ECON 4330
Seminar 2

1 Short look at OLG

This might not be covered in class, but the model should be known so we do a simple, but instructive, exercise.

1. We use a simple OLG model where consumers live for two periods. They have exogenous labor income only in the first period and save for old age at a world interest rate r. The instantaneous utility function is log utility and the problem of generation t is given by:

$$\max_{c_t^y, c_{t+1}^o} U_t = log(c_t^y) + \beta log(c_{t+1}^o)$$

$$s.t \qquad c_t^y + b_{t+1} = y_t \tag{1}$$

$$c_{t+1}^o = (1+r)b_{t+1}$$

(a) Solve for the optimal savings for generation t.

Assume that income grows by a fixed rate, g, every period such that $y_t = (1+g)^t y_0$. Solve for the net savings(savings of the young minus dis-savings of the old) in each period.

(b) How does the time-profile of the net savings of the country depend on the growth rate?

(c) How does this compare to the infinite horizon case from the last seminar?

2 Transfer problem(Revised version of Exam 2015)

Consider a model with only tradable goods, but differentiated by country. Consumption c in the home country is the aggregate of home produced good

c_h and foreign good c_f,
$$c = \xi c_h^\omega c_f^{1-\omega}, \tag{2}$$
where $1 \geq \omega \geq 0.5$ and $\xi = \omega^{-\omega}(1-\omega)^{-(1-\omega)}$. Households derive utility $u(c)$ from consuming c. The home country produces y_h of the home good at price p_h. The price of the foreign good is p_f, where the foreign good is the numeraire, so that $p_f = 1$. The budget constraint is then equal to

$$p_h y_h = p_h c_h + p_f c_f. \tag{3}$$

1. Write down the household's optimization problem and solve for the demmand of the two goods.

2. Show that the share of home goods in total consumption is:

$$\psi = \frac{\omega}{p_h^{1-\omega}} \tag{4}$$

such that

$$c_h = \psi c, \tag{5}$$

3. Consider now the case of two symmetric countries, that is the foreign country has the preferences

$$c^* = \xi(c_f^*)^\omega (c_h^*)^{1-\omega} \tag{6}$$

and produces y_f goods. Write down the foreign household's optimization problem and solve for foreign consumption of the two goods.

4. Find the share of home goods (produced in the home country) in total foreign consumption, ψ^*, such that:

$$c_h^* = \psi^* c^*, \tag{7}$$

5. Solve for the price p_h. Hint: Use the goods market clearing condition $c_h + c_h^* = y_h$.

6. Now suppose that the home country receives a transfer T from the foreign country, so that it can spend now $p_h y_h + T$ and the foreign country can spend $p_f y_f - T$. Solve for the price p_h. Hint: Use again the goods market clearing condition $c_h + c_h^* = y_h$.

7. How does the price p_h depend on T?

8. Is there a value of ω where p_h is independent from T? Why?

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9. What happens with ψ and ψ^*?

10. What happens to total consumption of thegood produced in the home country?

11. Is the change in the price p_h a burden or a benefit for the foreign country?

3   Discussion

Please send me suggestions for discussion topics at nicolai.ellingsen@econ.uio.no.