**Problem set 5 (October 8, 2015)**

Consider the model of bank competition discussed in class and based on Helmann, Murdock and Stiglitz (2000). Assume that banks can use their own capital, together with deposits, to buy assets. Let's measure the own capital invested as a share of the deposits, and denote it $kD(r_i, r_{-i})$. The total amount invested by bank $i$ is $(1+k)D(r_i, r_{-i})$. Bank owner have also an alternative use of their capital that ensures a return equal to $\rho$. This implies that the opportunity cost of capital is $\rho$. The prudent asset ensures per-period profit $\pi_P(r_i, r_{-i}, k) \equiv m_P(r_i, k)D(r_i, r_{-i})$. When the bank gambles, the per-period profit is $\pi_G(r_i, r_{-i}, k) \equiv m_G(r_i, k)D(r_i, r_{-i})$. $m_P(r_i, k)$ is the effective profit margin earned on each unit of deposit if the bank chooses the prudent asset, while $m_G(r_i, k)$ is the effective profit margin earned on each unit of deposit if the bank chooses the gambling asset.

(1) What are $m_P(r_i, k)$ and $m_G(r_i, k)$ equal to?

Consider first an unregulated market in which banks are free to choose any level of $k$. Each period can be divided in 3 stages:

Stage 1: banks decide simultaneously their interest rates,

Stage 2: each bank decides how much own capital to invest,

Stage 3: each bank decides whether to invest in the prudent asset or in the gambling asset.

The **no-gambling condition** in this case has the form $r \leq \hat{r}(k)$ ($\hat{r}$ is now a function of $k$).

(2) What is $\hat{r}(k)$ equal to?

In a candidate symmetric equilibrium in which all the banks choose the prudent asset the first order condition requires $m_P(r_P, k) = \frac{D(r_P, r_P)}{(\frac{\partial D(r_P, r_P)}{\partial r_i})}$. This condition can be written as $r_P = f(\alpha, \rho, k, \epsilon)$ (where $\epsilon$ is the elasticity defined in the class notes).

(3) What is $f(\alpha, \rho, k, \epsilon)$ equal to?

(4) If $\rho > \alpha$ what is the optimal $k$ for the bank?

Suppose a regulation is introduced, requiring banks to invest an amount of own capital equal to a share $k^R$ of the deposits raised.

(5) What is the effect of an increase in $k^R$ on $r_P$ and on $\hat{r}$? Are there values of $\delta$ for which $\frac{\delta \hat{r}}{\delta k}$ is smaller than 0?

(6) Assume now that $\hat{r}(k)$ is increasing in $k$. Draw $r_P$ and $\hat{r}$ as a function of $k$.

Let $\underline{k}$ be defined by the equation $r_P(\underline{k}) = \hat{r}(\underline{k})$. Consider a regulation, called regulation 1, that imposes $k^R > \underline{k}$ and ensures that banks choose the prudent investment. The regulator could ensure that banks choose the prudent project also by imposing a different regulation: a cap on the interest rate that requires bank to offer at most rate $r_P(k^R)$ and a own capital requirement equal to $k_0$, where $k_0$ satisfies $\hat{r}(k_0) = r_P(k^R)$ (call this regulation 2).

(7) If you pick a $k^R > \underline{k}$ can you show where $k_0$ lies in the graph you drew for question (6)? Which of the two policies would you suggest to the regulator?