6th sem - ECON 4335

Question A

1.1 The fundamental value of an asset price is commonly defined as the sum of future expected dividends from holding the asset, discounted by a risk-free interest rate. Expected future dividends from holding a stock may be one example, house prices reflecting the expected future cash flows from renting a property instead of buying another.

1.2 The main point is that actors have perfect foresight and are able to distinguish between fundamental and 'bubble' value. As the list in Rødseth's slides from 24/4 show (henceforth just referred to as the "slides"), these conditions are:

1. Has to be infinitely many investors that the bubble can be passed on to for investors to cash their profits from holding the asset.
2. Expected return must be equal to or exceed the risk-free interest rate. From (4) in the slides, the continuation value is given as $b_t^c = R^{-1}(1-p)b_{t+1}^c$ which can be solved for b_{t+1}^c: $b_{t+1}^c = \frac{Rb_t^c}{1-p} \geq R$, where $p =$ P(Bubble bursting). If $b_{t+1}^c < R$, then investors would buy the risk-free instrument instead of the bubbly asset.
3. Investable funds cannot grow faster than economy forever
4. Expected return must not exceed the growth rate of the economy. If the opposite was true, eventually there would not be enough credit to buy all the assets. Investors with perfect foresight would foresee this, and then it would be optimal to sell the assets in period before the bubble bursting for all investors, making the bubble burst anyway. So then they would sell one period earlier, and the same would be true until there's no bubble to begin with.
5. Rational bubbles can only exist if interest rate is below growth rate. Since $b_{t+1}^c > R$ then $g \leq R \Rightarrow g \leq b_{t+1}^c$ which violates the condition 4.
6. Bubbles cannot be negative (assuming free disposal)
7. The assets must not be easily reproducible making the supply price inelastic. If the assets were easily reproducible, an increase in asset prices would stimulate higher supply in a relatively short amount of time, depressing prices. Examples where the supply is relatively inelastic, at least in the short term, is housing (and apparently, tulips).

1.3 In the OLG equilibrium, savings must equal investments, $k_{t+1} + b_t = s_t$, which is divided among real investment and asset holdings. If we assume that the economy is characterized by a standard CD, function $y_t = k^\alpha$ (assuming $l = 1$), the total return on capital investment should equal $(1+r) = R$ for one unit invested. Then, if $b_{t+1}^c > R$, the consumer will substitute away some of the real investment in favor of increased asset holdings. This reduces overall real investment, so-called "crowding out".

1.4 Building on the setup in Martin & Ventura, expansionary bubbles can appear where the value of the bubble increases investments, making the capital of the economy expand. The main mechanism is a relaxation of the credit constraint due to higher value of firms that are connected to the bubble, and an amplification through a multiplier effect that makes more credit available after increased investment. Basically, it's proposed that this increase will have two separate effects on the economy. As described in the question above, some of it will be channeled to increased savings in bubbly assets. Another effect is that more of the real investment is done by firms who are the most efficient, increasing the overall growth of the capital stock. The net effect on the economy is determined by the relative strengths of the opposing effects. If the latter is the strongest, the capital stock increases along with lower interest rates.

A possible relevance for banking crises, is that an expansionary bubble increases the total volume of lending on the premise that the asset prices are equal to the fundamental values. The extension of credit to a number of firms, which become unable to repay all their obligations to the bank when it's revealed that fundamental value is substantially lower than market value, may introduce systemic risk in that sense the many banks have bought into the bubble, due to its size. When it bursts, banks may become insolvent, or the banks that remain solvent might want to restrict their lending since they do not have full information on who's got claims on firms with bursting asset prices, drying up the interbank market. This may lead to a shortage of liquidity that further exacerbates a potential crisis.

In the late 80's, Finland, Norway and Sweden saw soaring asset prices and an increase in investment before their bubbles burst. Particularly Finland and Sweden saw large increases in housing prices.

1.5 One of the syllabus articles, Fuster et. al., investigates which dynamics might be observed with rational and semi-rational actors responding to a shock in asset prices and finds that rational actors correctly predict and adjust their investment levels according to the underlying economic process. The semi-rational actors overestimate the persistence of growth causing asset prices to rise/fall too much. However, in Martin & Ventura the rational actors are fully aware that they invest in a bubble, and their actions increases investment and credit growth. It's hard to determine if there is some difference in the investment levels in the two cases, but since they act in the same way as individuals that are not fully rational, our guess is that it's hard to distinguish rational from semi-rational actors, and then rational from semi-rational bubbles.

In the article by Borio & Drehman, which is also on the reading list, they look at several indicators for the likelihood of crises. These are measured as gaps between asset prices and their long-run trend. They find that indicators including credit-to-GDP gap, property price gap, and equity price gap are able to somewhat efficiently forecast the occurrence of crises in a long-term perspective.

In equation (4) from the slides, the probability stays the same for all periods. The article by Borio & Drehman implies that the probability can change over time. Then the continuation value would fall as the probability of a crisis increases. Maybe then investment and credit growth would become somewhat lower as p increases if it was a rational bubble?

1.6 Allen & Gale ch. 9 puts forth a theory where an agency problem is present. They argue that many investors in asset markets obtain their funds from external lenders which may not be capable of effectively monitoring the riskiness of the investors portfolio. Often, the investors are protected by limited liability which results in choosing a more risky portfolio than creditors would like provided that the return on the risky asset is sufficiently high. This is called risk-shifting. If one assumes that the risky assets are in fixed supply, the increased demand stemming from the agency problem will increase the equilibrium price of the risky asset such that it exceeds the fundamental value, creating a bubble. A crucial determinant here is the initial level of credit. During a period of financial liberalization, the total volume of credit is often increased, raising the likelihood of a bubble, and 'fuels' its size.

In Fuster et. al., they model actors with semi-rational expectations about the future and simulate their reactions to news about increased dividends, modeled as shocks to input-response functions. They find that investors overestimate the persistence of shocks, i.e. overestimate the state which the economy reverts to. This makes asset prices fluctuate too much relative to the fundamental prices, making a bubble possible. In the Allen & Gale setup, the increase in asset prices is partly explained within the model, whereas in Fuster et. al. it's exogenous. However, if a financial liberalization causes a shock to asset prices the latter model may explain a mechanism that strengthens the growth of the bubble.

Question B

2.1 Let $X_i, i \in \{1,2\}$ be a stochastic variable where, i designates the mortages. Let, $X_i = 1$ denote that the i-th mortage is repaid, and $X_i = 0$ the opposite. The sample space can then be defined as $S = \{11,10,01,00\}$. Also, let $Y = X_1 + X_2$. From the text, we assume that r is related to the probabilities in the following way:

$$P(X_i = 1|X_j = 1)_{i\neq j} = pr \geq p \Leftrightarrow r \geq 1$$

and similarly for $X_i = 0$. Then

$$P(Y = 2) = P(X_i = 1|X_j = 1)_{i\neq j} \times P(X_j = 1) = p^2 r$$

$$P(Y = 0) = P(X_i = 0|X_j = 0)_{i\neq j} \times P(X_j = 0) = (1-p)^2 r$$

So if a mortage fails or not, the probability that the other mortage does the same is greater than the individual probability of failure or repayment. This implies that if one mortage fails (repays), the likelihood that the second will be repaid (fails) is lower than the individual probability: $P(X_i = 1|X_j = 0)_{j\neq i} < p$ and $P(X_i = 0|X_j = 1)_{j\neq i} < (1-p)$.

Let's say $X_j = 1$ then, from the definitions above $P(X_i = 1|X_j = 1)_{i\neq j} = P(X_i = 1) \times r = pr$. Since the conditional distribution of X_i given X_j must sum to one in probabilities, we know that $pr + \tilde{p} = 1$, where $\tilde{p} = P(X_i = 0|X_j = 1)_{j\neq i}$. Solving this for $\tilde{p}$ gives

$$\tilde{p}_{X_i=0|X_j=1} = 1 - pr.$$

Similarly

$$\tilde{p}_{X_i=1|X_j=0} = 1-(1-p)r$$

Using this, we find that

$$P(Y=1) = P\Big(X_i=1, X_j=0 \bigcup X_j=1, X_i=0\Big)_{j\neq i}$$

$$= P(X_i=0|X_j=1)_{j\neq i} \times P(X_j=1) + P(X_i=1|X_j=0)_{j\neq i} \times P(X_j=0)$$

$$= \tilde{p}_{X_i=0|X_j=1} \times p + \tilde{p}_{X_i=1|X_j=0} \times (1-p)$$

$$= (1-pr)p + \Big(1-(1-p)r\Big)(1-p)$$

$$= 1+2p(r-pr)-r$$

This is the same result you get by taking $P(Y=1) = 1-P(Y=2)-P(Y=0)$.

2.2

Senior class: $E(Senior) = 2P(Y=2)+P(Y=1)$

Junior class: $E(Junior) = 2P(Y=2)$

r/p	.8	.9
1	1.6	1.8
1.1	1.66	1.88
1.2	1.72	1.96

Expected payoff - Senior class

r/p	0.8	0.9
1	1.28	1.62
1.1	1.408	1.782
1.2	1.536	1.944

Expected payoff - Junior class

The senior class has a higher expected payoff since the probability of getting at least one mortage repaid is greater than the probability of two repayments. Also, a higher correlation increases the expected payoff since it's less likely that you get just one mortage repayment, and more likely that you get two.

2.3 The different parameters that needs to be estimated are the probability of default and the correlation between the mortages.

2.4 A fall in housing prices might decrease the probability that mortages are repaid since some buyers might really be investors dependent on rising asset prices to repay. This would correspond to a lower p. The fall would decrease the expected return on the junior and senior class assets. However, the fall in p has a stronger effect on the junior class' expected payoff compared to the senior, since it simultaneously becomes more likely that you observe *one* mortage repayment.

Question C

This will probably depend somewhat on which rules are followed when setting the policy rate. Norges Bank uses a rule in which the policy rate (deposit rate at Norges Bank) is set equal to the bottom of the interest rate corridor. Liquidity is then supplied in an amount that secures redistribution of liquidity through the interbank market, forcing the interbank rate to become equal to the policy rate. Naturally, an increase in the policy rate will raise the interbank rate, making it more expensive to obtain liquidity. This may increase the funding costs of banks, causing a shift in the spread between their deposit and lending rate. An increased lending rate with equal deposit rate could be a possible outcome. If the "margin" is defined as this spread, an increase in the CB rate, might give rise to higher margins.

An important question is how the increased funding costs of intermediation is shared between borrowers and "their" banks. This is quite hard to answer, but one factor might be the elasticity of loan demand. If this is somewhat inelastic, banks can "push" more of the increased funding costs on the lenders without loosing too much demand for their product. An example might be housing markets. Here there are considerable transaction costs involved when selling your home, including fees on real estate agent, moving expenses and search costs when trying to find a new place to live. This might lead consumers to reduce other types of consumption before selling their home, making them less responsive to interest rate increases. Probably, this in turn depends on the income of the borrowers and their leverage.