

**Problem set 2 (September 10, 2015)**

Consider the one-good, two-types, three-dates economy of Diamond and Dybvig. There are infinitely many, *ex ante* identical, individuals, each endowed with one unit of the good at  $t = 0$ . Consumption takes place either at  $t = 1$  or  $t = 2$ . With probability  $\lambda$  a consumer needs to consume at  $t = 1$ , and with probability  $1 - \lambda$  at  $t = 2$ . There is an independent draw for each agent. Ex post the consumers can be divided into group 1, impatient consumers, and in group 2, those who will wait until  $t = 2$  (patient consumers). An individual's type is private information. The utility function of a consumer is  $u(c) = \frac{c^{1-s}}{1-s}$  where  $c$  refers to the level of consumption in the period in which the consumer needs to consume, with  $s > 1$ . There is no discounting.

The economy has two ways of transferring resources between periods: storage (called a short-term project) with gross return equal to 1, and a long-term investment project, with gross return at  $t = 2$ , equal to  $R > 1$ , per unit invested at  $t = 0$ . If necessary, the long-term project can be liquidated or stopped prematurely at  $t = 1$ , with a return  $L \in (0, 1)$ .

- (1) Derive the allocation that maximizes social welfare, as given by expected utility. How is initial wealth allocated between the two investment opportunities? Will there be any liquidation?
- (2) Let optimal consumption be  $C_1^*$  for a type 1-individual, and  $C_2^*$  for a type 2-individual. Who will have the higher consumption? Explain why an uneven distribution can be optimal. How is the optimal consumption profile affected by  $s$ ?
- (3) Assume that in the economy there is a competitive banking sector, where individuals can deposit their unit wealth at  $t = 0$ . The banks have the same investment opportunities as above. Suppose the banks offer the depositors the opportunity to withdraw at  $t = 1$  or at  $t = 2$ . Explain why and under what circumstances the optimal allocation can be realized as an equilibrium.

- (4) When banks offer the deposit contract  $\{C_1^*, C_2^*\}$ , explain why there are two (Nash) equilibria that are consistent with rational behavior for all individuals; one where only the early consumers withdraw at  $t = 1$ , and another one where everyone withdraws at  $t = 1$ . What will the individual consumption level be in the latter equilibrium if you assume  $L = 1$ ?
- (5) Suppose the banking sector offers the contract  $\{C_1^*, C_2^*\}$  to depositors at  $t = 0$ . Imagine that a financial (or a bond) market is opened at  $t = 1$ . A bond is here a promise to have one unit consumption at  $t = 2$ . Late consumers are offered to buy bonds at a price  $p = \frac{1}{R}$ . Will  $\{C_1^*, C_2^*\}$  still be a Nash equilibrium? Explain!