ECON 4335 Economics of Banking, Fall 2019

Final Exam: Grading Guidance

Working Draft *7 November 2019*

1. Are the following statements true, false, or uncertain? Briefly explain (48 points)

a) (12 points) Uncertain. This depends on the initial money market rate: if the initial money market rate is above the initial interest rate paid on reserves (the "floor"), cutting the interest rate paid on reserves does not affect the money market rate; if the initial money market rate is equal to the interest rate paid on reserves, cutting the interest rate paid on reserves implies a fall in the money market rate, implying an expansionary monetary policy.

b) (12 points) False. Credit rationing may be persistent, too. It may arises from adverse selection in the credit market: although raising loan rate increases a bank's profit, it also leads to a higher concentration of risky borrowers in the borrower pool, increasing the likelihood of borrowers' default and reducing banks' profit; therefore, profit-maximizing banks would choose a relatively lower lending rate and ration credit supply to ensure more prudent borrowers in the customer pool and less loss from the risky borrowers.

c) (12 points) False. People's borrowing capacity is limited by the value of their collateral, which is much affected by macroeconomic conditions and financial market. Maximizing consumption by borrowing with binding constraints implies that changing collateral value will add feedback to consumption, generating excessive volatility in consumption. This is especially true in the downturn: with falling consumption and aggregate demand, the collateral value falls, so as borrowers' borrowing capacity. Borrowers will be forced to deleverage and repay debt, which further hurts their consumption.

d) (12 points) Uncertain. With higher deposit rate from the tighter competition, banks do have the incentive to take more risk in their own investments to increase the net interest margin due to limited liability; on the other hand, in the long run, banks may also be willing to coordinate to be prudent, if the benefit of being prudent in the long run outweighs the profit from gambling in the short run, as long as the competition is not too tight and bank's net interest margin is not too low.

2. Adverse selection and credit market (52 points)

(A) (13 points) With perfect information, the bank can observe true types of entrepreneurs, so that it is able to charge different loan rates from different entrepreneurs. The profit-maximizing bank's problem is to choose the optimal loan rates R_G / R_B for good / bad entrepreneurs.

$$\begin{aligned}
\max_{R_G,R_B}\ & \frac{1}{2}R_G p + \frac{1}{2}R_B q - 1, \\
s.t.\ & 2p - R_G p \geq 0, (PC - G) \\
& 3q - R_B q \geq 0, (PC - B)
\end{aligned}$$

$$\frac{1}{2}R_G p + \frac{1}{2}R_B q - 1 \geq 0.(PC - Bank).$$

Solve to get $R_G = 2$, $R_B = 3$ and (PC-Bank) holds.

(B) (13 points) If the good entrepreneurs are willing to accept the contract, their participation constraint (PC-G) must hold, or, $2p - Rp \geq 0$, $R \leq 2$. If the bad entrepreneurs are willing to accept the contract, their participation constraint (PC-B) must hold, or, $3q - Rq \geq 0$, $R \leq 3$. To serve both types, it must be $R \leq 2$.

To serve both types, the profit-maximizing bank's problem is to choose the optimal loan rates R

$$\max_{R\leq 2} \frac{1}{2}Rp + \frac{1}{2}Rq - 1,$$
$$s.t.\ \Pi_1 = \frac{1}{2}Rp + \frac{1}{2}Rq - 1 \geq 0.(PC - Bank).$$

The object function is linear in R, implying the optimal solution is $R = 2$. However, (PC-Bank) needs to hold at the same time, to make sure the bank is willing to serve both, i.e., $\Pi_1 = p + q - 1 \geq 0$.

(C) (13 points) That only bad entrepreneurs are willing to accept the contract implies that (PC-G) is violated, $R > 2$, and (PC-B) still holds, $R \leq 3$. To serve only bad entrepreneurs, the profit-maximizing bank's problem is to choose the optimal loan rates R

$$\max_{2<R\leq 3} \frac{1}{2}Rq - \frac{1}{2},$$
$$s.t.\ \Pi_2 = \frac{1}{2}Rq - \frac{1}{2} \geq 0.(PC - Bank).$$

The object function is linear in R, implying the optimal solution is $R = 3$. (PC-Bank) holds, too, as $\Pi_2 = \frac{3}{2}q - \frac{1}{2} > 0$.

(D) (13 points) The bank only wants to serve bad entrepreneurs if $\Pi_2 > \Pi_1$, i.e., $\frac{3}{2}q - \frac{1}{2} > p + q - 1$, i.e., $2p - 1 < q$.

In comparison, if the bank serves both types instead of bad type only, it gains $\frac{1}{2}Rp - \frac{1}{2}$ with $R = 2$ from including the good entrepreneurs; however, it also loses $\frac{1}{2}(3 - 2)q$ from the bad entrepreneurs for being unable to charge higher loan rate from them. The bank only wants to serve bad entrepreneurs if the loss outweighs the gain.

3. Bank run (50 points)

(A) (14 points) The social planner solves

$$\begin{aligned}
\max_{\alpha}\ & u = p\frac{1}{1-\gamma}c_1^{1-\gamma} + (1-p)\frac{1}{1-\gamma}c_2^{1-\gamma}, \\
s.t.\ & pc_1 = \alpha, \\
& (1-p)c_2 = (1-\alpha)R.
\end{aligned}$$

Solve to get $c_1^* = \frac{R}{pR+(1-p)R^{\frac{1}{\gamma}}}$, $c_2^* = \frac{R^{1+\frac{1}{\gamma}}}{pR+(1-p)R^{\frac{1}{\gamma}}}$, and $\alpha^* = pc_1^* = \frac{pR}{pR+(1-p)R^{\frac{1}{\gamma}}}$.

(B) (14 points) The bank solves

$$\begin{aligned}
\max_{\alpha}\ & u = p\frac{1}{1-\gamma}c_1^{1-\gamma} + (1-p)\frac{1}{1-\gamma}c_2^{1-\gamma}, \\
s.t.\ & pc_1 = \alpha, \\
& (1-p)c_2 = (1-\alpha)R.
\end{aligned}$$

which is exactly the same as the planner's problem.

The social planner's solution can be implemented in the banking economy because the banking allocation $\left(c_1^*, c_2^*\right)$ is (1) utility maximizing as it solves both the bank's and social planner's problems; (2) feasible as it fulfills the resource constraints as specified in the planner's problem; and (3) implementable: type 2 consumers won't mimic the patient ones because they cannot wait till $t = 2$ and the patient ones won't mimic the impatient ones because they are worse off ($c_1^* < c_2^*$) by mimicking, so that the deposit contract is incentive compatible and consumers self-select the proper outcomes.

(C) (11 points) If bank run happens at $t = 1$, the total demand for repayment is $c_1^* > 1$, and the total cash supply is $\alpha^* + (1-\alpha^*)\delta < 1 < c_1^*$. The banks are bankrupted, and expected payoff of each depositor is $\alpha^* + (1-\alpha^*)\delta > 0$, call it c_1^r.

To show this is a Nash equilibrium, we just need to show that there is no profitable unilateral deviation. Suppose one consumer deviates unilaterally, or, she waits until $t = 2$ instead of joining the bank run at $t = 1$. However, as the banks are bankrupted at $t = 1$, her consumption will be $c_2^r = 0 < c_1^r$, making her worse off. As there is no profitable unilateral deviation, the bank run is a Nash equilibrium.

(D) (11 points) With panicking type 2 consumers, total demand for repayment is $(p+f)c_1^* > \alpha^*$, so that an amount L of long assets needs to be liquidated until $\alpha^* + \delta L = (p+f)c_1^*$, $L = \frac{fc_1^*}{\delta}$. For the remaining type 2 consumers, their expected return (if they wait till $t = 2$) is $\tilde{c}_2 = \frac{1-\alpha^*-L}{1-p-f}R = \frac{1-\alpha^*-\frac{fc_1^*}{\delta}}{1-p-f}R$. A bank run only happens if f is large enough so that $\tilde{c}_2 < c_1^*$, incentivizing type 2 consumers to mimic type 1 consumers and withdraw early (it is sufficient to reach here). Further, we can compute the threshold $\underline{f}$ from $\tilde{c}_2 = \frac{1-\alpha^*-\frac{fc_1^*}{\delta}}{1-p-f}R < c_1^*$, or, $f >$

$$\frac{R(1-\alpha^*)-(1-p)c_1^*}{\left(\frac{R}{\delta}-1\right)c_1^*} = \underline{f}.$$