Jon Vislie; August 2014
Hand-out ECON 4335 Economics of Banking

On Credit Risk and Contracting

In my Introductory Note I mentioned that banks are exposed to risk of various types. In the two first lectures we have studied risk issues related to liquidity risk (random supply of deposits (Tobin) and idiosyncratic liquidity demand (Diamond-Dybvig). Now we turn to market or credit risk as being identified on the asset side of a bank's balance sheet. (When there is no risk in making losses on loans, there should be no need for equity or bank capital, as was the case for the good bank equilibrium in the D-D-model.) In addition to contribute to the payment system in an economy another very important task for a bank is to pool or collect (short-term) deposits and make long-term loans – the so-called maturity transformation. On providing, granting or extending loans the bank will face the risk of making losses on these loans. That they in fact can face such risk should be rather obvious – some of a bank's borrowers might get into trouble because of a recession (due to unemployment, a borrower will not be able to repay the debt, etc.) or because of bad governance among the borrowers. That type of risk is part of the game. On the other hand banks are also better prepared for bearing risk than most of their borrowers; households and firms. The majority of borrowers is small (compared to banks) and they are risk averse (dislike lotteries) whereas banks can act in a more risk neutral fashion because they are able to diversify their loan portfolios so as to reduce the overall risk for making losses. (This is true whenever the projects that are financed have independent returns and have finite variance – The Law of Large Numbers.) Banks are, along with insurance companies, among the risk-sharing institutions of a modern society. That does not mean that banks are not worried about losses; they want, we hope, to provide loans to the right (profitable) projects and to write and enforce contracts with borrowers so that the amount of money lent to them is used in a way that is compatible with the lender's interest – and also for the benefit of the society.

In an ideal world a loan contract should specify in great detail the obligations of the parties in any contingency or any state of the world, how the money is used, the amount repaid by the borrower at a date agreed upon ex ante, under what conditions a loan is rolled over or renewed in the future, and what role the lender should have if the borrower should default. We will not go into all these issues – some of them are dynamic and very complicated – but we will take a less ambitious step by looking at some issues in a borrower-lender relationship, where the lender is a bank, and the borrower is an entreprenur who is exposed to some risk of making losses. (To make things simple, we assume, for a moment that a borrower has not access to other financing options.)

As noted above, a bank can experience a loss on a loan that can occur either from "a bad draw or simply due to being unlucky" – the loss is due to a random event leading to losses, not under control of the borrower – the borrower has a bad or unprofitable project, or the loss is due to a deliberate action (in the worst case, fraud), usually not observable by the lender, taken by the borrower so that the probability of success (high return) is reduced.

Before we come the transaction costs caused by "informational issues", let me just mention one important result: Efficient or first-best Pareto-optimal risk-sharing between two agents; one being risk averse and the other risk neutral, given that the risk cannot be affected by any party, is characterized by the risk-averse agent being fully insured, by having the same income or return in any state of the world, whereas the risk neutral agent should bear all risk. If such an insurance contract should be offered to a risk-averse agent when she is in a position to affect the probability for some event, say by exerting some costly and unobservable effort to increase the probability for high return or high output (the good state), full insurance will not induce the agent to exert effort. Full insurance will normally have bad incentive effects, because the agent now has no incentive to exert effort as the income is the same whatever he does and whatever state will be realized. Hence, in a more realistic setting, when such unobservable actions can be taken by the risk-averse agent, we have to trade off incentives and risk sharing, by linking the agent's income to realized output. Then we have to give up first-best, and instead look at second best risk-sharing. We say that the relationship is flawed by moral hazard. This type of principal-agent relationship is taken to the borrower-lender relationship. We want to design the loan contract so that a borrower will be induced or motivated to take proper actions (from both a social and the lender's point of view), while being insured against purely unlucky states of the world outside the borrower's control. This issue is the main topic when we get to moral hazard or to problems related to ex post non-verifiable actions that affect stochastically the outcome (success or failure) of undertaking a project. Another result is that we can ignore the issue of risk sharing, if both parties are risk neutral. In such a case the agent taking some unverifiable action can be given incentives for good performance by being made *residual claimant*. This means that the marginal effort in some sense is fully paid off or the agent will reap, at the margin, the full benefit from her action. If a borrower should be made residual claimant in a borrower-lender relationship, the bank will require a fixed repayment for the loan whereas the borrower will reap the remaining profits, but will also bear the losses if that should occur – but now at no cost – due to risk neutrality. But if the borrower is protected by limited liability, as we will assume, then we will have an interesting problem which involves important trade-offs between providing incentives along with how the repayment, constrained by limited liability, will vary with the realization of the stochastic return. (This moral hazard problem when the borrower and the lender are protected by limited liability is taken up in the second part of this note.)

Another informational issue is related to one where the borrower has some ex ante private exogeneous information about her type (the borrower's quality; good or bad, honest or dishonest). Because we find this problem "simpler" from an analytical point of view, we will start with contracting under pure private information. (The problem is similar to Akerlof's famous "Lemon Problem".)

1. Contracting under private ex ante information

Consider a lending-borrowing relationship where a borrower or an entrepreneur wants to finance a project that costs I = 1. The project has a random return Y that is type-dependent because the probability for success depends on whether the borrower is a good type or a bad type. (The type assignment is an intrinsic feature of the borrower that cannot be altered. Also, this information is known only to the entrepreneur, and in general a lender cannot distinguish between the various entreprenuers.) The lotteries seem rather simple, but we get the main idea even with this simple structure. We suppose that:

If a borrower is good; she has the lottery: $Y = \begin{cases} R & with\ prob\ p \in (0,1) \\ 0 & with\ prob\ 1-p \end{cases}$

If she is bad: $Y = \begin{cases} R & with\ prob\ q < p \\ 0 & with\ prob\ 1-q \end{cases}$

It is common knowledge that the fraction of good investors, here equal the probability for meeting a good entrepreneur is α (= the fraction of good investors in the population) and the probability of meeting a bad one is is $1-\alpha$. Define the lenders' prior probability of success as $m := \alpha p + (1-\alpha)q \in [q,p]$, which is the average probability of success in the population. Suppose that both parties are risk-neutral and that the borrower is protected by limited liability – which means that no repayment is made if failure; i.e., should the event $Y = 0$ be realized, she has no assets to pay bach her debt.

Suppose the lender offers a loan at some rate of interest. In that case, when borrowing one unit, a borrower's payoff is: $V^B = \pi[R-(1+i)]$, where $\pi \in \{q,p\}$, with i being the rate of interest charged (and repaid only if success) by a competitive banking industry having a fixed gross funding cost (equal to the opportunity cost – what can be achieved by lending abroad) per unit, as given by $1+f$.

A representative lender/bank has then the following payoff: $V^L = \pi(1+i)-(1+f)$, if lending out I = 1 to a borrower of unknown type. (If the project fails the bank will not get the loan repaid.)

We assume that the banking industry is competitive – in Lectures 5 and 6 we will discuss other aspects of competititon. We also assume that the good project is socially efficient in the sense that: $pR-(1+f)>0$. We analyze two cases as regarding the creditworthiness of a **bad** entrepreneur:

- One where $qR-(1+f)>0$; the bad project is creditworthy
- Another where the bad project is not creditworthy; $qR-(1+f)<0$.

a) Symmetric and perfect information (SI)

Let us have this case as a benchmark. The bank knows true type of a borrower. Suppose that the bad type is not creditworthy; hence we require $qR-(1+f)<0$. Then in a competitive equilibrium only the good entrepreneur gets his project financed, with a competitive rate of interest satisfying:

$V_{SI}^L = 0 = p(1+i_{SI}^G)-(1+f) \Leftrightarrow 1+i_{SI}^G = \dfrac{1+f}{p}$, which is the principal plus the interest rate charge required from a good entrepreneur. Hence in this case the payoff to the borrower is $V_{SI}^B = p(R-(1+i_{SI}^G)) = pR-(1+f)>0$; equivalent to the social surplus.

If, on the other hand the bad project is creditworthy as well, we have another type of loan, offered only to bad types, with $1+i_{SI}^B = \dfrac{1+f}{q} > 1+i_{SI}^G = \dfrac{1+f}{p}$. The rate of interest charged on bad loans is higher than the one charged on good loans if the bad project is profitable.

b) Asymmetric information

Let us turn to the asymmetric information case where only the borrower knows her true type. Suppose now that projects undertaken by both types are creditworthy, and suppose also that the banks, rather naïvely, offer the two contracts to a privately informed borrower. The borrower is then given the opportunity to select. (It is like a menu within which a guest at a restaurant can choose between two identical dishes but one with a higher price!) Then because the rate of interest is higher for a bad project than a good project, any rational entrepreneur will choose the contract intended for the good type. The bad type will mimick the good one, and obtain an expected payoff $qR - \frac{q}{p}(1+f) > qR - (1+f)$, because $p > q$. Such a contract is not information-revealing. We also say that this menu will not induce truth-telling!

The bank will in this case no break even, because with $1 + i_{SI}^{G} = \frac{1+f}{p}$, the expected profit to the bank is:

$$\alpha \underbrace{\left[p(i + i_{SI}^{G}) - (1+f)\right]}_{=0} + (1-\alpha)\underbrace{\left[q(1 + i_{SI}^{G}) - (1+f)\right]}_{=neg} = \alpha \cdot 0 + (1-\alpha)(1+f)\left[\frac{q}{p} - 1\right] < 0$$

Now, because of asymmetric information, the bank cannot, within this setting, do anything else than to offer one common (or pooling) rate of interest to any borrower; call this $\hat{i}$, such that expected profits in the banking industry becomes zero: $m(1+\hat{i}) = 1 + f \Rightarrow 1 + \hat{i} = \frac{1+f}{m} \in \left(\frac{1+f}{p}, \frac{1+f}{q}\right)$. The common rate of interest under asymmetric information will then be between the two we had under symmetric and perfect information; in particular it will exceed the one offered to good types under full information.

Our first conclusion then is that asymmetric information will increase the cost of capital for the best projects. This higher cost reflects an externality on the good types due to the presence of bad types in the population.

Question: What will be the impact of this rate setting?

A borrower of the two different types will have the following payoffs:

$$V^B(\hat{i}, good) = p\left[R-(1+\hat{i})\right] = pR - \frac{p(1+f)}{m} = \frac{p}{m}\left[mR-(1+f)\right]$$

$$V^B(\hat{i}, bad) = q\left[R-(1+\hat{i})\right] = qR - \frac{q}{m}(1+f) = \frac{q}{m}\left[mR-(1+f)\right]$$

We then observe, when taking into account that we have already assumed $pR-(1+f)>0$, that **if** we $mR-(1+f)<0$, then we must have $qR-(1+f)<0$, and **no borrower of any type will want to undertake the project at such a rate of interest leaving the banks with expected profits equal to zero.**

Even if good projects are socially desirable, along with bad projects not being creditworthy, the equilibrium (pooling) rate of interest $1+\hat{i} = \frac{1+f}{m}$, will prevent any type of borrower from outside financing; an extreme form for credit rationing. This is an example of Gresham's law: Good projects are driven out by bad projects.

Under what circumstances will this situation occur? This unfortunate case will occur if $\alpha < \alpha^0$, i.e. if the fraction of good projects is below some critical value, where $\left[\alpha^0 p + (1-\alpha^0)q\right]R = 1+f$. (Note that $m = q + \alpha(p-q)$ is increasing in α.) Hence, if $\alpha < \alpha^0$, no lending will take place if $qR-(1+f)<0$. Or, what is the same, if the fraction of bad – and not creditworthy – entrepreneurs is too high, then the financial market will break down. The good borrowers are hurt by the presence of bad borrowers, which will create a social cost, as we then have underinvestment. The banking industry will then not be able to fulfill its main task.

If on the other hand, $mR \geq 1+f$, then either both projects are creditworthy or the bad project is not, but in that case we have $\alpha \geq \alpha^0$ (a high fraction of good projects).

If the borrower is **good**, we have the following payoffs:

For the bank or lender:

$$V^L = p(1+\hat{i}) - (1+f) = p\frac{1+f}{m} - (1+f) = (1+f)\left[\frac{p}{m} - 1\right] > 0 \text{ because } p > m,$$

And for the (good) borrower:

$$V^B = p(R - (1+\hat{i})) = pR - \frac{p(1+f)}{m} = \frac{p}{m}\left[mR - (1+f)\right] \geq 0$$

On the other hand if the borrower is a bad type:

The payoff to the lender is:

$$V^L = q(1+\hat{i}) - (1+f) = (1+f)\left[\frac{q}{m} - 1\right] < 0 \text{ because } m > q$$

And for the borrower:

$$V^B = q(R - (1+\hat{i})) = \frac{q}{m}(mR - (1+f)) \geq 0$$

Hence, if the fraction of good projects is sufficiently high, along with bad projects being not creditworthy; $qR < 1+f$, then a rate of interest charged by the banking industry so as to make zero expected profits from lending, will lead to **overinvestment** as also the socially undesirable bad projects are undertaken along with the good projects. Ex post, the lenders lose money on the bad types, but makes money on the good types. There is **cross-subsidization**. Adverse selection will therefore appear here as lower average quality on the loans given to entrepreneurs – too many (bad) projects are implemented

The **required** rate of return on project financing has been higher because we have: If lending is socially desirable, i.e. if $mR \geq 1+f$ (if not; no lending at all), this condition can be expressed as:

$$mR \geq 1+f \Leftrightarrow \left[1-(1-\alpha)\frac{p-q}{p}\right]pR := (1-\Lambda)pR \geq 1+f$$, where Λ is called an index of adverse selection; cf. Tirole.

What does the index $\Lambda := (1-\alpha)\dfrac{p-q}{p}$ indicate? The threshold for accepting a good project has now become higher because of asymmetric information. We observe that the criterion can be written as: $pR \geq (1+f)+\Lambda pR$. The additional cost operates like the cost due to a negative externality caused by the presence of bad entrepreneurs. (These hurt the good borrowers.) A fraction $1-\alpha$ of the entrepreneurs is bad. The likelihood ratio is $\dfrac{p-q}{p}$ shows the relative reduction in success probability for a bad type. Per unit expected gross return from a good project (pR), the externality cost is given by $(1-\alpha)\dfrac{p-q}{p}$. This term captures the additional charge put on a borrower of good type so as to compensate for the unprofitable projects that are implemented due to asymmetric information. The good borrowers have to bear the additional cost.

The information-adjusted payoff to a borrower having a good project is then:

$$V^B = pR - p(1+\hat{i}) = pR-(1+f)+(1+f)-\frac{p(1+f)}{m} = pR-(1+f)-(1+f)\left[\frac{p}{m}-1\right]$$

The standard Net Present Value investment criterion, $pR-(1+f)$, is becoming stricter, with an additional term as given by:

$(1+f)\left[\dfrac{p}{\alpha p+(1-\alpha)q}-1\right]=(1+f)(1-\alpha)\dfrac{p-q}{m}=\dfrac{p}{m}(1-f)(1-\alpha)\dfrac{p-q}{p}$. Use the fact that $\dfrac{m}{p}=1-\Lambda$; hence we have that a good borrower's net payoff under asymmetric information is $V^B(good)=\underbrace{pR-(1+f)}_{std\ NPV}-(1+f)\dfrac{\Lambda}{1-\Lambda}\leq pR-(1+f)$.

The good borrowers are incurred an additional cost due to the presence of bad borrowers in the market. The inability for banks to distinguish between good and bad projects is translated into a higher rate of interest charges on loans, reducing the expected payoff for good entrepreneurs or borrowers. (In this model the good entrepreneurs have no way of signalling their quality – this would require a model with more stages, where for instance a good entrepreneur could take a costly action ex ante so as to signal her true type, an action that under some conditions will not be mimicked by a bad entrepreneur. This is a more complex model where one needs a more demanding equilibrium concept – like a Perfect Bayesian Equilbrium.)

2. Contracting and moral hazard

Consier now a very important problem with moral hazard, within a simple setting even though the analytics can seem hard. The goal is to design a debt contract by a lending bank, where the borrower can take an unverifiable action that affects the likelihood for success of a project; and hence the payoff to the lender, as well as the borrower.

The more general background for such an issue is: If I, in the role as principal, delegate a task to a person, called an agent, then I want the agent to take actions in my interest; hence I would like to reward good performance or compensate for high effort of the agent. An important feature of these models is that higher effort is beneficial to the principal but costly to the agent. In most cases only output, not input or effort, can be observed or verified. The relationship between output and input is then flawed by noise, which normally can be affected by the agent himself. (Without such noise, the level of effort could be directly inferred by observing output – hence no contractual problem. You can then design a contract so that the right effort would be exerted.) Output is then a stochastic variable. Hence, one might have a situation where the observable output is satisfactory in some sense, but the agent has shirked – he has been very lucky (in a relationship between a tenant farmer and a landlord, the weather was very good so the output became high even if the farmer was lazy), and then rewarded for wrong reason.

We will take this kind of problem to the borrower-lender relationship, and derive a debt contract when the borrower has limited liability and can take an unverifiable action that affects the probability distribution for the return from the project. Both the lender and the borrower will be risk neutral. We also assume that the lender is protected by limited liability.

The gross return from the project is Y being verifiable, defined on $\left[0,\infty\right)$, with a (continuously increasing and differentiable) cumulative probability distribution $F(y\left|e\right)$ that is conditioned on effort, e, chosen by the borrower. (y is some realization of the stochastic variable Y.) We can think of working more or less on the project will affect the return structure. We assume that effort will stochastically increase the return from the project by the first-order stochastic dominance relation, or more effort will produce an improvement in the return structure in a first-order stochastic way, in the sense that the distribution $F(y\left|e_2\right)$ dominates strictly in the first-order stochastic sense the distribution $F(y\left|e_1\right)$ for $e_2 > e_1$. This is the case if $F(y\left|e_2\right) < F((y\left|e_1\right)$. The probability distribution with $e = e_2$ puts more weight or mass in the upper tail than the one with the lower effort. (This assumption can be translated into $\frac{\partial F(y\left|e\right)}{\partial e} := F_e(y\left|e\right) < 0$.) The conditional density function is

$f(y|e) = \dfrac{\partial F(y|e)}{\partial y}$, with $\int_0^\infty f(y|e)dy = 1$ for any $e \geq 0$. We now make another important assumption, called *the monotone likelihood ratio property (MLRP)*, which can be stated more generally as $\dfrac{\partial}{\partial y}\left[\dfrac{f_e(y|e)}{f(y|e)}\right] > 0$, saying that the ratio $\dfrac{f_e(y|e)}{f(y|e)}$ increases with *y*, or with $e_2 > e_1$, says that the ratio $\dfrac{f(y|e_2)}{f(y|e_1)}$ is an increasing function of y. What this assumption says is that high return signals high effort. The posterior probability for high effort is higher the higher is *y*.

To make the problem as simple as possible, suppose that the borrower can either work $(e = 1)$ or not, with $e = 0$. According to first-order stochastic dominance we then have $F(y|e = 1) := F_1(y) < F(y|e = 0) := F_0(y)$. The corresponding conditional densities are $f_1(y)$ and $f_0(y)$. In addition, the *MLRP* can be expressed as $\dfrac{f_1(y)}{f_0(y)}$ being increasing in *y*. On observing a large output is evidence for the agent having chosen to work ($e = 1$), rather than not working.

Suppose the loan is of some given size I, granted by a bank, which is funded at a unit cost $(1 + f)$. To simplify, we put *I* = *0* without losing the main idea; this is just a constant. The bank's task now is to design "the best" repayment schedule, debt contract or repayment function, $R(y)$, as a function of the realized return from the project. (That must of course require that realized return is verifiable.) But at the same time, the lender will offer a contract so that the borrower is induced to exert the "right" amount of effort on the project.

The borrower has a cost function for effort as given by $c(e)$, which, in general, is twice differentiable, strictly increasing, and strictly convex for any $e > 0$, with $c(0) = 0, c'(0) = 0$ and $c'(\infty) = \infty$. (For the problem to be interesting, effort must be chosen before the stochastic return is realized.) With our binary effort options, we let

$c(1) = c > c(0) = 0$. We just impose that the lender wants the borrower to choose $e = 1$ rather than $e = 0$. The preferred probability distribution is therefore $F_1(y)$. For some realized value of the return, the borrower will have a net profit $y - R(y) - c(e)$ for some effort choice, whereas the repayment to the bank is $R(y)$. The main problem for the lender is to determine the entire repayment schedule (or the entire function) so as to maximize expected return, subject to a participation constraint for the agent (PC), an incentive constraint (IC) which induces the agent to choose the lottery preferred by the lender, and a limited liability constraint (LL) as we assume that the borrower has no assets and therefore cannot bear any loss. With Y being continuous, we will solve the following problem:

$$Max_{\{R(\cdot)\}} \int_0^\infty R(y) f_1(y) dy$$

s.t. $\int_0^\infty \left[y - R(y)\right] f_1(y)dy - c \geq 0$ (PC); 0 is the outside option - reservation utility)

$\int_0^\infty \left[y - R(y)\right] f_1(y)dy - c \geq \int_0^\infty \left[y - R(y)\right] f_0(y)dy$; (IC)

$y - R(y) \geq 0$ for any $y \geq 0$ (LL)

Our first observation is that (PC) is automatically satisfieid if (IC) and (LL) hold. The problem can be refomulated to be written as:

$$Max_{\{R(\cdot)\}} \int_0^\infty R(y) f_1(y) dy \text{ s.t. } \begin{cases} \int_0^\infty \left[y - R(y)\right][f_1(y) - f_0(y)]dy - c \geq 0 \\ R(y) \in \left[0, y\right] \quad for\ any\ y \geq 0 \end{cases}$$

Let λ (a positive constant) be a multiplier on (IC), which is binding. Then we can write a restricted Lagrangan for the problem without LL, as:

$$L = \int_0^\infty \{R(y) + \lambda[\{y - R(y)\}\frac{f_1(y) - f_0(y)}{f_1(y)} - c]\}f_1(y)dy$$

Without going into too many mathematical details, one should use the intuition to verify that an optimal repayment schedule must obey:

For any value of $y > 0$: $1 - \lambda\frac{f_1(y) - f_0(y)}{f_1(y)}\begin{cases} > 0 \Rightarrow R(y) = y \\ < 0 \Rightarrow R(y) = 0 \end{cases}$

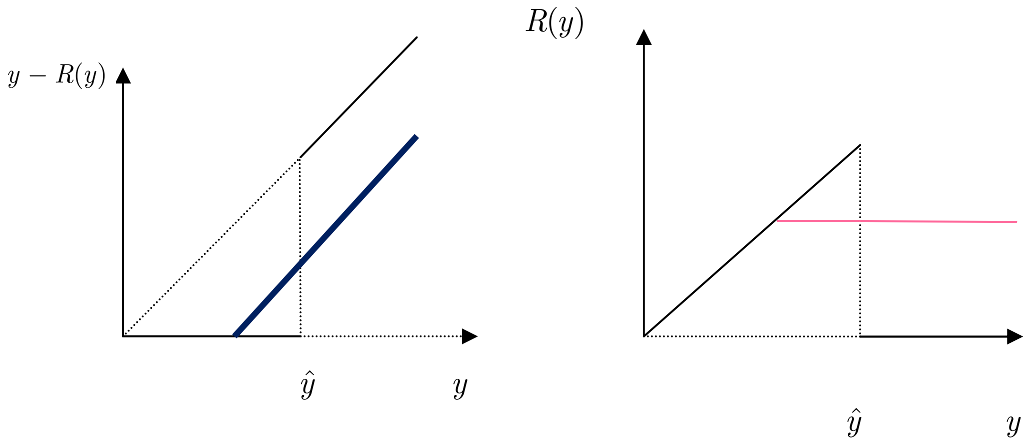
Because of *MLRP*, saying that $\frac{f_1(y)}{f_0(y)}$ is increasing in y, we have

$\frac{f_1(y) - f_0(y)}{f_1(y)} = 1 - \frac{f_0(y)}{f_1(y)}$ will increase with *y*; so $1 - \lambda\frac{f_1(y) - f_0(y)}{f_1(y)}$ will be declining in *y*. Hence there will exist a critical return value, $\hat{y}$ so that:

The lender: $R(y) = \begin{cases} y & \forall y < \hat{y} \\ 0 & \forall y > \hat{y} \end{cases}$ and the borrower: $y - R(y) = \begin{cases} 0 & \forall y < \hat{y} \\ y & \forall y > \hat{y} \end{cases}$

In this debt contract, nothing is repaid if the return is sufficiently high; i.e. the borrower takes all the return in cases where it is strong evidence that the borrower has worked hard. For all values below this critical value, the return accrues to the lender. This contract gives maximal reward to the borrower equal to y when $y > \hat{y}$ and has maximal penalty if realized return is below this critical value.

This reward structure is depicted below:



This repayment structure suffers from the following problem: Suppose the borrower has a return close to the critical value, but "just below it". Then on colluding with a third party by borrowing money so as to manipulate the accounting system to get a reported return just above the critical value, the borrower takes all the return. (The gain to be shared between the borrower and the helper can then be rather significant!) To avoid such behaviour, the lender can offer the borrower to reap the full return, with some consequence for the lender's payoff as well. One therefore requires that the repayment schedule $R(\cdot)$ to be everywhere non-decreasing, with no downwards jump as in the original contract.)