ECON 4335 Economics of Banking, Fall 2018

Final Exam: Grading Guidance

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1. Are the following statements true, false, or uncertain? Briefly explain (60 points)

a) There is no bubble in the rational expectation equilibrium, because rational people would never invest in bubbles as long as they know the prices for bubbles are higher than their fundamental values and bubbles will burst someday. (12 points)

False. Bubble can still arise in the rational expectation equilibrium, so long as people believe that the price growth rate of the bubble is no less than the risk-free rate in the economy. This also holds even if people believe that the bubble will burst with a positive probability, so long as people believe that the price growth rate of the bubble exceeds the risk-free rate so that the expected return rate from the bubble is no less than the risk-free rate.

b) If each individual bank is financially sound, the stability of entire banking system is guaranteed. (12 points)

False. Beyond the idiosyncratic risks on the bank level, there are also systemic risks arising from the correlations and co-movements of banks. Therefore, even each individual bank is financially sound, the banking system may still fail due to the systemic risks.

c) Tighter competition in the banking sector reduces stability in the financial system. (12 points)

Uncertain. Whether competition reduces financial stability or not depends on who are taking the risks. If the borrowers are taking the risks, tighter competition in the credit market reduces banks' loan rate and borrowers' incentives of moral hazard; this in fact increases financial stability (moral hazard hypothesis). However, if the banks are taking risks, higher competition in the deposit market forces banks to make riskier investments for higher yields; this indeed reduces financial stability (franchise value hypothesis).

d) For a central bank that conducts monetary policy using corridor system, reducing discount rate implies a rise in monetary base, or, the central bank is moving towards expansionary monetary policy. (12 points)

Uncertain. This depends on the initial money market rate: If the initial money market rate is below the initial and new discount rates, cutting discount rate does not affect the money market rate or monetary base; if the initial money market rate is equal to the discount rate, cutting discount rate implies a fall in the money market rate which encourages the banks to borrow more from the central bank, implying a rise in monetary base and expansionary monetary policy.

e) Credit rationing implies there is a systematic shortage of credit supply in the credit market. Therefore, optimal policy shall aim for market clearing such that 100% of the demand from those borrowers who can afford the banks' loan rates is fulfilled. (12 points)

False. Credit rationing arises from adverse selection in the credit market: Raising loan rate *leads to a higher concentration of risky borrowers in the borrower pool, increasing the likelihood of borrowers' default and reducing banks' profit; therefore, profit-maximizing banks would choose a lower lending rate and ration credit supply to ensure more prudent borrowers in the customer pool. Any policy aiming for market clearing will effectively increase the loan rate, increase the share of risky borrowers and reduce banks' profit.*

2. Moral hazard and the risk-taking incentive of banks (60 points)

Consider a one-period economy with a monopoly profit-maximizing bank that can invest in either of the following two types of assets:

(1) Good assets: One unit of safe asset yields a gross return G with probability p_G and 0 otherwise at the end of the period;

(2) Bad assets: One unit of bad asset yields a gross return B with probability p_B and 0 otherwise at the end of the period.

It is known that $G < B$, $p_G > p_B$ and $p_G G > p_B B > 1$. The bank has no capital and totally relies on deposits for its investment.

There is no deposit insurance available in this economy. Depositors are risk neutral and they take deposit contracts from the bank at the beginning of the period. Depositors get repaid at the end of the period with a gross interest rate R if the bank's assets return, otherwise depositors get nothing. There is no asymmetric information. To ease the computation, we further assume that the bank's participation constraint is always fulfilled.

(A) Provide a graphical illustration of how the payoffs of the two types of assets vary with R, and compute the critical value of R, denoted by $\hat{R}$, below which the bank only invests in the good assets and above which the bank only invests in the bad assets. Compute $\hat{R}$. (15 points)

The payoff for the good project is $p_G(G - R)$, while for the bad is $p_B(B - R)$, as illustrated by Figure 1. The critical value of $\hat{R}$ makes $p_G(G - R) = p_B(B - R)$, solve to get $\hat{R} = \frac{p_G G - p_B B}{p_G - p_B}$.

(B) Recall that there is no deposit insurance available in this economy; this implies that depositors are happy to deposit as long as their expected gross return from deposits is no lower than 1. As the bank is a monopoly, depositors earn zero profit from deposits. For the values of p_G, G, p_B, B, find a condition under which depositors are willing to deposit and the bank invests only in the bad project. (10 points)

To convince the depositors to participate in a bank running only bad project, $p_B R \geq 1$. Depositors earn zero profit from deposits, $p_B R - 1 = 0$. Therefore, $R = \frac{1}{p_B}$. Using the result of (A), a bank's running only bad project implies $R > \hat{R} = \frac{p_G G - p_B B}{p_G - p_B}$, i.e., $\frac{1}{p_B} > \hat{R} = \frac{p_G G - p_B B}{p_G - p_B}$.

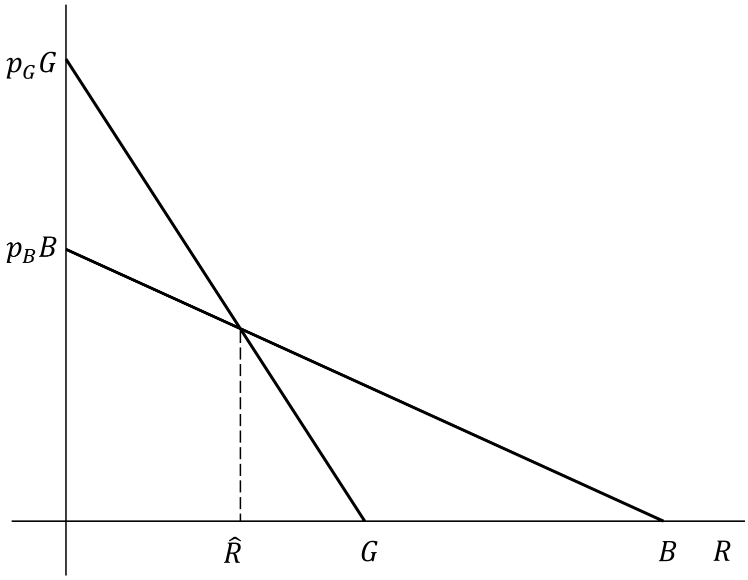

Fig. 1. Payoffs of projects

(C) Now the bank has a choice to hold an amount of capital k for each unit of asset. Therefore, for each unit of the bank's asset, the bank has to raise k from the shareholders and $1 - k$ from the depositors. The bank has to guarantee a return to equity (ROE) of ρ ($\rho > 0$) for the shareholders, and we assume that the bank pays a gross return R for each $1 - k$ unit of deposit.

(C1) Suppose there is an R under which depositors are willing to deposit and the bank invests only in the bad assets. Compute R and the optimal level of capital for the profit-maximizing bank. (10 points)

With capital holding k, to convince the depositors to participate in a bank running only bad project, $p_B R \geq 1 - k$; depositors earn zero profit from deposits, $p_B R - (1 - k) = 0$. Therefore, $R = \frac{1-k}{p_B}$. A profit-maximizing bank running only bad project chooses to

$$\max_k p_B \left(B - \frac{1-k}{p_B} \right) - (1 + \rho)k$$

which is linear in k. The optimal k is 0, call it k_B.

(C2) Suppose, instead, there is an R under which depositors are willing to deposit and the bank invests only in the good assets. Compute R and the optimal level of capital for the profit- maximizing bank. (10 points)

With capital holding k, to convince the depositors to participate in a bank running only good project, $p_G R \geq 1 - k$; depositors earn zero profit from deposits, $p_G R - (1 - k) = 0$. Therefore, $R = \frac{1-k}{p_G}$. A profit-maximizing bank running only good project chooses to

$$\max_k p_G\left(G - \frac{1-k}{p_G}\right) - (1+\rho)k$$

which is linear in k. The optimal k should be as small as possible. On the other hand, the fact that the bank is only running good project implies that $R \leq \hat{R}$*. Now* $\hat{R}$ *is achieved under* $p_G(G-R)-(1+\rho)k = p_B(B-R)-(1+\rho)k$*, compute to get* $\hat{R} = \frac{p_G G - p_B B}{p_G - p_B}$*. Combining* $R = \frac{1-k}{p_G}$ *and* $R \leq \hat{R}$ to get $k \geq 1 - \left(\frac{p_G G - p_B B}{p_G - p_B}\right) p_G$. The optimal capital level is thus $k = 1 - \left(\frac{p_G G - p_B B}{p_G - p_B}\right) p_G$, call it k_G.

(C3) Show there exists a threshold $\hat{\rho}$, such that for any $\rho < \hat{\rho}$, the profit-maximizing bank chooses an optimal capital level k^* and invests only in the good assets. Compute $\hat{\rho}$ and k^*. (15 points)

Given the results from (C1) and (C2), a profit-maximizing bank will either (1) run good project only, with k_G*, or (2) run bad project only, with* k_B*. If the bank chooses only good project, she must earn higher profit, or*

$$p_G\left(G - \frac{1-k_G}{p_G}\right) - (1+\rho)k_G > p_B\left(B - \frac{1-k_B}{p_B}\right) - (1+\rho)k_B.$$

Solve to get $\rho < \frac{p_G G - p_B B}{k_G} = \frac{p_G G - p_B B}{1-\left(\frac{p_G G - p_B B}{p_G - p_B}\right)p_G} = \hat{\rho}$*, and* $k^* = k_G = 1 - \left(\frac{p_G G - p_B B}{p_G - p_B}\right) p_G$.

3. Financial intermediation, fragility, and unconventional monetary policy (60 points)

Consider a one-good, three-date economy: There are infinitely many ex ante identical consumers (whose population is normalized to 1), each endowed with one unit of resource at $t = 0$. Consumption may take place either at $t = 1$ or $t = 2$, while each consumer's timing preference of consumption only gets revealed at $t = 1$: With probability p ($0 < p < 1$) a consumer is an impatient one (type 1 consumer), who only values consumption at $t = 1$, while with probability $1 - p$ a consumer is a patient one (type 2 consumer), who only values consumption at $t = 2$. A consumer's type is private information and only known to herself.

Let c_i denote the consumption of a type $i = 1, 2$ consumer. At $t = 0$, without knowing her type, a consumer's expected utility from consumption is $u = p\sqrt{c_1} + (1-p)\sqrt{c_2}$.

The economy has two technologies of transferring resources between periods: storage technology with gross return equal to 1, and a long-term investment technology with a constant gross return $R > 1$ at $t = 2$ for every unit invested at $t = 0$. If necessary, an on-going long-term project can be liquidated or stopped prematurely at $t = 1$, with a return $0 \leq \delta < 1$.

(A) Suppose at $t = 0$ a social planner allocates all resources in this economy to maximize each consumer's expected utility: At $t = 0$ the planner collects $0 \leq \alpha \leq 1$ from each consumer and invest in the storage technology, and the rest — $(1-\alpha)$ from each consumer — will be invested in the long-term technology. At $t = 1$ the total proceeds from the storage technology will be evenly distributed among impatient consumers (whose population is p), and at $t = 2$ the total proceeds from the long-term technology will be evenly distributed among patient consumers (whose population is $1-p$). Show that the optimal solution to type $i = 1, 2$ consumer's consumption is $c_1^* = \frac{1}{p+(1-p)R}$, $c_2^* = \frac{R^2}{p+(1-p)R}$, and the resource invested in storage technology is $\alpha^* = \frac{p}{p+(1-p)R}$. (15 points)

The social planner solves

$$\begin{aligned} &\max_{\alpha} \; p\sqrt{c_1} + (1-p)\sqrt{c_2}, \\ &s.t. \; pc_1 = \alpha, \\ &\quad (1-p)c_2 = (1-\alpha)R. \end{aligned}$$

Solve to get $c_1^* = \frac{1}{p+(1-p)R}$, $c_2^* = \frac{R^2}{p+(1-p)R}$, *and* $\alpha^* = \frac{p}{p+(1-p)R}$.

(B) Suppose that the economy is in autarky such that every consumer has to allocate her endowments between two technologies by herself at $t = 0$. Show that consumers cannot achieve the optimal solution defined in (A). (10 points)

Suppose at $t = 0$*, without knowing her type, one consumer invests* $0 \le \alpha \le 1$ *on short assets and the rest on long assets. At* $t = 1$*, her type gets revealed.*

(1) *If she is type 1, she gets the storage and liquidates the rest at* $t = 1$, $c_1^a = \alpha + (1-\alpha)\delta \le 1$;
(2) *If she is type 2, she waits and gets returns from both assets at* $t = 2$, $c_2^a = \alpha + (1-\alpha)R \le R$.

Given that $c_2^a \le R < c_2^*$*, certainly the allocation* $\left(c_1^a, c_2^a\right) \neq \left(c_1^*, c_2^*\right)$ *is inferior to the planner's solution. Note: Here the relative rate of risk aversion is smaller than 1, which is different from what is assumed in the standard Diamond-Dybvig model. As a result* $c_1^* < 1$ *and* $c_2^* > R$*, therefore, it is crucial here to show that* $c_2^* > R$ *so that it never falls in the range of* c_2^a *and* $\left(c_1^a, c_2^a\right) \neq \left(c_1^*, c_2^*\right)$ *for sure.*

(C) Suppose there is a competitive banking sector in the economy, in which banks take consumers' endowments as deposits at $t = 0$ and allocate between the two technologies. Consumers withdraw c_i at $t = i$ according to their type i. Show that banks can implement the optimal solution achieved in (A) in the following way: (1) Banks invest α^* of deposits in storage technology, $1-\alpha^*$ of deposits on long-term technology; (2) consumers who withdraw at $t = 1$ get c_1^* each, and consumers who withdraw at $t = 2$ get c_2^* each; (3) impatient consumers all withdraw at $t = 1$ and patient consumers all withdraw at $t = 2$. (10 points)

The social planner's solution can be decentralized in the banking economy because the banking allocation $\left(c_1^*, c_2^*\right)$ *is (1) utility maximizing as it solves the social planner's problem; (2) feasible as it fulfills the resource constraints as specified in the planner's problem; and (3) implementable: Impatient consumers won't mimic the patient ones because they cannot wait till* $t = 2$ *and the patient ones won't mimic the impatient ones because they are worse off*

($c_1^* < c_2^*$) by mimicking, so that the deposit contract is incentive compatible and consumers self-select the proper outcomes.

(D) Banking sector in this economy is fragile: Patient consumers may demand their deposits at $t = 1$, which leads to bank run. However, whether this happens or not crucially depends on the value of δ. Show that there exists a threshold $\underline{\delta}$, such that as long as $\delta > \underline{\delta}$ and banks do the same as described in (C), there will never be bank runs. (15 points)

The bank run outcome is not equilibrium only if it is profitable for a patient consumer to deviate unilaterally given that all other consumers run on the bank; this is equivalent to saying that there is resource left even after all consumers run on the bank, so that it is profitable for a patient consumer to unilaterally deviate, wait till $t = 2$ and get better off: $pc_1^* + \left(1 - pc_1^*\right)\delta > c_1^*$, or, $\delta > \frac{(1-p)c_1^*}{1-pc_1^*} = \frac{1}{R} = \underline{\delta}$ (using c_1^* from (A). However, it is sufficient to reach $\delta > \frac{(1-p)c_1^*}{1-pc_1^*} = \underline{\delta}$).

(E) During the 2007-2009 global financial crisis, several central banks purchased huge volume of securities from the financial market, hoping to prevent the prices of long assets from falling too much. Using your finding in (D), explain why such unconventional policy helps eliminate panics in banking sector. (10 points)

As long as the asset purchasing program can maintain $\delta > \underline{\delta}$, bank runs are fully eliminated as explained in (D).