Problem Set 2

Tuesday, 12 September 2017 11:44

D. D. ← credit risk — screening, monitoring, ...

← no equity : conflict of interest

1. asymmetric information : P.-A.

(1) adverse selection

(2) moral hazard

α : G: return $\begin{cases} G, & \text{prob. } p \\ 0, & 1-p \end{cases}$

$1-\alpha$: B: return $\begin{cases} B > G, & \text{prob. } q < p \\ 0 & 1-q \end{cases}$

$pG > qB > 1$ socially desirable

1. Frictionless econonomy : no asy. inf.

Bank:

$$\max_{R_G, R_B} \Pi = \alpha\left[p(R_G - 1) - (1-p)1\right] + (1-\alpha)\left[q(R_B - 1) - (1-q)1\right]$$

s.t.

$p(G - R_G) \geq 0$ (P.C.- G)

$q(B - R_B) \geq 0$ (P.C.- B)

$\left.\begin{array}{l} pR_G - 1 \geq 0 \\ qR_B - 1 \geq 0 \end{array}\right\}$ (P.C. - Bank)

P.C.- G $\Rightarrow$ $R_G = G$

P.C - B $\Rightarrow$ $R_B = B$

P.C Bank ✓ ✓ $p \cdot R_G - 1 = pG - 1 \overset{\text{ass.}}{>} 0$

$q \cdot R_B - 1 = qB - 1 > 0$

2. Bank: R

$$\max_{R} \pi = \pi_G + \pi_B$$

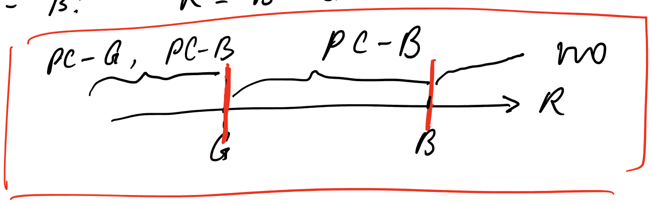
s.t. $\Pi \geq 0$ P C - Bank

$p(G - R) \geq 0$ P.C. - G

$q(B-R) \geq 0$ p.c. − B

→ p.c. − G: $R = G$

→ p.c. − B: $R = B > G$



$R \in (-\infty, G]$: G. B.

$R \in (G, B]$: B

$R \in (B, +\infty)$: no

$R^* = G$ optimal $R^* = B$

pooling eq.

→ P. C. − Bank ?

(A) $R \in (-\infty, G]$: $R^* = G$

$$\begin{cases} G & p(G-R) = 0 \\ B & q(B-R) > 0 \end{cases}$$

info rent

$$\Pi(G) = \alpha p R + (1-\alpha) q R - 1$$

$$= [\alpha p + (1-\alpha) q] G - 1$$

Bank part. only if $\Pi(G) \geq 0$, $R = G$

doesn't part. if $\Pi(G) < 0$, no market

(B) $R \in (G, B]$: pc − G ✗ pc − B ✓

PC − Bank: $R^* = B$ $\Pi(B) = (1-\alpha)(qB - 1) > 0$

B ← separating eq.

> 0

(C) $R \in (B, +\infty)$ pc − G ✗ pc − B ✗ no market

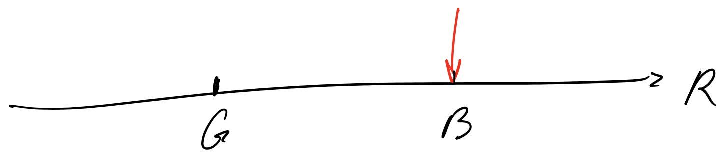
3. PC − Bank: $\Pi(G) \gtrless 0$ $\Pi(B) > 0$

pc − G $R^* = G$

PC − B $\qquad R^* = G \qquad R^* = B$

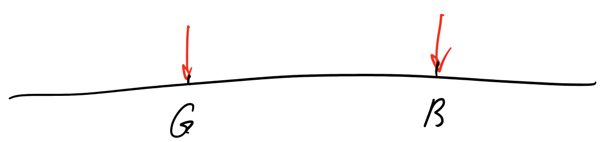
Optimal R for the bank:

(1) $\qquad \pi(G) < 0 \qquad \pi(B) > 0$



$R^* = B \longrightarrow$ PC − B ✓ $\qquad B$

(2) $\qquad \pi(G) > 0 \qquad \pi(B) > 0$



$\pi(G) > \pi(B)$

$\Downarrow$

$R^* = G$,

G, B

$$\underbrace{\alpha p G + (1-\alpha) q G - 1}_{\pi(G)}$$

$$> \underbrace{(1-\alpha)(q B - 1)}_{\pi(B)}$$

$\pi(G) < \pi(B)$

$\Downarrow$

$R^* = B$

B

$$\Rightarrow \underbrace{\alpha(pG - 1)}_{\text{gain from } G} > \underbrace{(1-\alpha) q \boxed{(B - G)}}_{\text{info. rent}}$$

4. Previous: Bank $\quad R \leftarrow 1$

now: $\quad R \leftarrow 1 - (w)$

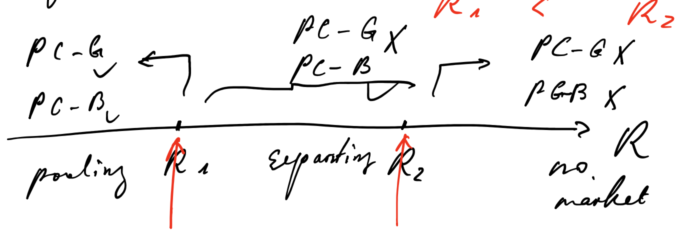
4.

now: $\quad R \leftarrow 1 - \underbrace{w}_{\text{internal funding}}$

PC-G: $p(G - R) - w \geq 0 \Rightarrow R \leq G - \frac{w}{p} \equiv R_1$

PC-B: $q(B - R) - w \geq 0 \Rightarrow R \leq B - \frac{w}{q} \equiv R_2$

<u>Ass:</u> $w < \dfrac{B - G}{\frac{1}{q} - \frac{1}{p}} \Rightarrow \underbrace{G - \frac{w}{p}}_{R_1} < \underbrace{B - \frac{w}{q}}_{R_2}$



<u>PC-Bank?</u>

$$\underset{\text{pooling}}{\Pi(R_1)} = \underbrace{\alpha\left[pR - (1-w)\right]}_{\text{profit } G} + \underbrace{(1-\alpha)\left[qR - (1-w)\right]}_{\text{profit } B}$$

$$= \left[\alpha p + (1-\alpha) q\right]\left(G - \frac{w}{p}\right) - (1-w) \overset{?}{\underset{<}{>}} 0$$

$$\underset{\text{sep.}}{\Pi(R_2)} = (1-\alpha)\underbrace{\left(qR^{(=R_2)} - (1-w)\right)}_{\text{profit } B}$$

$$= (1-\alpha)(\underbrace{qB - 1}_{>0}) > 0$$

Bank's optimization problem

(1) $\Pi(R_1) < 0 \qquad \Pi(R_2) > 0$

$\longrightarrow R^* = R_2, \quad B \text{ (sep.)}$

(2) $\Pi(R_1) > 0 \qquad \Pi(R_2) > 0$

$$R^* = R_1 ? \quad R_2 ?$$

$\Pi(R_1) > \Pi(R_2) \Rightarrow R^* = R_1$	$\Pi(R_2) > \Pi(R_1) \Rightarrow R^* = R_2$
$\Downarrow$	$\Downarrow$
pool. G. B.	sep. B

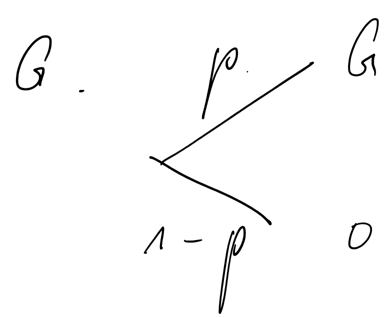
$$\underbrace{[\alpha p + (1-\alpha) q]\left(G - \frac{W}{p}\right) - (1 - W)}_{\Pi(R_1)}$$

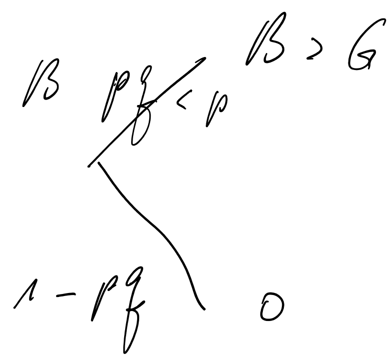
$$> \underbrace{(1-\alpha)(qB - 1)}_{\Pi(R_2)}$$

$$\Rightarrow \boxed{\underbrace{\alpha(pG - 1)}_{\text{profit from } G} + \underbrace{(1-\alpha)\left(1 - \frac{q}{p}\right)}_{\text{cost of failure}} W} > \underbrace{(1-\alpha) q (B - G)}_{\text{info. rent } B}$$

W: stake of borrowers $\Rightarrow$ "skin-in-the-game"

2. Moral hazard





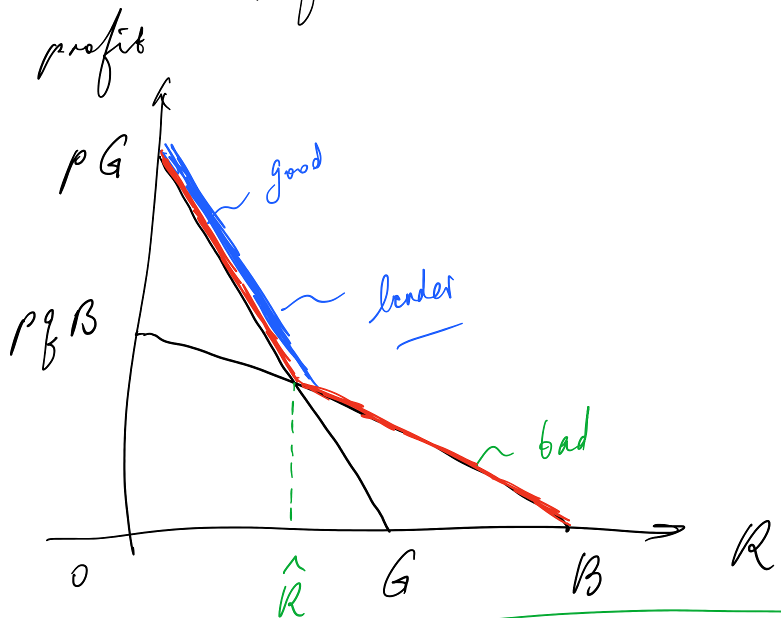
$$pG > 1 \underset{\text{not des.}}{>} pqB$$

soc. desirable

1. 1 with $\underset{\sim}{R}$

PC - G : $\underbrace{p(G-R) \geq 0}$

PC - B : $pq(B-R) \geq 0$



Lender : PC - l

$$\boxed{\begin{array}{ll} R \leq \hat{R} \to \text{good} & p\hat{R} - 1 \geq 0 \\ \Rightarrow R^* = \hat{R} & \end{array}}$$

$R > \hat{R} \to$ bad $\quad \underline{pqB - 1 < 0}$

2. charge high $\underline{R}$

good. $R \to \hat{R}$

PC-G : $pR - 1 \underset{= 0}{\geq} 0$

$\Rightarrow R = \boxed{\frac{1}{p}}$ $\Bigg\} \Rightarrow$

$$\Rightarrow \hat{R} = \boxed{\frac{1}{p}}$$

$= 0$

$$\frac{1}{p} \leq \hat{R}$$

$$p(G - \hat{R}) = pq(B - \hat{R})$$

$$\Rightarrow \hat{R} = \frac{G - qB}{1 - q}$$

$$\boxed{p \geq \frac{1 - q}{G - qB} = p_1}$$

3. Bank: monitoring with c

pc-bank: $\pi^{bank} \geq 0 \Rightarrow pR^b - c - 1 \geq 0$

Good

pc-borrower:

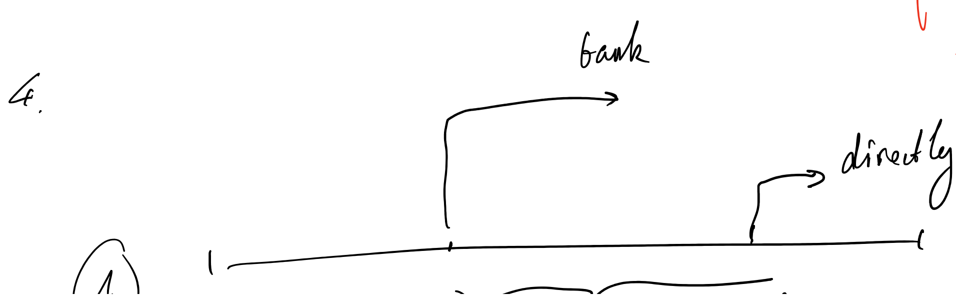
$$\pi^b = p(G - R^b) \geq 0$$

competition

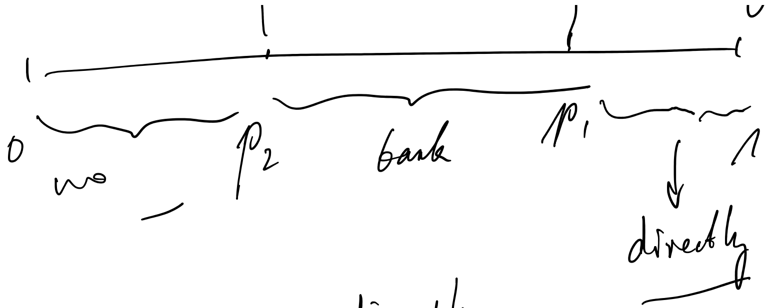
$> R$

$$\Rightarrow R^b = \boxed{\frac{1 + \boxed{c}}{p}} \leq G \rightarrow \boxed{p \geq \frac{1 + c}{G} = p_2}$$

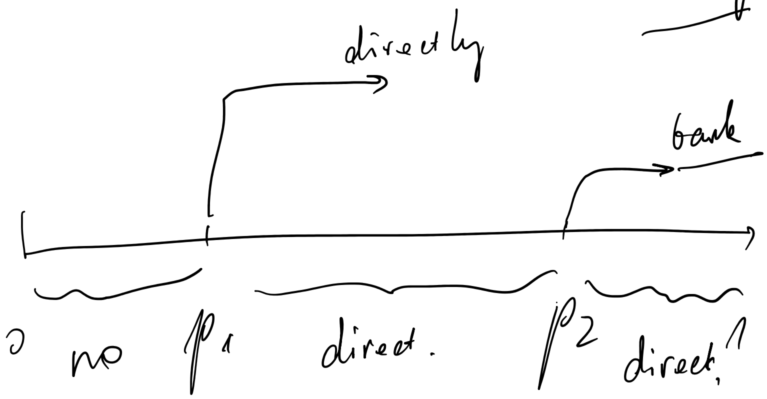
4.



1


0
no
p2
bank
p1
1
directly

2


directly
bank
0
no
p1
direct.
p2
direct.
1