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   (a) Balance sheet equation at $t = 0$ is

   $$L + P = D + E \tag{4}$$

   Replace P by ϕD and solve.

   (b) Upper limit is reached when $P = E$ or since $P = \phi D$, when $D = E/\phi$. When $P = E$, (4) tells that $L = D$ Hence, $L^{max} = E/\phi$.

2. From the balance sheet equation at $t = 1$

   $$\tilde{V} = \tilde{L} + \tilde{S} - D \tag{5}$$

   If $\tilde{L} \geq D$, then $\tilde{S} = 0$ and $\tilde{V} = \tilde{L} - D$.
   If $\tilde{L} < D$, then $\tilde{S} = D - \tilde{S}$ and $\tilde{V} = 0$.
   Use (1) to replace D and you get

   $$\Pi = \tilde{V} - E = \begin{cases} \tilde{L} - \frac{L-E}{1-\phi} - E & \text{if} \quad \tilde{L} \geq D \\ -E & \text{if} \quad \tilde{L} < D \end{cases} \tag{6}$$

3. Worst case is $\tilde{L} = (R - \Delta)L$. No default in this case if

   $$\tilde{L} = (R - \Delta)L \geq \frac{L - E}{1 - \phi} = D \tag{7}$$

   Hence, critical level of L is when

   $$(R - \Delta)L = \frac{L - E}{1 - \phi}$$

   Solving for L yields L^C.

4. If $L < L^C$:

   $$\mathtt{E}\Pi = \frac{1}{2}[(R + \Delta) + (R - \Delta)]L - \frac{L - E}{1 - \phi} - E = RL - \frac{L - E}{1 - \phi} - E \tag{8}$$

   If $L \geq L^C$:

   $$\mathtt{E}\Pi = \frac{1}{2}\left[(R + \Delta)L - \frac{L - E}{1 - \phi}\right] - E \tag{9}$$

   - Expectation is independent of Δ for $L < L^C$, increasing in Δ for $L > L^C$ (and L^C is lowered when Δ increases).
   - Expectation is increasing in L and more so when $L > L^C$
   - Limited liability encourages risk taking

5. Choose as much risk as possible: $\Delta = 1$ (see Q3) and $L = L^{max} = E/\phi$ This implies

$$\mathtt{E}\Pi = \frac{1}{2}\left[(R+1)L^{max} - \frac{L^{max} - E}{1-\phi}\right] - E = \frac{RE}{2\phi} - E \tag{10}$$

   Expected rate of return on equity is

$$\frac{R}{2\phi} - 1$$

   Positive if $R > 2\phi$.

6. Higher ϕ reduces loan volume and in that way risk-taking.

7. Competition may wipe out profits. Equalization of expected returns to bank between loans with different known levels of risk.