ECON 4335 Economics of Banking, Fall 2015

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Hellmann, Murdock and Stiglitz (2000)

Hellmann, Murdock and Stiglitz (2000) construct a dynamic model to consider the effect of competition in the banking industry. They show that, under limited liability, intense competition induces banks to invest in risky assets.

Here is a summary of the model. N banks operate for T periods. Each period every bank $i \in N$ offers interest rate r_i while other banks offer interest rates $\mathbf{r}_{-i}$ ($\mathbf{r}_{-i}$ is a vector composed of $N-1$ elements). As a result, bank i receives an amount of deposits equal to $D(r_i, \mathbf{r}_{-i})$, where $D(.)$ is increasing in r_i and decreasing in $\mathbf{r}_{-i}$.[2]

A bank has two alternatives uses for the funds raised with the deposits. The bank can invest in a prudent asset with return α, or in a risky asset that returns γ with probability θ and returns 0 with probability $1-\theta$, where $\gamma > \alpha > \theta\gamma$. If the risky asset is chosen and the return happens to be 0, the bank fails and the license to run the bank is taken away by the regulator. When a bank loses its license, it loses its **franchise value** (= discounted value of future profits), but bank stakeholders get to keep the part of their capital that they did not invest in the bank (= owners enjoy limited liability).

The prudent asset ensures per-period profit $\pi_P(r_i, \mathbf{r}_{-i}) \equiv (\alpha - r_i)D(r_i, \mathbf{r}_{-i})$ where $\alpha - r_i$ is the profit margin earned on each unit of deposit. When the bank gambles, the per-period profit is $\pi_G(r_i, \mathbf{r}_{-i}) \equiv \theta(\gamma - r_i)D(r_i, \mathbf{r}_{-i})$. Banks maximize the discounted sum of payoffs, and $T \to \infty$. The rate of time discount is $\delta \in (0, 1)$. As long as the bank does not fail, every period is identical, and therefore the expected return from choosing the prudent asset in every period is $V_P(r_i, \mathbf{r}_{-i}) = \sum_{t=0}^{\infty} \delta^t \pi_P(r_i, \mathbf{r}_{-i})$. The expected return from the gambling asset is instead $V_G(r_i, \mathbf{r}_{-i}) = \sum_{t=0}^{\infty} (\delta\theta)^t \pi_G(r_i, \mathbf{r}_{-i})$. Whenever the bank chooses the gambling asset, in each period in which the bank is still active on the market, there is a probability $1-\theta$ that the bank

[1]This note is almost entirely based on the note written by Jon Vislie for the Fall 2014 Banking course.

[2]The volume of deposits depends only on the rate offered and not on the probability that banks are able to repay the deposits, because depositors are insured.

will fail. Therefore, if the bank chooses the gambling asset, the discount factor is $\delta\theta$, as the bank earns $\pi_G(r_i, \mathbf{r}_{-i})$ in period t only if the risky asset returned γ in every previous period. This event occurs with probability θ^{t-1}. The expected returns are can be expressed in a more manageable form if we use the following property, which holds for any $x \in (0,1)$:

$$\sum_{t=0}^{\infty} x^i = \frac{1}{1-x}$$

. To see that the property holds, notice that

$$\begin{aligned}(1-x)\sum_{t=0}^{\infty} x^t &= (1-x)(1+x+x^2+...)\\ &= 1-x+x(1-x)+x^2(1-x)+..\\ &= 1-x+x-x^2+x^2-x^3+..\\ &= 1\end{aligned}$$

Therefore the returns can be written as:

$$\begin{aligned}V_P(r_i, \mathbf{r}_{-i}) &= \frac{\pi_P(r_i, \mathbf{r}_{-i})}{1-\delta},\\ V_G(r_i, \mathbf{r}_{-i}) &= \frac{\pi_G(r_i, \mathbf{r}_{-i})}{1-\delta\theta}.\end{aligned}$$

Unregulated Market

We show that when competition is very intense there is no equilibrium in which all banks choose the prudent investment.

Each period can be divided in 2 stages:

Stage 1: banks decide simultaneously their interest rates,

Stage 2: each bank decides whether to invest in the prudent asset or in the gambling asset.

At stage 2 banks choose the prudent asset if and only if $V_G \leq V_P$, or equivalently:

$$\pi_G(r_i, \mathbf{r}_{-i}) - \pi_P(r_i, \mathbf{r}_{-i}) \leq (1-\theta)\delta V_P(r_i, \mathbf{r}_{-i}), \tag{1}$$

this is the **no-gambling condition**. It says that starting from a strategy of investment in the prudent asset, the one-period gain from deviating to gambling (left-hand side of (1)) is smaller than the present discounted value of future lost franchise value (right-hand side of (1)). If the bank fails, it loses the franchise value in the next period, therefore we discount V_P by δ. The franchise value is lost only if the gambling goes wrong, and this event occurs with probability $1-\theta$.

Substituting for $\pi_G(r_i, \mathbf{r}_{-i})$, $\pi_P(r_i, \mathbf{r}_{-i})$ and $V_P(r_i, \mathbf{r}_{-i})$ in (1), the **no-gambling condition** becomes equivalent to:

$$\begin{aligned}
\theta(\gamma - r_i)D(r_i, \mathbf{r}_{-i}) - (\alpha - r_i)D(r_i, \mathbf{r}_{-i}) &\leq (1-\theta)\delta\frac{(\alpha - r_i)D(r_i, \mathbf{r}_{-i})}{1-\delta} \rightarrow \\
\theta(\gamma - r_i) - \alpha + r_i &\leq \frac{(1-\theta)\delta(\alpha - r_i)}{1-\delta} \rightarrow \\
r_i &\leq \hat{r} \equiv (1-\delta)(\frac{\alpha - \theta\gamma}{1-\theta}) + \delta\alpha.
\end{aligned}$$

In stage 1, if banks anticipate choosing the prudent asset, they set the rate:

$$r_P = \arg\max_r \left\{V_P(r, \mathbf{r}_{-i})\right\}.$$

In a symmetric equilibrium (every bank chooses the same rate r_P), the first order condition requires

$$\begin{aligned}
FOC: \quad & \tfrac{\partial V_P(r, \mathbf{r}^P)}{\partial r}|_{r=r^P} = 0 && \rightarrow \\
& \tfrac{\partial \pi_P(r, \mathbf{r}^P)}{\partial r}|_{r=r^P} = 0 && \rightarrow \\
& -D(r^P, \mathbf{r}^P) + (\alpha - r^P)\tfrac{\partial D(r, \mathbf{r}^P)}{\partial r}|_{r=r^P} = 0 && \rightarrow \\
& r_P = \alpha\tfrac{\epsilon}{\epsilon+1}
\end{aligned}$$

where $\mathbf{r}^P$ is a vector with N-1 elements, all equal to r^P, and

$$\epsilon \equiv \frac{\partial D(r, \mathbf{r}^P)}{\partial r}|_{r=r^P} \frac{r^P}{D(r^P, \mathbf{r}^P)}$$

is the elasticity of deposits to interest rates. The elasticity of deposits to interest rates captures the level of competition for deposits. If banks are perceived to be close substitutes, competition is fierce and the amount of deposits in a bank varies a lot when the interest rate changes (large ϵ). If instead banks are perceived as differentiated (weak competition), then deposits are inelastic. As competition becomes more intense, $\epsilon \to \infty$ and therefore $r_P \to \alpha$. You can check that $\hat{r} < \alpha$. Therefore, for strong enough competition $r_P > \hat{r}$. This implies that if competition is sufficiently intense there is no equilibrium in which banks all choose the safe asset. In such equilibrium, banks would find it optimal to set an interest rate for which, in stage 2 they would choose the risky asset.

Regulations

Two regulations are potential candidates to solve this market failure.

(1) **Minimum capital requirements**: banks are required to invest an amount of their own capital that cannot be smaller than a set share of the deposits,

(2) **Deposit-rate controls**: a cap on the interest rate that banks can offer in stage 1.

The argument in favor of **minimum capital requirements** seems intuitive. The more capital, the more "skin in the game" or the more of the cost of gambling or downside risk is borne by the bank, inducing prudent behavior. This **capital-at-stake effect** should therefore induce no-gambling. It turns out that this is not necessarily true. If investing the capital of the bank shareholders was a profitable choice, **minimum capital requirements** would be ineffective. **Minimum capital requirements** are effective only if bank shareholders have alternative uses of capital that return on average more than α. In this case, **minimum capital requirements** reduce current and future profits and hence reduce the franchise value. A reduced franchise value in turn reduces the incentive to choose the prudent asset. Therefore either **minimum**

capital requirements have no effects on the investment choice, or have two effects on the choice of investment and these two effects go in opposite directions.

Deposit-rate controls are instead a way to curb the effect of competition for deposits. **Deposit-rate controls** increase the franchise value (I) and increase the expected payoff from a prudent investment more than they increase the payoff from a gambling investment (II). Both effect (I) and (II) ensure that the no gambling condition is satisfied by a larger set of parameters.