Econ 4335: Seminar 9 (Exam, spring 2013)

Problem A

1. **Assume that at time 0 the bank has a given level of equity, $\boldsymbol{E}$. Use the balance sheet for period 0 to show that a) if the bank lends $\boldsymbol{L}$ it needs to collect deposits:**

$$\boldsymbol{D = \frac{L - E}{1 - \phi}}$$

And b) the maximum amount that the bank can lend is:

a)

In the balance sheet we must have that assets are exactly equal to liabilities. This structure gives us the following relation:

$$L + P = D + E$$

Using that we know that $P = \phi D$, we get the following:

$$L + \phi D = D + E$$

$$L - E = D - \phi D$$

$$L - E = D(1 - \phi)$$

$$D = \frac{L - E}{1 - \phi}$$

b)

We know that assets are equal to liabilities. If the bank is able to expand its liabilities (increase its deposits or raise equity) it can lend more. We can express lending the following way:

$$L = E + D - P$$

Which indicates that lending increases with D & E, but decreases when P increases.

$$L = E + D - \phi D$$

$$L = E + D(1 - \phi)$$

Because $0 < \phi \leq 1$ we know that the term inside the bracket is positive. If the bank increases deposits with one unit, it can increase lending by $(1 - \phi)$ units, while if it increases equity with one unit, it can increase lending by one unit.

$$L = E + (1 - \phi)D$$

Taking into consideration that E is given, this equation tells us that a bank that wants to maximize its lending, must maximize D. As we have seen the level of deposits is given by:

$$D = \frac{L - E}{1 - \phi}$$

D is maximized when E is as little as possible. From the text we know that $E \geq P$, thus the smallest amount of equity the banks can have is $E = P$. This gives us:

$$L = E + D - P$$

$$L = P + D - P$$

$$L = D$$

$$L = \frac{P}{\phi}$$

$$L^{max} = \frac{E}{\phi}$$

2. **The payout from the insurance fund is:**

$$\boldsymbol{\tilde{S}} = \begin{cases} \mathbf{0} & \boldsymbol{if\ \tilde{L} \geq D} \\ \boldsymbol{D - L} & \boldsymbol{if\ \tilde{L} < D} \end{cases}$$

Show how you can express the net profits of the bank's owners, $\boldsymbol{\Pi = \tilde{V} - E}$ in terms of $\boldsymbol{L}$, $\boldsymbol{\tilde{L}}$ and $\boldsymbol{E}$ for each of the two cases in (2).

Again we can use the balance sheet to define:

$$\tilde{V} = \tilde{L} + \tilde{S} - D$$

This gives us the following payoff in the case where $\tilde{L} \geq D$

$$\Pi = \tilde{V} - E$$

$$\Pi = \tilde{L} + \tilde{S} - D - E$$

$$\Pi = \tilde{L} + 0 - \frac{L - E}{1 - \phi} - E$$

$$\Pi = \tilde{L} - \left(\frac{L - E}{1 - \phi}\right) - \frac{E(1 - \phi)}{1 - \phi}$$

$$\Pi = \tilde{L} - \left(\frac{L - E + E(1 - \phi)}{1 - \phi}\right)$$

$$\Pi = \tilde{L} - \frac{L + E(1 - \phi - 1)}{1 - \phi}$$

$$\Pi = \tilde{L} - \frac{L - \phi E}{1 - \phi}$$

In the case where $\tilde{L} < D$

$$\Pi = \tilde{V} - E$$

$$\Pi = \tilde{L} + \tilde{S} - D - E$$

$$\Pi = \tilde{L} + (D - \tilde{L}) - D - E$$

$$\Pi = D - D - E$$

$$\Pi = -E$$

We see that the losses of the bank owners are restricted to the loss of their equity. Thus we have a form of limited liability as the bank owners are not responsible for repaying the deposits when they make losses on loans. The deposits they are unable to repay themselves, are repaid by the insurance fund.

3. **Suppose the gross repayment on loans is $(\boldsymbol{R} + \boldsymbol{\Delta})\boldsymbol{L}$ with probability $\mathbf{1/2}$ and $(\boldsymbol{R} - \boldsymbol{\Delta})\boldsymbol{L}$ with probability $\mathbf{1/2}$. Assume that $\boldsymbol{R} > \mathbf{1}$ and that $\boldsymbol{R} - \mathbf{1} < \boldsymbol{\Delta} < \mathbf{1}$. Show that there is no risk that the bank needs to be bailed out by the insurance fund if it lends less than**

$$\boldsymbol{L^C} = \frac{\boldsymbol{E}}{\mathbf{1} - (\mathbf{1} - \boldsymbol{\phi})(\boldsymbol{R} - \boldsymbol{\Delta})} < \boldsymbol{L^{max}}$$

The bank is bailed out whenever the realized repayment on the loans in period 1 is less than the deposits $\tilde{L} < D$. Whenever the realized repayment is larger than the deposits $\tilde{L} \geq D$ the bank doesn't need to be bailed out. This implies that as long as the lowest repayment the banks can possibly achieve is higher than this level, there will be no risk that the bank will need to be bailed out.

$$(R - \Delta)L \geq D$$

$$(R - \Delta)L \geq \frac{L - E}{1 - \phi}$$

$$L(R - \Delta) - \frac{L}{1 - \phi} \geq \frac{-E}{1 - \phi}$$

$$L\left((R - \Delta) - \frac{1}{1 - \phi}\right) \geq \frac{-E}{1 - \phi}$$

$$L\big((R - \Delta)(1 - \phi) - 1\big) \geq -E$$

$$L(1 - (R - \Delta)(1 - \phi)) \leq E$$

$$L \leq \frac{E}{1 - (R - \Delta)(1 - \phi)}$$

The level of lending that can occur without any risk of needing to bail out the banks is given when L is less than or equal to the expression above. The critical level is given when this expression holds with equality. Notice that when the risk, Δ, increase, the critical value goes down. This means that when the risk is higher, the bank will surpass the critical value of "safe lending" at a lower level of lending than when the risk is lower.

4. **Given the same distribution of $\tilde{\boldsymbol{L}}$ as in question 3, what is the expected net profit of the bank's owners? How does it depend on $\boldsymbol{\Delta}$ and on $\boldsymbol{L}$? What general principle(s) does this example demonstrate?**

As we saw in question 3, when the lending of the bank is below the critical value, the return of lending will always be great enough to cover the deposits. When this is the case, the bank will not need to receive the insurance pay-out and it will not lose any equity. This means that when lending is less than or equal to L^C the return to the bank will be given as we found in 2 in the case where $\tilde{L} \geq D$

$$\Pi = \tilde{L} - \frac{L - \phi E}{1 - \phi}$$

This implies that the expected net payoff of the bank's owners will be:

$$\mathrm{E}(\Pi) = \frac{1}{2}(R+\Delta)L + \frac{1}{2}(R-\Delta)L - \frac{L-\phi E}{1-\phi}$$

$$\mathrm{E}(\Pi) = \frac{1}{2}RL + \frac{1}{2}\Delta L + \frac{1}{2}RL - \frac{1}{2}\Delta L - \frac{L-\phi E}{1-\phi}$$

$$\mathrm{E}(\Pi) = RL - \frac{L-\phi E}{1-\phi}$$

$$E(\Pi) = L\left(R - \frac{1}{1-\phi}\right) + \frac{\phi}{1-\phi}E$$

From this we see that when $L \leq L^C$ the expected payoff of the bank owners does not depend on the level of risk, when the risk is symmetrical. The payoff does, on the other hand, depend critically on the amount of lending. The payoff is higher the higher is lending and is thus maximized when $L = L^C$.

$$\frac{dE(\Pi)}{dL} = R - \frac{1}{1-\phi}$$

As long as $R \geq \frac{1}{1-\phi}$ the payoff of the bank owners is larger the more they lend. It is reasonable to assume that this condition holds, otherwise there would be no incentive for banks to lend at all.

However, the expected payoff of the bank owners is different if $L^C < L \leq L^{max}$. Once lending crosses this threshold, obtaining the payoff of $(R-\Delta)L$ will imply that the banks return on lending is not enough to cover the amount of deposits they hold. This will of course imply that they will need the insurance pay-out and that the bank will lose equity.

Using the payoffs for the different states (from 2) and realizing that these each occur with 50% probability, we find that the expected pay-off when $L^C < L \leq L^{max}$ is:

$$E(\Pi) = \frac{1}{2}\left(\tilde{L} - \frac{L-\phi E}{1-\phi}\right) + \frac{1}{2}(-E)$$

$$E(\Pi) = \frac{1}{2}\left((R+\Delta)L - \frac{L-\phi E}{1-\phi}\right) + \frac{1}{2}(-E)$$

$$E(\Pi) = \frac{1}{2}\left[(R+\Delta)L - \frac{L}{1-\phi} + \frac{\phi E}{1-\phi} - E\right]$$

$$E(\Pi) = \frac{1}{2}\left[L\left((R+\Delta) - \frac{1}{1-\phi}\right) + \frac{\phi E}{1-\phi} - E\right]$$

Thus, the expected profit of the bank owners does depend on the level of risk when $L^C < L \leq L^{max}$.

$$\frac{d\Pi}{dL} = (R+\Delta) - \frac{1}{1-\phi}$$

We see that the profit is increasing with the amount of lending as long as $(R+\Delta) > \frac{1}{1-\phi}$. From this expression we also see that the return of every unit the bank extends in loans is greater the greater is the risk. In other words; the bank has incentives for great risk taking when the lending crosses the critical threshold. Once the critical limit is crossed, the bank maximizes its profits by selecting as high lending as possible, in this case L^{max}.

- This problem illustrates the problem with moral hazard due to limited liability. Once lending exceeds the critical value, the bank is incentivized to gamble. The bank can increase the return it receives when its investments pay off by maximizing lending and risk. If their investments don't pay off they only lose their equity. In other words there is a potentially huge upside and a given downside. When profit maximizing banks make the most of this situation it leads to excessive risk taking.

5. **Suppose the bank can choose the level of risk, Δ, and the volume of loans L, freely within the range permitted by the assumptions above. What levels would it choose if it starts with a given level E? What rate of return on equity would this choice result in? Is the net rate of return positive?**

The bank will select the level of risk and lending that maximizes profits. As we have seen in problem 4, the level of lending that maximizes profits in the two cases is L^C and L^{max}. Between these two, the bank will select the one that maximizes profits. The bank lends L^{max} when:

$$E\left(\Pi^{high}\right) = \frac{1}{2}\left[L\left((R+\Delta) - \frac{1}{1-\phi}\right) + \left(\frac{\phi}{1-\phi} - 1\right)E\right] \geq L\left(R - \frac{1}{1-\phi}\right) + \frac{\phi}{1-\phi}E = E(\Pi^{low})$$

$$L\left((R+\Delta) - \frac{1}{1-\phi}\right) + \left(\frac{\phi}{1-\phi} - 1\right)E \geq 2L\left(R - \frac{1}{1-\phi}\right) + \frac{\phi}{1-\phi}2E$$

$$L\left(R - \frac{1}{1-\phi}\right) - 2L\left(R - \frac{1}{1-\phi}\right) + L\Delta + \left(\frac{\phi}{1-\phi}\right)E - \frac{\phi}{1-\phi}2E - E \geq 0$$

$$-L\left(R - \frac{1}{1-\phi}\right) + L\Delta - \left(\frac{\phi}{1-\phi}\right)E - E \geq 0$$

$$-L\big((1-\phi)R - 1\big) + L(1-\phi)\Delta - \phi E - (1-\phi)E \geq 0$$

$$-L\big((1-\phi)R - 1 + (1-\phi)\Delta\big) - E \geq 0$$

$$-L\big((1-\phi)(R+\Delta) - 1\big) - E \geq 0 \quad |(-1)$$

$$L\big((1-\phi)(R+\Delta) - 1\big) + E \leq 0$$

$$E^C \leq L\big(1 - (1-\phi)(R+\Delta)\big)$$

We see that the level of lending that the bank chooses depends on the level of the initial value of the equity. If their equity is below E^C, then they will find it profitable to cross L^C and lend L^{max}. If, on the other hand, the banks initial equity is larger than E^C the bank will prefer to lend only L^C. The reason for this behaviour is that once the equity is large enough, the amount the bank stands to lose if their investments don't pay off (if the state $(R-\Delta)L$ is realized) is larger than the benefit of extending the additional loans. When this is the case they will prefer to invest such that their equity is not jeopardized – in other words lending L^C.

In the case where $L > L^C$ the return on equity is $\frac{\phi}{1-\phi} - 1$, while in the case where $L \leq L^C$ the return on equity is $\frac{\phi}{1-\phi}$. In the first case, the return may be negative, while in the second it is always positive.

6. Will the size of ϕ influence risk taking? If so, in what way?

The size of ϕ should contribute towards less risk taking. This is because a higher ϕ increases the amount of insurance premium that the bank pays, $P \uparrow$. In turn, this increases the minimum level of equity that the bank must hold. When the equity held by the bank increases, the banks are more likely to have incentives that make them prefer L^c to L^{max}.

Question B

Banks seem to get more attention from governments than most other industries. What makes banks special?

As we have seen already, limited liability in banking leads to problems with moral hazard. Banks are incentivized to take on high risk as this will lead high returns if the gamble pays off, but at most to a loss of equity if the gamble fails. Now, as we know, firms may also exhibit moral hazard. If their project fails they don't repay their loans to the bank and the owners of the firm lose their equity. However, there are at least two important differences between banks and firms.

The first is that banks tend to have very low levels of capital, while it is not uncommon that firms have around 40% capital. This means that firms have a lot more "skin in the game" and the moral hazard is thereby significantly lower in firms than in banks. With the very low levels of capital in banks, only small losses are required before the equity is wiped out and the bank is unable to fulfil its obligations.

The second has to do with the consequence of failure. If a shoe manufacturer fails, production of shoes will be lower, but isolated there should be no reason that demand for shoes goes down. This means that the other shoe manufacturers can increase their own production to replace the failed manufacturer. The other manufacturers gain market shares and do better due to the failed manufacturer. However, if a bank fails, the consequences are not as clear cut. In fact, the failure of a bank is likely to make the other banks worse off, rather than better off. The reason for this is the level of co-integration in the banking sector and contagion.

When a bank faces losses on its lending it is forced to rebalance its balance sheet in order to make sure it fulfils the required capital ratio. This can be done either by raising more equity or by selling off assets. It may be difficult for a bank that has just announced losses in its last quarterly report to acquire new equity. Then it must sell off assets. When the bank does this the supply of these assets goes up, leading the price of the assets to fall. If this were the end of the tale, it would all be good and well. However, the portfolios of different banks tend to be highly correlated. This means that when the assets sales of the failing bank leads the asset prices to fall, the other banks will also see that their assets drop in value. This may lead banks that initially where doing fine to have to sell off their assets as well, leading to additional price reductions and damage to the balance sheets. We get what is called a fire sale of assets.

When such fire sales or similar occur, the banks might become unsure about which banks are solvent and which are on the brink of bankruptcy. This might lead banks to become unwilling to lend to each other and the interbank market freezes up. This is bad for at least two reasons. The first being that when the interbank rates spike, banks that need to borrow become even more vulnerable to becoming insolvent due to the high rates. In other words, there is a self-fulfilling mechanism where banks that are near insolvency face higher rates which make them even less solvent. The second reason why frozen interbank markets are bad is because the banking sector after all is a very useful sector in the sense that it delivers maturity transformation which is valuable to the economy. When the interbank market freezes up, the banks are hindered from providing this valuable service.

When firms fail, the losses are incurred by the bank. But this is reasonable as the banks business is extending loans and assessing risk. On the other hand, when banks actually fail, when they are unable to repay their obligations, the government steps in with taxpayers money. This is of course part of what gives rise to the moral hazard in banking, but it is also bad simply because the government must use taxpayers money, that could have made good use elsewhere.