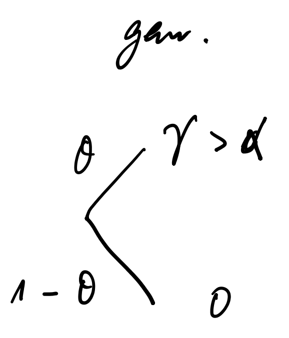
Problem Set 4
Thursday, 12 October 2017 07:59

1. pru. gam.

α



interest margin: for each unit of dep. raised.

prudent:

$$\alpha(1+k) - \rho k - r_i = m_p(r_i, k)$$

gamb:

$$\theta\left[\gamma(1+k) - r_i\right] - \rho k = m_G(r_i, k)$$

2. gambler: profit $\gamma \cdot \theta$, loss if fail. → out

if:

$$\underbrace{\Pi_G(r_i, r_{-i}, k)}_{\text{gambler profit}} + \theta \delta V_p(r_i, r_{-i}, k)$$

(all other bank → r_{-i}; survive → θ; discount → δ; "future" → franchise value)

$$\geq \underbrace{\Pi_p(r_i, r_{-i}, k)}_{\text{prud. profit}} + \delta V_p(r_i, r_{-i}, k) \quad ①$$

$$\rightarrow \Pi_G(\quad) = D(\,)\left(\theta[\gamma(1+k) - r_i] - \rho k\right) \quad ②$$

$$\rightarrow \Pi_p(\quad) = D(\,)\left(\alpha(1+k) - \rho k - r_i\right) \quad ③$$

$$V_p(\quad) = ?$$

$$= \Pi_p(\quad) + \delta \Pi_p(\quad) + \delta^2 \Pi_p(\,)$$

$$+ \ldots$$

$$= \frac{1}{1-\delta} \Pi_p(r_i, r_{-i}, k) \quad ④$$

① – ④ ⇒

gambler if

$$r > \frac{(1-\delta)(1+k)(\alpha - \theta\gamma)}{1-\theta} + \delta[\alpha(1+k) - \rho k] \equiv \hat{r}(k)$$

prud. if $\boxed{r \leq \hat{r}(k)}$

if prud. what r?

$$\max_{r_p} \Pi_p(r_p, r_p, k) = m_p(r_p, k) \cdot D(r_p, r_p)$$

(other.)

F.o.c.

$$\frac{\partial \Pi(\,)}{\partial r_p} = \underbrace{\frac{\partial m_p(\,)}{\partial r_p}}_{-1} \cdot D(\,) + m_p(\,)\frac{\partial D(\,)}{\partial r_p}$$

$$= 0$$

$$\Rightarrow m_p(\quad) = \frac{D(\quad)}{\frac{\partial D(\quad)}{\partial r_p}} \quad (*)$$

What if the bank deviates and becomes a gambler?

$$\max_{r_G} \Pi_G(r_G, r_p, k) = m_G(r_G, k) D(r_G, r_p)$$

F.o.c.

$$m_G(r_G, k) = \frac{D(r_G, r_p)}{\frac{\partial D(r_G, r_p)}{\partial r_G}}$$

3. $(*) \rightarrow m_p(\,) = \alpha(1+k) - \rho k - r_p = \dfrac{D(\,)}{\frac{\partial D(\,)}{\partial r_p}}$

$$\Rightarrow [\alpha(1+k) - \rho k - r_p] \cdot \frac{\partial D(\,)}{\partial r_p} \cdot \frac{r_p}{D(\,)} = r_p$$

$$\frac{\frac{\partial D(\,)}{D} \;?\%}{\frac{\partial r_p}{r_p} \;1\%} \doteq \varepsilon$$

$$\Rightarrow r_p = \frac{\varepsilon}{1+\varepsilon}[\alpha(1+k) - \rho k] \equiv f(\alpha, \rho, k, \varepsilon)$$

$$r_G = \frac{\varepsilon}{1+\theta\varepsilon}[\theta\gamma(1+k) - \rho k] \quad \text{— assume } \varepsilon \text{ is const.}$$

4. optimal $k \rightarrow \dfrac{dV}{dk}$

$$\underbrace{V_p(r_p, r_p, k)}_{m(a)} = \Pi_p(\quad) + \delta \Pi_p(\quad) + \delta^2 \Pi_p(\quad) \ldots$$

$$= \frac{1}{1-\delta} \underbrace{\Pi_p(r_p, r_p, k)}_{f}$$

(r_p → $x(a)$, k → a)

$$\frac{dV_p(\quad)}{dk} \stackrel{?}{=}$$

Envelope Theorem

For value function

$$m(a) = \max_x f(x(a), a)$$

$$\rightarrow \frac{dm(a)}{da} = \left.\frac{\partial f(x(a), a)}{\partial a}\right|_{x = x(a)}$$

$$\Rightarrow \frac{dV}{dk} = \frac{D(r_p, r_p)}{1-\delta} \underbrace{(\alpha - \rho)}_{<0}$$

optimal $k = 0$

5. $$\frac{\partial r_p}{\partial k} = \frac{\partial f(\alpha, \rho, k, \varepsilon)}{\partial k} = \frac{\varepsilon}{1+\varepsilon}\underbrace{(\alpha - \rho)}_{<0} < 0$$

$$\frac{\partial r_G}{\partial k} = \frac{\varepsilon}{1+\theta\varepsilon}(\theta\gamma - \rho) < \boxed{\frac{\partial r_p}{\partial k}} < 0$$

(threshold → $\hat{r}$)

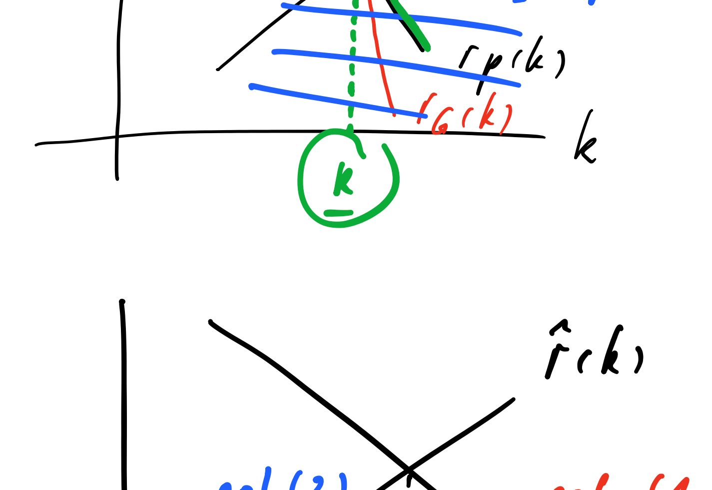
$$\frac{\partial \hat{r}}{\partial k} = ? \quad \hat{r} = \frac{(1-\delta)(1+k)(\alpha - \theta\gamma)}{1-\theta} + \delta[\alpha(1+k) - \rho k]$$

$$\frac{\partial \hat{r}}{\partial k} = \underbrace{\frac{(1-\delta)(\alpha - \theta\gamma)}{1-\theta}}_{>0} + \delta\underbrace{(\alpha - \rho)}_{<0} \; \stackrel{?}{\gtrless} \; 0$$

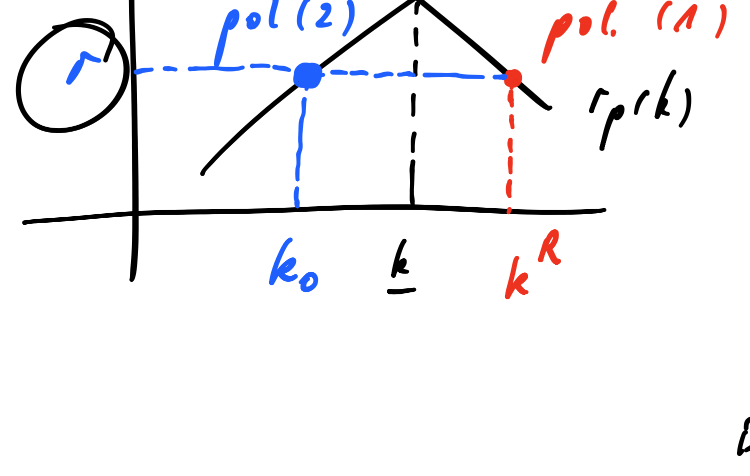
$$\text{if } \frac{\partial \hat{r}}{\partial k} < 0 \Rightarrow \delta > \frac{\alpha - \theta\gamma}{\rho(1-\theta) + \theta(\alpha - \gamma)}$$

↑ large enough, value more of future

6. if $\dfrac{\partial \hat{r}}{\partial k} > 0$, or δ is not large



7.



Welfare?

Banks: $k_0 < k^R$, $\dfrac{dV_p}{dk} < 0$

$$\Rightarrow V_p(k_0) > V_p(k^R)$$

Depositors? Same r

⇒ indifferent

pol. (1) → (2) Pareto improvement

Comment:

$\underline{\rho}$ — const. — capital is costly
$\underline{r}$ — from competition

⇐ risk premium!