Problem set 3 Econ 4335

(1) Borrower's reservation payoff:

$sF(k) - (1+r)k = 0$ for $s \in \{\underline{s}, \bar{s}\}$

$$\Rightarrow 1+r = s\frac{F(k)}{k}$$

Bank maximation problem then becomes:

$$\max_{\{k\}} V = (r-f)k \quad \text{s.t. } r = s\frac{F(k)}{k} - 1$$

$\Leftrightarrow$

$$\max_{\{k\}} sF(k) - k - fk$$

FOC: $sF'(k) - (1+f) = 0$

$$\boxed{F'(k) = \frac{1+f}{s}}$$ determines $\{\underline{k}, \bar{k}\}$ for $\{\underline{s}, \bar{s}\}$

While the interest rate is determined by:

$$r = s\frac{F(k)}{k} - 1$$

Determines $\{\underline{r}, \bar{r}\}$ for $\{\underline{s}, \bar{s}\}$ by using the FOC.

How will k & r depend on s?

Differentiate FOC wrt. s:

$$F''(k)\frac{\partial k}{\partial s} = -\frac{1+f}{s^2}$$

$$\Rightarrow \frac{\partial k}{\partial s} = -\frac{\overbrace{1+f}^{>0}}{\underbrace{s^2}_{>0}\underbrace{F''(k)}_{<0}} > 0$$

So being the good type increases the size of the loan.

What about r? Differentiate wrt s:

$$r = s\frac{F(k)}{k} - 1$$

$$\frac{\partial r}{\partial s} = \underbrace{\frac{F(k)}{k}}_{+} + s\cdot\underbrace{\frac{\partial}{\partial s}\left(\frac{F(k)}{k}\right)}_{-} \gtrless 0 \,?$$

We don't know what will happen to the interest rate. An increase in s will increase the interest rate directly, but also increase k. Because $\frac{F(k)}{k}$ is declining in k, this will lead to a decrease in r. In total, we don't know the sign of $\frac{\partial r}{\partial s}$ without further information about F(k).

(2) Asymmetric information: Bank offers two contracts: $\{\underline{k}, 1+\underline{r}\}$ & $\{\bar{k}, 1+\bar{r}\}$.

If the high-level firm chooses the contract $\{\bar{k}, 1+\bar{r}\}$, the payoff will be:

$$\bar{U} = \bar{s}F(\bar{k}) - (1+\bar{r})\bar{k}$$

Bank's optimization gives $\bar{U} = 0$.

If high-level firm chooses the other contract $\{\underline{k}, 1+\underline{r}\}$, their payoff will be:

$$\bar{U}_{low} = \bar{s}F(\underline{k}) - (1+\underline{r})\underline{k}$$

And since $\underline{U} \geq 0$ & $\bar{s}F(\underline{k}) > \underline{s}F(\underline{k})$, this implies that $\bar{s}F(\underline{k}) - (1+\underline{r})\underline{k} \geq 0$. The high-level firm will try to mimic the low-level firm in order to get a lower interest rate and get positive profits.

(3) The bank maximizes expected profits:

$$\max_{\{\bar{k}, \underline{k}\}} \; p(\underline{r} - f)\underline{k} + (1-p)(\bar{r} - f)\bar{k}$$

$$\text{st.} \quad \begin{array}{ll} (1) & \bar{s}F(\bar{k}) - (1+\bar{r})\bar{k} \geq 0 \\ (2) & \underline{s}F(\underline{k}) - (1+\underline{r})\underline{k} \geq 0 \end{array} \Bigg\} \text{ Participation constraints}$$

$$\begin{array}{ll} (3) & \bar{s}F(\bar{k}) - (1+\bar{r})\bar{k} \geq \bar{s}F(\underline{k}) - (1+\underline{r})\underline{k} \\ (4) & \underline{s}F(\underline{k}) - (1+\underline{r})\underline{k} \geq \underline{s}F(\bar{k}) - (1+\bar{r})\bar{k} \end{array} \Bigg\} \text{ Incentive constraints}$$

Only $\bar{s}$-type tries to mimic $\underline{s}$-type. Need only (3), not (4).

(2) & (3) implies (1). When $\bar{U}$ has to be bigger than some positive value, it certainly has to be bigger than 0.

So we are left with (2) & (3). They hold with equality in optimum. The problem becomes:

$$\max_{\{\bar{k}, \underline{k}\}} \; p\left(\underline{s}F(\underline{k}) - (1+f)\underline{k}\right) + (1-p)\left[\bar{s}F(\bar{k}) - \Delta s F(\underline{k}) - (1+f)\bar{k}\right]$$

where $\Delta s = \bar{s} - \underline{s}$

FOC'S: $p(\underline{s}F'(\underline{k})-(1+f)) + (1-p)(-\Delta S)F'(\underline{k}) = 0$

$$\cancel{(1-p)}\left[\bar{s}F'(\bar{k}) - (1+f)\right] = 0$$

$$\Rightarrow (5)\ F'(\bar{k}) = \frac{1+f}{\bar{s}}$$

$$(6)\ F'(\underline{k}) = \frac{p(1+f)}{p\underline{s} - (1+p)\Delta S} > \frac{1+f}{\underline{s}}$$

The bank now offer the $\{\underline{k}, 1+\underline{r}\}$ contract with a higher interest rate compared to the FB solution. Because of the incentive constraint, the $\bar{s}$-firm does not want to mimic the $\underline{s}$-firm anymore. The higher interest rate in the $\{\underline{k}, 1+\underline{r}\}$ contract scares the $\bar{s}$-firm off.

(4) • Higher $f \Rightarrow F'(k) \uparrow$ for $\{\underline{k}, \bar{k}\}$.

Which means that $k \downarrow$ & $r \uparrow$ for $\{\underline{k}, \bar{k}\}$.

- When the funding cost of the bank goes up, interest rate on loans also goes up and loan size down for both types.

• $\Delta S \uparrow$ does not affect $\bar{k}$ or $\bar{r}$ unless $\bar{s}$ goes up, but will affect $\underline{k}$ and $\underline{r}$ in any case.

As we see from the FOC wrt $\underline{k}$,

$$F'(\underline{k}) = \frac{p(1+f)}{p\underline{s} - (1-p)\Delta S}.$$

This means that if ΔS increases, $F'(\underline{k})$ goes up $\Rightarrow$ $\underline{k} \downarrow$ and $\underline{r} \uparrow$.

Because ΔS implies that the high-level type can achieve a bigger payoff by mimicking the low-level type. We then need a higher $\underline{r}$ in order to scare the $\bar{s}$-firm from choosing the $\{\underline{k}, 1+\underline{r}\}$ contract.

• $\underline{s} - \frac{1-p}{p}(\bar{s} - \underline{s}) \leq 0 \iff$

$$p\underline{s} - (1-p)\Delta S \leq 0$$

Look at the FOC again: $F'(\underline{k}) = \frac{p(1+f)}{p\underline{s} - (1-p)\Delta S}$

Which under this condition becomes negative, which is not possible! The bank cannot set high enough $\underline{r}$ in order to scare the $\bar{s}$-type off the $\{\underline{k}, 1+\underline{r}\}$ contract. The bank gets 0 profits.