

ECON 4335 – fall 2014

Problem set 4 – seminar #4 (September 23, 2014)

Problem 1

In an economy we have a large number of risk-neutral entrepreneurs who are protected by limited liability, and have no initial wealth. The entrepreneurs undertake projects at a cost $I = 1$. The financing of each project is done by a monopolistic and risk-neutral bank. Projects can be of two types: A “safe” – undertaken by “good” entrepreneurs, or “risky” – undertaken by “bad” entrepreneurs. The type of an entrepreneur is known only to the entrepreneur himself (private information). The fraction of “good” entrepreneurs in the economy is α ; $0 < \alpha < 1$. The bank’s funding cost is $1 + f$ per project, but we can assume here that $f = 0$. The safe project has a gross return G with probability p , with probability $1 - p$ it fails with gross return equal to zero. The risky project has a gross return B with probability q , and a zero return with probability $1 - q$. Assume that $1 > p > q > 0$, $B > G > 0$, and $pG = qB = \mu > 1$. Define also $m := \alpha p + (1 - \alpha)q$ as the average “success” probability in the economy. (The risky project has variance, $\mu(B - \mu)$, which exceeds the variance of the safe project $\mu(G - \mu)$.) All agents know the probability distributions; but only each entrepreneur knows (in general) his/her true type, but the distribution is common knowledge.

- a) Assume first complete information. Show then that the bank can extract all profits by extending loans at terms that depend on the type of the entrepreneur.
- b) Suppose next that only the entrepreneurs know their type. Illustrate how the bank’s expected gross return will vary with $1 + r$, where r is a rate of interest paid on a loan, and explain how the mixture of loan applicants will change with the rate of interest.
- c) What interest rate will a profit-maximizing monopolistic bank choose? What is critical to your answer?

Problem 2

Consider a risk-neutral firm, protected by limited liability, that wants to finance a project of size one. The firm has no initial wealth; hence to undertake the project the firm has to borrow. After a loan has been granted and the contract being signed

(claiming that R is to be repaid if the project succeeds), the firm might take an unverifiable action regarding the riskiness of the project. The lender knows that the borrower may affect the probability of repayment by such an action, and might want to induce the borrower to choose a safe project.

At the stage when the firm can make a choice, it has two options. The first one is to choose a safe project, as given by the lottery $\{(p, G); (1 - p, 0)\}$, telling us that with probability p the return is G ; but zero otherwise. The other option is to choose a risky project, as given by $\{(q, B); (1 - q, 0)\}$. We assume that $pG > 1 + i > qB$, and $B > G$, where i is the risk-free rate of interest. (Let $i = 0$ in the following.) Hence we have $p > q$. (The risky project has a higher gross return if success, but will succeed less often.) Suppose that only success can be verified (not the size of the return or what project has been chosen). For some repayment R expected net return to a borrower is $p(G - R)$ if the safe project is chosen and $q(B - R)$ if the risky project is chosen.

1. Provide a graphical illustration of how the payoffs to the firm vary with R , and derive a critical value of R , denoted \hat{R} , below (above) which the firm will choose the safe (risky) project.
2. What conditions have to be satisfied in a (competitive) credit market equilibrium with only direct finance (no banks)?

Let us introduce banks or monitors, and ask whether banks can save us from not undertaking profitable (safe) projects. A bank is assumed to have access to a monitoring technology such that by incurring a monitoring cost, c , it is able to prevent the firm from undertaking the risky project, and choose the safe one if that is profitable. Suppose that the banking sector is still competitive.

3. Derive the conditions for a competitive equilibrium with bank lending.
4. For what parameter values (values of p) will we have only direct finance, only bank lending and no lending, respectively?