the world credit market.

Income in period 1 is exogenous and equal to Y_1, while income in period 2 is $A_2F(K_2)$, where A_2 is the (stochastic) level of productivity and K_2 is the capital stock. Assume complete depreciation. This means that K_2 is equal to the level of investment in period 1. A_2 can take values between A_L and A_U and has probabilty density function $\pi(A_2)$ with $E(A_2) = 1$.

When period 2 arrives, the country may *default*. In that case we assume that the country incurs a default cost $\eta A_2F(K_2)$. Consequently, the country only repays the loan if the default cost exceeds D.

1. Let $V(D, K_2)$ be the *market value* of D in period 1. Explain why

$$V(D, K_2) = \eta F(K_2) \int_{A_L}^{\frac{D}{\eta F(K_2)}} A\pi(A)dA + D \int_{\frac{D}{\eta F(K_2)}}^{A_U} \pi(A)dA$$

Solution

The market value of debt $V(D, K_2)$ equals the expected repayment value of debt face value D. For low realizations of next period productivity A_2, i.e. when $A_2 < \frac{D}{\eta F(K_2)} = A^*$, the debtor defaults and the creditors receive

$$\eta A_2F(K_2) < D$$

and for high realization of productivity $A_2 \geq A^*$ the debtor repays the entire face value D. The integrals define the probabilities.

2. The value V depends on K_2, i.e., it depends on how much the country decides to invest. Assume that the country chooses K_2 to maximize $F(K_2) - K_2 - V(D, K_2)$. Find the first-order condition for optimal K_2. Give it an interpretation. HINT: Use the Leibniz rule.

Solution

Assuming that utility is linear in consumption, and $\beta(1+r) = 1$, expected utility maximization implies maximizing present value of income (we also set $r = 0$)The first order condition is:

$$F'(K_2) - 1 - \frac{\partial V}{\partial K_2} = 0$$

The first to terms represents the standard intertemporal trade off between current and future consumption, whereas the last term $\frac{\partial V}{\partial K_2}$ captures the effect on expected repayment from higher investment:

$$\begin{aligned}
\frac{\partial V}{\partial K_2} &= \eta F'(K_2) \int_{A_L}^{A^*} A\pi(A)dA + \eta F(K_2)A^*\pi(A^*)\frac{\partial A^*}{\partial K_2} - D\pi(A^*)\frac{\partial A^*}{\partial K_2} \\
&= \eta F'(K_2) \int_{A_L}^{A^*} A\pi(A)dA + (\eta F(K_2)A^* - D)\,\pi(A^*)\frac{\partial A^*}{\partial K_2} \\
&= \eta F'(K_2) \int_{A_L}^{A^*} A\pi(A)dA + \left(\eta F(K_2)\frac{D}{\eta F(K_2)} - D\right)\pi(A^*)\frac{\partial A^*}{\partial K_2} \\
&= \eta F'(K_2) \int_{A_L}^{A^*} A\pi(A)dA > 0
\end{aligned}$$

The default option acts as an implicit tax on investment, since creditors confiscate a fraction η of the return in the event of default, thereby depressing investment.

3. The first-order condition implicitly defines optimal K_2 as a function of D, $K(D)$. We will take for granted that $K' < 0$ (this is cumbersome to show). What does this mean in economic terms?

Solution

When debt is high, the probability of default is high, thereby reducing the incentive to invest, since expected return falls. Two extreme cases: (1) If debt is sufficiently low, implying a default probability of zero, the country receives the entire return on capital $F'(K_2)$. In this case, a marginal increase in debt would not affect K_2. If debt is sufficiently large, implying a certain default, the country only receives $(1 - \eta)F'(K_2)$.

4. Use the preceding answer to give intuition for why V is a *concave* function in D, once you take into account how K_2 depends on D.

Solution

The total derivative of $V(D, K(D))$ is

$$\frac{dV}{dD} = \underbrace{\int_{A_*}^{A_U} \pi(A)dA}_{\partial V/\partial D>0} + \underbrace{\left[\eta F'(K_2)\int_{A_L}^{A^*} A\pi(A)dA\right] K'(D)}_{(\partial V/\partial K_2)K'(D)<0}$$

the first term captures the effect that higher debt increases the market value, given the probability of default. The second term captures the effect that higher debt depresses investment, thereby lowering the repayment in the event of default, thus reducing the market value. It is possible that for a sufficiently high debt level, the second effect is so strong that the market value of debt falls when debt increases. Intuitively, the concavity follows because both the probability of default increases (higher A^*) and the repayment when defaulting falls, as the debt level increases. Note that if debt is very small and default risk is non-existing, then increases in debt will increase the value of debt one to one. When default is certain increased debt will not matter.

5. Now we want to evaluate the possibility of debt buybacks. We imagine a scenario where a country observes that its debt is traded at low values in the credit market. Can it be a good idea to purchase its own debt at a low value in period 1, rather than waiting for maturity in period 2? Let Q be the face value of the debt the country decides to re-purchase. We assume the country maximizes its expected net income:

$$Y_1 - K_2 + F(K_2) - pQ - V(D-Q, K_2)$$

where p is the *market price* of the debt. If Q must be determined before optimal investment, we know that K_2 is given by $K(D-Q)$. Further, if Q must be officially announced before it is bought and since p is the price of debt traded in a rational market, we know that the market price must equal

$$p = \frac{V(D-Q, K(D-Q))}{D-Q}$$

i.e. the price the government must pay for its debt is affected by how much it decides to buy.

(a) Without doing any differentiation, discuss what effects a debt buy-back will have on the country's expected net income.

Solution The price p follows from the fact the fact that V is the market value of the debt and p is the unit price of that debt. Hence, if total debt outstanding is D, the market value must be

$$\begin{aligned} V &= pD \\ p &= \frac{V}{D} \end{aligned}$$

If the country decides to buy back part of the debt $Q > 0$ there are several effects going on

1. Consider the term
$$-K_2 + F(K_2)$$
The reduction in outstanding debt $D - Q$ stimulates investment which raises net income via the term $-1 + F'(K_2)$. We know that this term is positive since as long as there is a positive default probability $F'(K_2) > 1$ (see question 2)

2. Consider the term
$$-pQ - V(D - Q, K_2)$$
holding the price p constant. Starting from $Q = 0$, when buying back a unit of debt, the country pays $p = \frac{V}{D}$ today, and reduces the expected next period repayment with dV/dD. Due to the concavity of V, the reduction in expected repayment is lower than the price p. The buyback is thus a net cost. This is also true for arbitrary $Q > 0$.

3. Consider the term
$$-pQ$$
The more debt the country buys back, the higher the price p, since the market value of debt will move closer to the face value, i.e. $p'(Q) > 0$. If the country buys back a sufficiently high amount, so that the default probability is eliminated, the market value equals the face value and $p = 1$. As long as there is a positive default risk, the market value is lower than the face value and $p < 1$. In other words, the market price is low when outstanding debt $(D-Q)$ is high, and high when outstanding debt is low.

4. As Obstfeld and Rogoff shows (p. 399), the effect of increasing Q, starting from $Q = 0$, is a reduction in the country's net income. The buyback in our simple model is therefore never optimal for the debtor country

2 Defcits and debts

1. Suppose you have the following information from the country's national accounts:

   (a) Net foreign debt : 80 per cent of GDP
   (b) Trade surplus: 2 per cent of GDP
   (c) Current account surplus: - 2 per cent of GDP
   (d) Surplus on the interest account - 4 per cent of GDP
   (e) Inflation rate 2 per cent per year
   (f) Real GEP growth per year 1 per cent

   Will the nominal value of the foreign debt be increasing? Will the real value be increasing? Will the ratio of debt to GDP be increasing?

Solution Let's use a current account surplus of -2 percent, and for simplicity set GDP to 100. The nominal value of debt grows with 2, implying a gross growth rate of $82/80 = 1.025$. Since nominal value of debt grows with 2.5 percent and inflation is only 2 percent, the real value of debt grows. The gross growth rate in nominal GDP is $1.01 * 1.02 = 1.0302$ which is large than the nominal growth in debt. Hence the debt to GDP ratio falls.

2. Use the simple risk premium model from class. Find the interest rate Greece pay on their sovereign debt. Find also a risk free alternative, ex. Germany, and use the model to see how likely the market think a Greek default is. How may the answer be different? What does the model miss?

   Solution The rates I found at the ECB is 0.42% for Germany and 7.07% for Greece. The no-profit condition is $p(1 + i)D = (1 + r)D$ giving $p = \frac{1+r}{1+i} = \frac{1.0042}{1.0707} = 0.938$. Investors think it's 94 % chance of Greece repaying it's debt. There are though some point that may change that answer:

   - Market frictions and costly entry makes it possible for lenders to make positive profits. This would lead to Greece paying a higher interest than the one reflecting the actual risk. With a "fair" interest rate, the probability of repayment should be higher.
   - Partial default. It may be that Greece would default on only half it's debt. If it performed a haircut of 50 %, treating all investors the same, in case of default. The value of default would be higher and thus the implied chance of full repayment lower.

- Institutional investors. A big part of Greece's debt is held by institutional investors like the IMF and other european countries. It's likely that these investors hold the debt with other motives than profit and might thus accept a lower interest rate. Implying that the chance of repayment is actualy lower.

Discussion