Seminar 2.

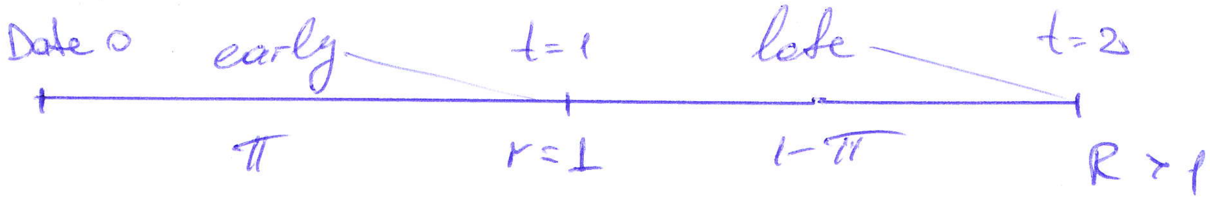
Diamond and Dybvig

one good

I ~ long term invest $t=0$
$1-I$ ~ short term invest. $t=0$

two-type (patient vs impatient) consumers

three date economy $t=0,1,2$ (ex-ante, interim, ex-post)



$t_1 = C_1 \Rightarrow$ impatient

$t_2 = C_2 \Rightarrow$ patient

$u(C) = \frac{1}{1-s} C^{1-s}$, $s>1$

$u'>0, u''<0, \; u'(0)=\infty$

Short term project $\Rightarrow t=1 \Rightarrow r=1$

Long term project $\Rightarrow t_2 = R>1$

if liquadated at $t_1 \Rightarrow 0 \le L_1 \le 1$

1. Derive the allocation that maximizes social welfare

$$\max_{C_1, C_2} U = \pi u(C_1) + (1-\pi) u(C_2)$$

$$\text{s.t} \begin{cases} \pi C_1 = 1 - I \\ (1-\pi) C_2 = RI \end{cases}$$

$$\pi C_1 = 1 - \frac{1-\pi}{R}$$

$$\Rightarrow \pi C_1 + \frac{1-\pi}{R} C_2 = 1 \quad (\text{book } i_1 + i_2 = 1)$$

$$\mathcal{L} = \pi u(c_1) + (1-\pi)u(c_2) - \lambda\left[\pi c_1 + \frac{1-\pi}{R}c_2 - 1\right]$$

$$\frac{\partial \mathcal{L}}{\partial c_1} = \cancel{\pi}\, u'(c_1) - \lambda \cancel{\pi}$$

$$\frac{\partial \mathcal{L}}{\partial c_2} = \cancel{1-\pi}\, u'(c_2) - \lambda \frac{\cancel{1-\pi}}{R}$$

$$\frac{u'(c_1^*)}{u'(c_2^*)} = R \qquad MRS$$

$$\frac{d\,u(c)}{dc} = c^{-s}$$

Inserting this into MRS

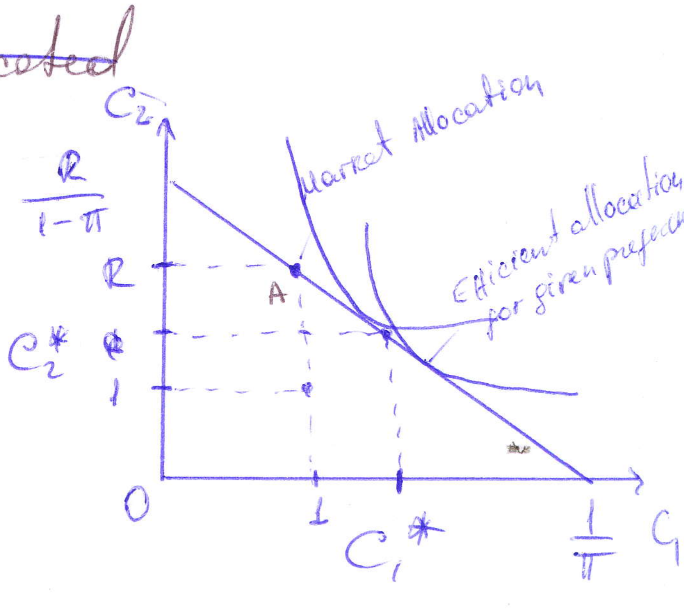
$$\boxed{\frac{c_2}{c_1} = R^{\frac{1}{s}}}$$

$\cancel{R > c_2^* > c_1^* > 1}$

~~How is the initial wealth allocated~~

$$c_1 = \frac{1-I}{\pi}$$

$$c_2 = \frac{RI}{1-\pi}$$



$$\frac{C_2}{C_1} = \frac{\frac{RI}{1-\pi}}{\frac{1-I}{\pi}} = R^{1/s}$$

$$\frac{RI}{1-\pi} \cdot \frac{\pi}{1-I} = R^{1/s}$$

$$\frac{RI}{1-I} = \frac{1-\pi}{\pi} R^{1/s}$$ to find I

$$\frac{1-I}{RI} = \frac{\pi}{1-\pi} R^{-\frac{1}{s}}$$

$$\frac{1}{I} - 1 = \frac{\pi}{1-\pi} \cdot R^{-\frac{1}{s}} \cdot R^{-1}$$

$$\frac{1}{I} = \frac{\pi}{1-\pi} \cdot R^{-\frac{1-s}{s}} + 1$$

$$I = \frac{1}{\underbrace{\frac{\pi}{1-\pi} \cdot R^{-\frac{1-s}{s}} + 1}_{>1}} < 1$$

$$\pi \in (0;1), \quad R > 1, \quad s > 1$$

$$\boxed{R > C_2^* > C_1^* > 1}$$

Since the coefficient of relative risk aversion is $s > 1$ then consumptions at two dates $\{C_1^* C_2^*\}$ are very cl
Which means that yield curve is downwardsloping

making interest on long term project very close to the short term one.

Will there be any liquadation ?

$$\frac{d}{dC}(C u') < 0 \qquad \text{and} \qquad R > 1$$

$$1 \cdot u'(1) > R \cdot u'(R) \qquad (C_1 = 1 \quad C_2 = R)$$

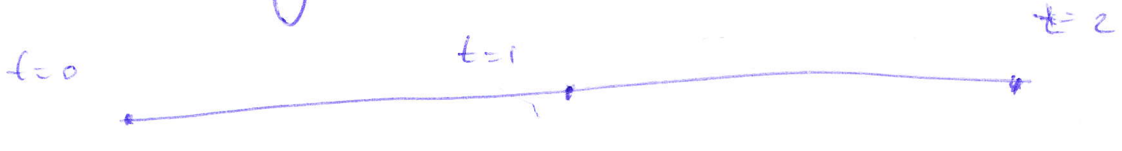
$$\frac{u'(1)}{u'(R)} > R \Rightarrow \boxed{R > C_2^* > C_1^* > 1}$$

Since $u'' < 0$ $\quad R > 1$

$$u'(C_1^*) > u'(C_2^*) \Rightarrow C_1^* < C_2^*$$

So incentive compatibility is met

No late consumer will have desire to pretend as an early consumer. if they do will face loss of utility.



2. Who will have higher consumption?

Optimal consump. $\{C_1^* \; C_2^*\}$

$s \to \infty$ RRA moves yield curve downward sloping. Uneven distribution will be optimal

$$\lim_{s \to \infty} \frac{C_2}{C_1} = \lim_{s \to \infty} R^{1/s} = 1.$$

In other word ~~revenue~~ interest on long term deposit is compounded but still does not exceed shor term ?!

3. Banks. ⇒ optimal allocation ⇒ equilibrium.

the same envirement slightly different than the previous one having been intermediary which allocates consump. to reach equilibrium = optimality. Due to competetiveness and free entry in Banking sector, bank profits are zero Banks will match maturity structure and liquidity needs for ~~dep~~ people. (consumers). Now Soc. Pl.

$$\max_{C_1 C_2} \left[\pi u(C_1) + (1-\pi) u(C_2)\right] \quad \text{s.t } (1-\pi) C_2 = (1-\pi C_1) R$$

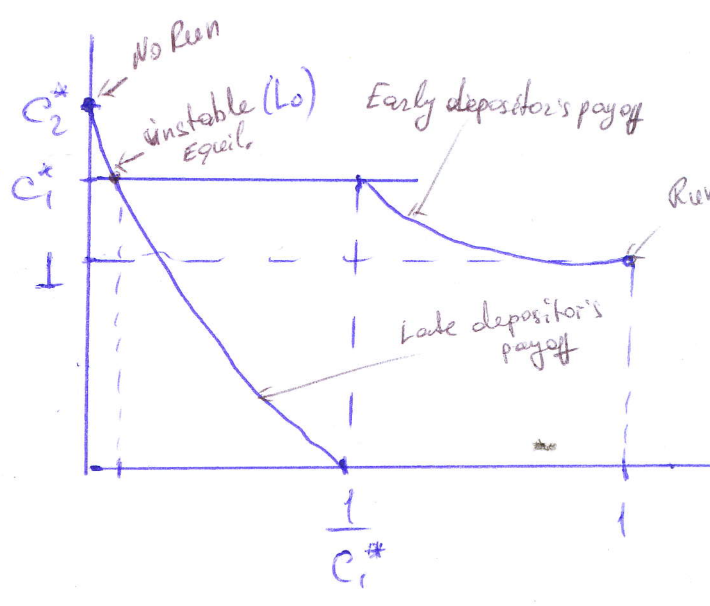
Zero-profit condition says that what is left for late depositors is equal to the long term return from lending taking into considezation liquidity reserves for early consumers (depositors. therefore, the amount

that is lent out at $t=0$ gives gross return R.
thus, under perfect competition in banking sector C_1^*, C_2^* can be realized as an equilibrium having ZP condition as a constraint. ~~Taking~~ Deriving the FOC. will give

$$\frac{u'(C_1)}{u'(C_2)} = R$$ the same result as before

Note: Another requirement this equilibrium to hold can be beliefs on who is early & late consumer.

4. Why there are two Nash eq. when Banks offer $[C_1^* \ C_2^*]$?
As we know NE states no party (here early vs late) has an incentive to deviate from his or her chosen strategy considering an opponent's choice. In this particular "Bank runs" model ~~even~~ either everybody is an early consumer withdrawing at $t=1$ or where the only early consumers withdraw at $t=1$ and we still have left late (patient) depositors. Former case is the famous problem in "Bank Run" model hearing some late depositor's liquadation action all other late depositor also run to withdraw at $t=1$.
Since $L=1$ everybody is early withdrawer (bad NE)



5. Financial Market. $(t=1)$ Still NE ?

Same contact $\{C_1^* \; C_2^*\}$ offered at $t=0$

Bonds pric p for late consumers $pR = 1 \Rightarrow R = \frac{1}{p}$

To understand the basic intuition behind bad market [JV p6.7]

Each unit bond sold gives p unit of consuming at $t=2$

$$C_1 = 1 - I + pRI \quad \text{for Early}$$

$$C_2 = RI + \frac{1}{p}\cdot(1-I) = \frac{1}{p}[1 - I + pRI] \quad \text{late cons.} \quad \text{since } R = \frac{1}{p}$$

$\frac{1-I}{p}$ ~ converted surplus for late consumer.

Now $C_1 = pC_2$ Necessary conditions for bond market;

$$p \leq 1 \Leftrightarrow \frac{1}{p} \geq 1$$

- if $p \geq 1$ or $\frac{1}{p} \leq 1 < R$ then no incentive for buy/sell bonds since rate of return ~~investing in~~ illiquid assets is higher
- Since in this model Investors are rational then they choose $I = 1$ which makes bond market very attractive at $t=1$. (No Equilibrium yet)

- if $\frac{1}{p} > R$ then $I = 0 \Rightarrow$ Excess supply of good - bond shortage

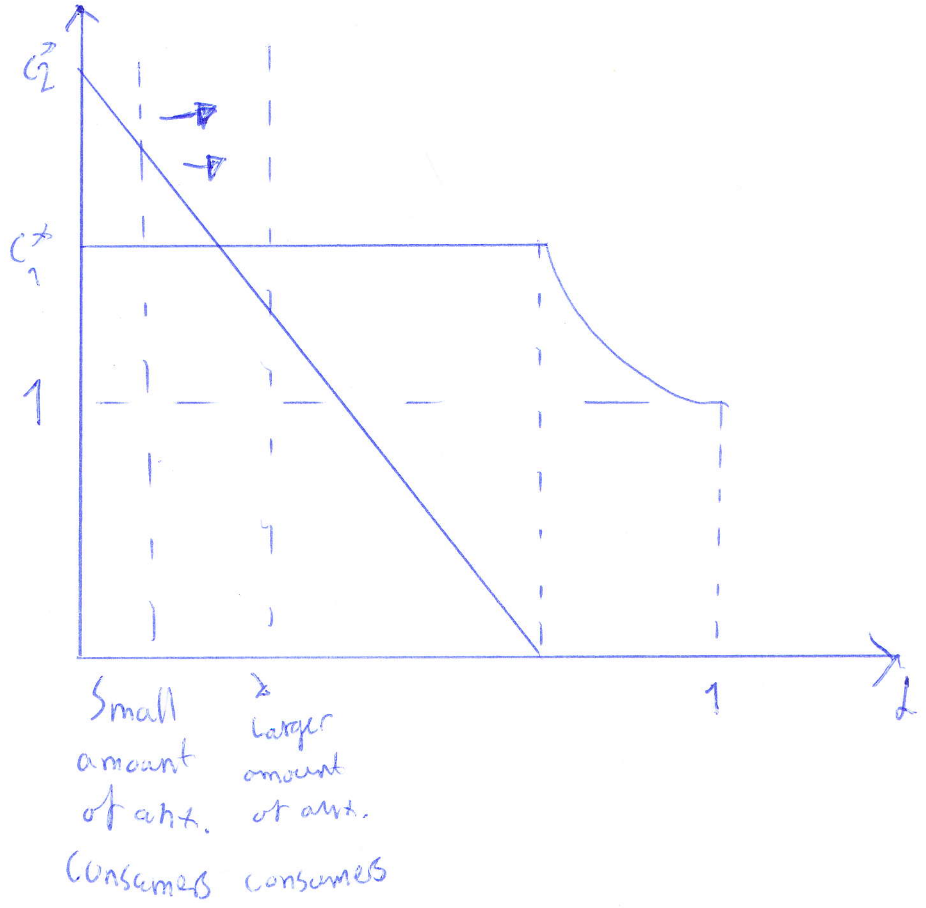
Return for bonds > Return for ~~good~~ illiquid project

No Eq. in both cases

Only $1 < \frac{1}{p} = R > C_2^*$ Markets are clear, Therefore

$$\pi R I = (1-\pi)\frac{1-I}{p} \qquad \text{Supply of Bonds} = \text{Demand of Bonds}$$

Problem 6



Explained: If the amount of anxioces individuals are sufficently small then there will be two Nash-equilibria as stated in earlier problem, But if the amount would be higher them critical point (~~m~~ dotted line to the right) there would only exist one equilibrium, the "bad" Nash-equilibrium.