ECON 4335 – Economics of Banking – problem set #3 – seminar week 41

Question 1 (Both groups)

Return to the problem at the first seminar where we now introduce banks or financial intermediaries, as an additional source for financing projects. (See also F&R; 2.5.3.) Under direct finance, and with an opportunity cost for investors (equal to the money market rate of interest), $1 + r$, we can, as analyzed in Problem Set 1, derive a condition on initial cash holdings which will warrant direct finance.

In order to fully capture the new environment, with three types of agents – entrepreneurs, investors and banks – we reformulate the model from seminar 1 so as to be a kind of "profit-sharing"-model. (This is nothing new; it is just a simpler way to capture what each party should be entitled to in case of success.)

a) If we let R_i now be the gross return to investors (under direct finance) if success, in exchange for an initial investment I_i, when the "working hard with no private benefit" – project is induced, with $p_H(R - R_i) \geq p_L(R - R_i) + B$ or

$$\Delta p \cdot R_e \geq B \text{ or } R_i \leq R - \frac{B}{\Delta p} \text{ satisifed where } R_e = R - R_i \text{ is the return or}$$

profit to the entrepreneur/ borrower in case of success, show that competitive equilibrium is characterized by $p_H R_i \geq (1 + r)I_i \Rightarrow I_i \leq \dfrac{p_H R_i}{1 + r} \leq \dfrac{p_H}{1 + r}[R - \dfrac{B}{\Delta p}]$.

b) Show then that the project can be financed directly if the entrepreneur has sufficient initial cash holdings so that

$$I_i + A \geq I \Leftrightarrow A \geq I - I_i \geq I - \frac{p_H}{1 + r}[R - \frac{B}{\Delta p}] := \overline{A}(r). \text{ Note that } \overline{A}(r) \text{ is}$$

increasing in r.

Now introduce bank finance and let competitive banks act as monitors as well as lenders. Due to moral hazard there is still a private benefit to the entrepreneur from choosing the "bad" project, but by incurring a monitoring cost c the bank can reduce the private benefit to $b < B$; interpreted as a reduction in the entrepreneur's opportunity cost of not shirking. Banks therefore alleviate the moral hazard problem. The probability of success is the same as before. For any financial contract the entrpreneur will prefer the B – project to the b – project. However, monitoring is privately costly in the sense that the cost c is unverifiable – hence the bank faces a moral hazard problem too. The bank must then be induced to monitor the borrowers. The problem is therefore to see how the financial contracts (specifying how much

each party should invest and how much should be paid if success) should be altered if we introduce banks as monitors and their incentive to monitor.

A bank that monitors can now help a capital-constrained entrepreneur (i.e. one with less initial cash than $\bar{A}(r)$) to invest. The bank may therefore alleviate the financing problem, by *offering a supplement* to external direct finance. Along with direct finance the entrepreneur can, through a contract (R_i, I_i) with external investors, in addition borrow I_b from a bank while promise to pay back R_b if success, where we have

$$R = R_e + R_i + R_b.$$

    c) Explain why the incentive compatibility constraint for the entrepreneur to work hard, and the incentive condition for a bank to monitor, now become:

$$(i) \quad p_H(R - R_i - R_b) \geq p_L(R - R_i - R_b) + b \Leftrightarrow R_e \geq \frac{b}{\Delta p} \ or \ R_i + R_b \leq R - \frac{b}{\Delta p}$$

$$(ii) \quad p_H R_b - c \geq p_L R_b \Leftrightarrow R_b \geq \frac{c}{\Delta p}.$$

    d) Define the expected (gross) rate of return demanded by a bank, as $(1 + \beta)$, so that with (ii) as an equality, we must have:

$$p_H R_b \geq (1 + \beta)I_b \Rightarrow I_b \leq \frac{p_H R_b}{1 + \beta} = \frac{p_H c}{(1 + \beta)\Delta p} := I_b(\beta), \text{ where } I_b(\beta) \text{ is decreasing}$$

in β. What is the interpretation of $I_b(\beta)$? Show that $\beta > r$, and why? (Hint: Because of (ii), we have a positive second period profit $p_H R_b - c > 0$. This surplus will be reduced – due to competition in the first period – by the fact that banks put money in the entrepreneur's project, but not beyond $I_b(\beta)$.)

    e) Direct finance from external investors will then be $I_i = I - I_b(\beta) - A$, whenever this is positive. Show that for external investors to provide finance one must have: $p_H R_i \geq (1 + r)I_i \Leftrightarrow p_H(R - \frac{b + c}{\Delta p}) \geq (1 + r)\left[I - I_b(\beta) - A\right]$. Then if $A + I_i + I_b(\beta) \geq I$, the project will be financed. Show that this condition defines another critical level on initial cash holdings, below which no project will get finance: $\underline{A}(r, \beta) = I - I_b(\beta) - I_i = I - \frac{p_H c}{(1 + \beta)\Delta p} - \frac{p_H}{1 + r}(R - \frac{b + c}{\Delta p})$.

Hence only firms with $A \geq \underline{A}(r, \beta)$ will be financed. The function $\underline{A}(r, \beta)$ is increasing in both r and β. What is the interpretation? Suppose that $\underline{A}(r, \beta) < \bar{A}(r)$.

f) Turn to market equilibrium. If total bank capital available for credit is K_b (an exogenous variable), and if the distribution of cash holdings in the economy can be described by the cumulative distribution function $G(\cdot)$, then total demand for bank credit is $\left[G(\bar{A}(r)) - G(\underline{A}(r,\beta)) \right] \cdot I_b(\beta) := D_b(r,\beta)$. How will D vary with its arguments? If r is exogenous, then the condition $K_b = D(r,\beta)$ will determine β. What is the impact of a lower value of K_b (a "credit crunch")?

g) (A bit harder; may be skipped – can serve as a supplement to F&R). Suppose that supply of direct finance is an increasing function $S(r)$, whereas demand for direct finance is $D_i(r,\beta) = \int\limits_{\underline{A}(r,\beta)}^{\bar{A}(r)} \left[I - A - I_b(\beta) \right] dG(A) + \int\limits_{\bar{A}(r)}^{\infty} \left[I - A \right] dG(A)$

What can be said about $D_i(r,\beta)$? Market clearing for direct capital will then be given by $S(r) = D_i(r,\beta)$. (For any value of β, we'll have a unique market-clearing rate of interest $r(\beta)$.) Show that the two market-clearing conditions can be combined to give one condition stating that total demand for capital being equal to total supply of external capital: $\int\limits_{\underline{A}(r,\beta)}^{\infty} \left[I - A \right] dG(A) = K_b + S(r)$.

This can be used for comparative static analysis. What impact on equilibrium will a lower supply of banking capital have ("a credit crunch"), and what impact will a negative (or inward) shift in the supply of capital, $S(r)$, outside banks have ("a savings squeeze")?

Question 2 (both groups)
Discuss briefly, without any formal model, how the ideas from the Diamond-Dybvig model can help us to rationalize the interbank market.