## Problem set 8 (29 October, 2015)

(Please first read the supplementary note for Macro Lecture 1.)

We are looking at a closed economy. The balance sheet in table 1 shows the structure of the asset markets. The line for interbank loans is there to remind us that they are present in the background but net out in the aggregate. The central bank's policy rate,  $i_p$ , is equal to the rate on deposits there. The central bank can lend to the other banks in two different ways. F-loans are sold to the banks that make the highest bids at an auction. The central bank decides the amount of these loans,  $L_{cb}$ . The banks can get H-loans in the amount they want, but the interest rate is far above the policy rate. Hence, H-loans are normally not used. The markets for bank loans and bank deposits are competitive. The interest rates banks pay on loans and deposits are  $i_l$  and  $i_d$  respectively. Unless otherwise noted, you can assume that there is no default risk anywhere.

### Question 1

#### Sectoral Balance Sheet

Instrument	Government	Central Bank	Banks	Firms	Households	Sum
Deposits at CB	0	$-D_{cb}$	$D_{cb}$	0	0	0
Loans from CB	0	$L_{cb}$	$-L_{cb}$	0	0	0
Deposits at banks	0	0	$-D_h$	0	$D_h$	0
Loans from banks	0	0	L	-L	0	0
T-bills	-B	$B_{cb}$	$B_b$	0	$B_h$	0
Interbank loans	0	0	0	0	0	0
Real capital	0	0	0	QK	0	QK
Sum = Net assets	$W_g$	$W_{cb}$	$W_b$	$W_f$	$W_h$	QK

In this question we are interested in what goes on at a point in time (a period so short that saving and investment in real capital does not have time to affect stocks significantly).

- (1) Explain how the interbank interest rate can be kept close to the policy rate.
- (2) Suppose banks want to keep deposits at the central bank in fixed proportion to their customer deposits:

$$D_{cb} = \phi D_h$$

Show that this implies a linear relation between  $D_{cb}$  and L

- (3) What is the minimum value of F-loans  $(L_{cb})$  that the central bank needs to lend if banks are to get their desired reserves? What will happen to the interbank interest rates, if the central bank does not lend enough in this way? What will happen if the central bank offers more F-loans than needed to satisfy the banks?
- (4) Suppose the policy rate is increased. Will this have any immediate effect on the balance sheet of the banks?

# Question 2

We now turn from looking at a point in time, to discussing changes over time.

- (1) Suppose that over a year firms borrowed  $\Delta L$  which was used to finance net investment in fixed capital  $I = \Delta L$ . How does the balance sheet change from the beginning to the end of the year? You are free to assume that all saving takes place in the household sector, and the net assets of government and central bank are unchanged at the end of the year and net incomes of banks and firms are distributed to the households.
- (2) Now we go beyond the balance sheet and look at the determination of investment and saving. Suppose for simplicity the following model consisting of a consumption function, an investment function, a national accounting equation and a relation stating that the value of the existing capital depends on the interest rate:

$$C = \overline{C} + (1 - s(i_d))Y;$$
  
 $0 < s(i_d) < 1; s' > 0;$   
 $I = I(Q) I' > 0;$ 

$$Y = C + I;$$

$$Q = Q(i_l); Q' < 0.$$

Suppose that at the beginning of a year the policy interest rate is increased. Discuss briefly the effect this will have over time on investment, saving and output in the period.

## Question 3

We go back to looking at the financial side of the economy in isolation. We now open up for possible defaults in the existing loan portfolio of banks. Suppose a certain fraction of the outstanding loans will not be repaid. The banks have added a premium to  $i_l$  that covers the expected losses. However, individual banks can be hit unevenly, and in rare cases a bank will default. Which bank will default, and when, is unpredictable. Depositors therefore become sensitive to the interest rate difference between T-bills and deposits. Suppose they are also risk-averse. Then their deposits and holdings of T-bills will be something like

$$D = g(i_d - i_b - x)W_h; \ B_h = (1 - g(i_d - i_b - x))W_h$$

where x is expected loss from bank default and g' > 0, g(-x) = 0. Suppose the volume of loans is given (loans are long term).

(1) Show that the equilibrium in the markets for deposits and t-bills requires that

$$D_h = g(i_d - i_b - x)W_h = \frac{1}{1 - \phi}(L + B_b - L_{cb} - W_b)$$

This equation determines the interest rate differential between deposits and t-bills and makes it positive.

(2) Default risk makes the interbank market work less smoothly. Suppose banks react by raising  $\phi$ . How will this affect the cost to extend a loan? What is the effect of this on  $i_l$ ?