ECON 4335 – fall 2014

Problem set 3 – seminar #3 (September 16, 2014)

The following problem is an extension of the asymmetric information contracting problem with a competitive banking industry, presented in class on September 1. Rather than having only one instrument, as in the simplest case, we now turn to a situation where a monopolistic bank has two instruments: the size of a loan and the rate of interest charged on borrowers. This provides an opportunity for the bank to screen loan applicants.

Consider a borrower-lender relationship between a monopolistic lender – a bank – and a borrowing firm, where the bank provides the firm with a loan. The loan is to be used to buy new equipment (with a price set equal to one). Let the size of the loan be k. If the bank's funding rate is f, and the rate of interest charged per unit of a loan, is r, the bank's net return from extending or granting a loan of size k to some entrepreneur will be $V = \left[(1+r)-(1+f)\right]k = (r-f)k$.

The borrowing firm has a profit or rent as given by $U = sF(k)-(1+r)k$, where $F(k)$ is a standard (neo-classical) production function; with $F(0)=0$, $F'(k)>0$, $F''(k)<0$ with $\frac{F(k)}{k}$ being declining in k. The parameter s is private information (known only by the firm) and can be interpreted as a type-parameter or a productivity shock, with $s \in \{\underline{s}, \overline{s}\}$. It is common knowledge that the probability distribution over types is given by $\Pr(s=\underline{s})=p$, for being a low-type, and $\Pr(s=\overline{s})=1-p$, for being a high-type; $\overline{s} > \underline{s} > 0$.

Assume that the bank's objective is to maximize expected profits. However, the lender cannot tell the potential borrowers apart. The borrower will only accept a contract, stipulating a pair $\left\{k, 1+r\right\}$, leaving her with a non-negative profit.

1. What would be the lender's optimal contract under complete and symmetric information; as given by a pair $\left\{k, 1+r\right\}$, one for each type of the borrower, when a borrower's payoff is driven down to zero (her reservation payoff)? How will the size of the loan and the rate of interest offered depend on s? Illustrate how the "spread" is determined.

2. When only the borrower knows her true type ex ante, as given by a value of s, show that if the first-best contract set above should be offered, a $\overline{s}$-firm would pretend to be a low-type firm. Explain why!

3. How can the lender, by properly designing the set of second-best contracts, induce the high-type to choose the contract designed for her? Interpret your results!

(Hint: The “trick” is to offer a menu of contracts, $\left\{(\underline{k},1+\underline{r}),(\overline{k},1+\overline{r}\right\}$, one for each type, so as to get each type of borrower to select the one designed for her. This is the incentive constraint – one for each type. Because only one of the types, check which one of them, will pretend to be the other type if offered the first-best contract, you have to put up a self-selection constraint (like an incentive compatibility constraint) for that type, leaving her with a contract not inferior to accepting the one offeed to the other type.

4. How is the optimal second best contract affected by:
   - a higher funding cost for the bank,
   - a greater dispersion in the distribution of s, via a higher $\overline{s}-\underline{s}$,
   - having $\underline{s}-\dfrac{1-p}{p}(\overline{s}-\underline{s})\leq 0$?