## ECON 4335 Economics of Banking, Fall 2017 Problem Set 5

October 3, 2017

## 1. Reserve banking and money multiplier

A closed economy starts to establish its monetary system. First, a central bank is established and it supplies  $M_0$  cash for the economy. The cash is lent to the first bank created in the economy — call it First Bank — to start banking service for the economy. Every bank in this economy is required to keep a fraction k of deposits in reserves, First Bank thus holds  $M_0$  deposits and provides  $(1 - k)M_0$  loans. Suppose all borrowers of First Bank buy goods from producers who are depositors of the second bank created in the economy — call it Second Bank, then after the purchase the producers deposit all their income, this allows Second Bank to start its lending business. Following the same procedure, Second Bank lends to borrowers, and they buy goods from producers who are depositors of Third Bank, and the chain goes on till Nth Bank is created.

- 1. Compute the total deposits in this economy.
- 2. When  $n \to +\infty$ , show that the total money supply is  $\frac{M_0}{k}$ .

## 2 Bank capital, deposit insurance and risk taking

Consider a risk-neutral bank that makes loans at time 0. The loans mature at time 1. The bank has limited liability. Deposits are insured. The market for deposits is competitive. For simplicity we assume that the going interest rate on insured deposits is zero. Figure 1 shows the balance sheets of the bank in period 0 and period 1. The insurance premium is proportional to the level of deposits:  $P = \phi D \ (\phi > 0)$ . The premium has to be paid upfront, which means that equity must at least be sufficient to cover the insurance premium,  $E \ge P$ .

Period 0			Period 1		
Assets	Liabilities		Assets	Liabilities	
Loans L	Deposits D		Loan repayments $\tilde{L}$	Deposits D	
Insurance premiums P	Equity E		Insurance payments $\tilde{S}$	Net value $\tilde{E}$	

Figure 1: Bank's balance sheets

- 1. Assume that in period 0 the bank has a given level of equity, *E*. Use the balance sheet for period 0 to show that
  - (a) If the bank lends L, it needs to collect deposits as much as

$$D = \frac{L-E}{1-\phi};$$

- (b) The maximum amount that the bank can lend is  $L^{max} = \frac{E}{\phi}$ .
- 2. The payout from the insurance fund is

$$\tilde{S} = \begin{cases} 0 & \text{if } \tilde{L} \ge D \\ D - \tilde{L} & \text{if } \tilde{L} < D \end{cases}$$
(1)

Show how you can express the net profits of the bank's owners,  $\Pi = \tilde{E} - E$ , in terms of *L*,  $\tilde{L}$  and *E* for each of the two cases in (1).

3. Suppose the gross repayment on the loans is  $(R + \Delta)L$  with probability  $\frac{1}{2}$  and  $(R - \Delta)L$  with probability  $\frac{1}{2}$ . Assume R > 1 and  $R - 1 < \Delta < 1$ . Show that there is no risk that the bank needs to be bailed out by the insurance fund if it lends less than

$$L^C = \frac{E}{1 - (1 - \phi)(R - \Delta)} < L^{max}$$

- 4. Given the same distribution of  $\tilde{L}$  as in the question 3, what is the expected net profit of the bank's owners? How does it depend on  $\Delta$  and L? What general principle(s) does this example illustrate?
- 5. Suppose the bank can choose the level of risk,  $\Delta$ , and the volume of loans *L*, freely within the range permitted by the assumptions above. What levels would it choose if it starts with a given equity level *E*? What rate of return on equity would this choice result in? Show that the net rate of return is negative for some parameter values. Why do you think this can be the case?

6. Will the size of  $\phi$  affect risk taking? If so, in what way?