ECON 4335 Economics of Banking, Fall 2017
Problem Set 6

October 18, 2017

Overborrowing, externalities, and systemic risk

Consider a small open economy with a tradable goods sector and a nontradable goods sector. Only tradable goods can be traded internationally; nontradable goods have to be consumed domestically. The economy is populated by a continuum of identical households of measure one, living for 2 periods $t = 0, 1$, with preferences given by $u(c) = \ln c_0^T + \ln c_0^N + \ln c_1^T$, in which c_0^T, c_0^N, c_1^T are consumption of tradable goods at $t = 0$, consumption of nontradable goods at $t = 0$, consumption of tradable goods at $t = 1$, respectively.

A representative household starts with initial asset b_0 at $t = 0$, and ends after $t = 1$ with zero asset, i.e., $b_2 = 0$. Note that b_0 can be positive or negative: when $b_0 < 0$, the household starts with initial debt. The timeline of events is as follows:

- At $t = 0$, the representative household receives both an endowment of tradable goods y_0^T and an endowment of nontradable goods y_0^N for consumption. y_0^T is a random variable that is drawn from a distribution with cummulative distribution function $F(y)$, while y_0^N is constant. After $\left(y_0^T, y_0^N \right)$ is revealed, the household can also borrow from abroad by purchasing a one-period, non-state contingent foreign bond denominated in units of tradables that demands a fixed interest rate r — normalized to be 0, determined exogenously in the world market. Normalize the price of tradables to 1 and denote the price of nontradable goods by p_0^N. In addition, the household's debt is securitized such that its total debt cannot exceed a fraction $0 < \kappa < 1$ of its total income from tradables and nontradables;

- At $t = 1$, starting with total asset b_1 the representative household only receives an endowment of tradable goods y_1^T for consumption. y_1^T is constant.

1. Specify the representative household's budget constraints, borrowing constraint, and lifetime optimization problem.

2. Compute the first order conditions for the household's optimization problem:

   (a) Derive the first order conditions with respect to c_0^T and c_0^N, then determine p_0^N;

   (b) Derive the first order conditions with respect to c_1^T and b_1, then determine the Euler equation. Why is the borrowing constraint *occasionally* binding?

3. Determine c_0^T:

   (a) Under what condition(s) is the borrowing constraint not binding? In this case, use the results from exercise 2(b) to determine c_0^T;

   (b) When the borrowing constraint is binding, compute c_0^T.

4. Consider two situations at $t = 1$: the economy can be either in normal state $y_1^T = \bar{y}$, i.e., the household receives a mean value $\bar{y}$, or crisis state $y_1^T = \bar{y} - 1$, i.e., y_1^T is below the mean. Suppose the household knows the true state of $t = 1$ at $t = 0$.

   (a) If borrowing constraint is not binding in both states, how does c_0^T react to the crisis, compared with c_0^T in the normal state?

   (b) If borrowing constraint is binding in both states, how does c_0^T react to the crisis, compared with c_0^T in the normal state?

☞ Bianchi, J. (2011), Overborrowing and systemic externalities in the business cycle, *American Economic Review* 101, 3400-3426.