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Contracting under private ex ante information

1. *Introduction*

An important issue in allocating capital from a bank (the lender) to an entrepreneur (the borrower) is related to private information about the project the entrepreneur wants to undertake through external finance. The borrower will normally have better information about the project than the lender – the entrepreneur is better informed about, say, the return structure, the probability of success, the quality of the project, the quality of the management etc. When a borrower knows more about a project than the lender, before the project has started is said to have *private information ex ante*. This is a special case of asymmetric information. The capital market is to a large extent "hurt" by such asymmetric information and it is therefore important whether this will cause efficiency loss. As we demonstrate, asymmetric information may produce social costs in the sense that we have too little investment (too few projects get finance) or too much investment (too many projects are accepted).[1]

We will consider a very simple lending-borrowing relationship:[2] A borrower or an entrepreneur wants to finance a project with a fixed cost (fixed size of the project). The project has a random return Y that is type-dependent in the sense that the probability of success depends on whether the borrower/entrepreneur is a *good* type or a *bad* type. The borrrower's type is assigned before the loan is extended, and is known only to the borrower herself. We say that one's type is private information. (The informational issue that is analyzed in Holmstrom & Tirole (1997), is called *moral hazard*. Such an informational problem occurs because the borrower can take an unverifiable action after the contract has been signed. Hence, depending on how the contract is designed, the agent's (borrower's) incentive to take the actions wanted by

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[1] This problem is related to Akerlof's famous "Lemon Problem".

[2] See chapter 6 in Jean Tirole "The Theory of Corporate Finance"; Princeton University Press, 2006.

the lender will normally be distorted. In Holmstrom & Tirole the issue of moral hazard is related both to the borrower and to the lender.)

2. *The Model*

The project itself can be regarded as a lottery with two outcomes, depending on the entrepreneur's type, where "prob" means probability:

If a borrower is *good*; she is facing the lottery: $Y = \begin{cases} R & \text{with prob } p \in (0,1) \\ 0 & \text{with prob } 1-p \end{cases}$

If she is *bad*, she faces the lottery: $Y = \begin{cases} R & \text{with prob } 0 < q < p \\ 0 & \text{with prob } 1-q \end{cases}$

The return of the project if success is *R* is the same across types, but the probability of succes is higher for the good type. The cost of the project is I and set equal to one, and we assume that the entrepreneur has no wealth or cash.[3] The probability of being a good entrepreneur is α (here assumed to be equal to the fraction of good entrepreneurs in the population) and the probability of being a bad entrepreneur is $1-\alpha$. (The model can be interpreted either as a pure single-borrower-single-lender relationship or as a description of a market with a large number of entrepreneurs, of which a fraction α is considered good, and where the lender cannot distinguish between the borrowers when he is asked to finance a project.)

Define the lenders' prior probability of success as $m := \alpha p + (1-\alpha)q$, which is the *average* probability of success in the population. For $\alpha \in (0,1)$, we have $q < m < p$. Both probability distributions are common knowledge; known by everyone, and known that everyone has this knowledge and so on.

[3] If the entrepreneurs were endowed with (different) wealth or cash, «wealthy good» entrepreneurs could signal their quality to the lender by investing own wealth in the project and by then increase her own stake in the project; a strategy that might not be mimicked by a "bad" or "poor" entrepreneur.

We also assume that both parties are risk-neutral, and that the entrepreneur/ borrower is protected by limited liability – which means that no repayment is made if the outcome is "failure" with a zero gross return.

Consider then the expected payoffs when a lender offers to finance the project by lending one unit at a gross rate of interest $1+i$, when the cost of the project $I=1$, as assumed. Then a borrower's expected payoff will be $V^B = \pi(R-(1+i))$, where $\pi \in \left\{q,p\right\}$, depending on type. Assume also that the banking industry is competitive and has a gross funding cost per unit $1+f$. A representative bank has then the following expected payoff: $V^L = \pi(1+i)-(1+f)$ when lending one unit to a borrower of unknown type, with $\pi \in \left\{q,p\right\}$.

One central assumption for the subsequent discussion is that at least the good project is socially efficient in the sense that $pR-(1+f)>0$. (We may say that only the good project qualifies for being called creditworthy. Then there are two cases to be analyzed regarding the quality of a bad project:

- One where $qR-(1+f)>0$; the bad project is creditworthy as well
- Another where the bad project is not creditworthy; $qR-(1+f)<0$.

We analyze first the benchmark case with symmetric and perfect (full) information. Thereafter we consider the impact of asymmetric or private information.

3. *Full Information*

Suppose that any bank knows the true type of any borrower. Because we have a competitive banking industry the expected payoff to any lender is equal to zero in equilibrium.

Suppose first that only the good type is creditworthy; hence we have $qR-(1+f)<0$ by assumption. Then in a competitive equilibrium only the good entrepreneur gets the project financed, with a competitive gross rate of interest $1+i_{FI}^{G}$, as determined from the zero-expected profit condition:

(1) $$V_{FI}^{L}=0 \Leftrightarrow p(1+i_{SI}^{G})-(1+f)=0 \Leftrightarrow 1+i_{SI}^{G}=\frac{1+f}{p}$$

The gross rate of interest in a full information competitive equilibrium will yield zero expected profit to any bank as loans are extended only to good entrepreneurs. (If expected profit was positive, new banks would enter the market; if negative banks would exit.)

The expected payoff to a good entrepreneur will then be $V_{FI}^{B}(G)=p(R-(1+i_{SI}^{G}))=pR-(1+f)>0$, being equal to the expected social surplus from undertaking a good project.

If the bad project is creditworthy as well, i.e. if $qR-(1+f)>0$, the loan offered to bad types will in equilibrium require a gross rate of interest as determined from

(2) $$1+i_{SI}^{B}=\frac{1+f}{q}>1+i_{SI}^{G}$$

This condition follows from the zero expected profit condition to banks from extendig loans to bad types, and when types can be distinguished from each other. If the bad project is creditworthy the rate of interest charged on bad loans will be higher than the one charged on good loans.

4. *Asymmetric Information*

In the full information case with no private information, the banks can offer separate contracts, one for each type as outlined above, with a higher rate of interest required from entrepreneurs with bad projects. Let us now relax the assumption that each borrower’s type is known and instead assume that only an entrepreneur knows her true type, whereas the lenders know the probability distributions. Also the return itself is verifiable.

Suppose first that both project types are creditworthy, hence we have $pR-(1+f)>0$ as well as $qR-(1+f)>0$. Let us assume that the banks, rather naïvely, offer the two contracts derived above to a privately informed borrower who then is given the opportunity to select. Because the rate of interest is higher for a bad project than for a good one, any rational entrepreneur will pretend to be a good type. Hence, any bad entrepreneur will mimick a good one, and obtain an expected payoff exceeding the one she could have obtained by revealing her true type:

(3) $$q\left[R-(1+i_{FI}^{G})\right]= qR-\frac{q}{p}(1+f)>qR-(1+f)>0\text{, because } p>q.$$

The full information competitive loan contracts will not reveal private information; any bad entrepreneur will pretend to be good.

In this case the banks will not break even; they will in fact run with a deficit. Because any entrepreneur will take the low-rate-of-interest-loan, $1+i_{FI}^{G}=\dfrac{1+f}{p}$, the bank will break even on the good borrowers but will suffer a loss on the bad ones; hence any bank's expected profit will now be negative, as seen from:

(4) $$\alpha\underbrace{\left[p(i+i_{FI}^{G})-(1+f)\right]}_{=0}+(1-\alpha)\underbrace{\left[q(1+i_{FI}^{G})-(1+f)\right]}_{-}=(1-\alpha)(1+f)\left[\frac{q}{p}-1\right]<0$$

A fraction α of the borrowers has good projects. These borrowers pay the rate of interest that will break even. Because the bad entrepreneurs take the same loan contract, a fraction $1-\alpha$ will pay, in fact, a too low rate of interest; hence the expected profit to the banks from these entrepreneurs is negative. Hence, this contract will not survive as an equilibrium contract – banks go bankerupt.

The question is then: What can the competitive banking sector do? Due to asymmetric information, no bank can, within this setting, do anything else than to offer one common contract (called a pooling contract) with a common rate of interest offered to any borrower if success; call this gross rate of interest $1+\hat{i}$, so as to make expected profits in the banking industry equal to zero. Hence the loan contract must obey:

$$(5) \qquad m(1+\hat{i}) = 1+f \Rightarrow 1+\hat{i} = \frac{1+f}{m} \in \left(\frac{1+f}{p}, \frac{1+f}{q}\right)$$

The rate of interest under asymmetric information will then be between the two we have derived under full information; in particular it will exceed the one offered to good types under full information.

Our first conclusion is that asymmetric information will increase the cost of capital for good projects. This higher cost of capital for good projects is reflecting a kind of externality cost inflicted on the good types caused by the presence of bad types.

The next question we may ask is: What will be the impact on the mixture of loan applicants of this interest rate setting?

When being offered this loan contract with a gross rate of interest $1+\hat{i} = \dfrac{1+f}{m}$, a borrower's expected payoff will depend on her type, but as seen below, it will either be positive or negative for both types:

$$(6-i) \qquad V^B(\hat{i}, good) = p\left[R-(1+\hat{i})\right] = pR - \frac{p(1+f)}{m} = \frac{p}{m}\left[mR-(1+f)\right]$$

$$(6-ii) \qquad V^B(\hat{i}, bad) = q\left[R-(1+\hat{i})\right] = qR - \frac{q}{m}(1+f) = \frac{q}{m}\left[mR-(1+f)\right]$$

We then have the following important conclusion, when taking into account our previous assumption that $pR-(1+f)>0$:

If $mR-(1+f)<0$, then we must have $qR-(1+f)<0$. In that case **no borrower of any type will want to undertake the project at the gross rate of interest charged, as in (5), so as to make the expected profits in the banking industry equal to zero.** If being offered this contract, no firm will earn a non-negative expected profit. In this case there is no lending.

Even though good projects are socially desirable, along with bad projects not being creditworthy, the equilibrium rate of interest $1+\hat{i}=\frac{1+f}{m}$, will exclude any type of borrower from outside financing. This is a kind of Gresham's law: Good projects are driven out by bad projects. This is a market failure as projects that should have been realized, are not implemented in equilibrium with a gross rate of interest as derived in (5).

Under what circumstances will such a market failure occur? If the fraction of good borrowers is low; $\alpha<\alpha^0$, where the critical value α^0 is defined so as to make ex ante expected payoff to the bank equal to zero; i.e. $\left[\alpha^0 p+(1-\alpha^0)q\right]R=1+f$, then $mR-(1+f)=\left[\alpha p+(1-\alpha)q\right]R-(1+f)$ is negative for $\alpha<\alpha^0$.

Therefore, no lending will take place if $mR-(1+f)<0 \Rightarrow qR-(1+f)<0$. The banking industry is not fullfilling its task in allocating capital to profitable projects. In the rpesent case the market breaks down as good projects are not implemented at all. (Good borrowers are hurt by the presence of bad borrowers.) Because the banks cannot separate a good borrower from a bad one, the break even rate of interest charged by the banks will, if $\alpha<\alpha^0$, be too high for any good borrower to undertake the project. There is a social cost as there is now **underinvestment.**

If on the other hand, $mR\geq 1+f$, then either both projects are creditworthy or the bad project is not, but in that case we have $\alpha\geq\alpha^0$ (a high fraction of good projects).

If the borrower is good, we have the following payoffs:

For a bank:

$$(7-i) \quad V^L = p(1+\hat{i}) - (1+f) = p\frac{1+f}{m} - (1+f) = (1+f)\left[\frac{p}{m} - 1\right] > 0$$

because $p > m$, and for a good borrower:

$$(7-ii) \quad V^B = p(R - (1+\hat{i})) = pR - \frac{p(1+f)}{m} = \frac{p}{m}\left[mR - (1+f)\right] \geq 0$$

If the borrower is a bad type, the expected payoff for a bank is:

$$(8-i) \quad V^L = q(1+\hat{i}) - (1+f) = (1+f)\left[\frac{q}{m} - 1\right] < 0$$

because $m > q$, and an expected payoff for a bad borrower, as given by

$$(8-ii) \quad V^B = q(R - (1+\hat{i})) = \frac{q}{m}(mR - (1+f)) \geq 0$$

Hence, if the fraction of good projects is sufficiently high among all entrepreneurs, along with bad projects being **not** creditworthy; $qR < 1+f$, then a rate of interest charged by the banking industry so as to make zero expected profits from lending, will lead to **overinvestment**. The socially undesirable bad projects are undertaken along with the good ones. Ex post, the lenders lose money on providing bad entrepreneurs with loans, but will make profit on good types; hence there is some **cross-subsidization**. Adverse selection will therefore appear here as lower quality among the borrowers – too many bad projects are implemented.

The required rate of return on financing is now increased: If lending is desirable, i.e., if $mR \geq 1+f$ (if not; no lending at all, and we have underinvestment if $pR > 1+f$), this condition can be rewritten as:

(9)
$$
\begin{aligned}
&mR \geq 1+f \Leftrightarrow \left[\alpha p+(1-\alpha)q\right]R \geq 1+f \Leftrightarrow \\
&\left[1-(1-\alpha)\frac{p-q}{p}\right]pR := (1-\Lambda)pR \geq 1+f
\end{aligned}
$$

where Λ, according to Tirole, is *an index of adverse selection*. (We have only used the fact that $m = m+p-p = p-(1-\alpha)(p-q)$.)

What does $\Lambda := (1-\alpha)\dfrac{p-q}{p}$ indicate? The threshold for accepting a good project is now higher due to asymmetric information, as compared to full information. The investment criterion under asymmetric information can be written as

(10) $$pR \geq (1+f) + \Lambda pR$$

The additional cost term– the last term on the RHS of the inequality in (10) – operates like a cost inflicted due to a negative externality caused by the pure presence of bad entrepreneurs. A fraction $1-\alpha$ of the entrepreneurs is bad. The likelihood ratio is $\dfrac{p-q}{p}$ shows the relative reduction in success probability for a bad type. Per unit expected gross return from a good project (pR), the externality cost is given by $(1-\alpha)\dfrac{p-q}{p}$. This term captures the additional charge put on a borrower of good type so as to compensate for the unprofitable projects that are implemented due to asymmetric information; see (*7-i*) and (*8-i*). The good borrowers take the cost by being charged a higher rate of interest on the loan under asymmetric information than what they would ave been charges under full information; i.e. $1+\hat{i} = \dfrac{1+f}{m} > \dfrac{1+f}{p} = 1+i_{SI}^{G}$ from (*1*).

The information-adjusted payoff for a good project is then:

$$(11)\quad \begin{aligned} V^B(good) &= p[R-(1+\hat{i})] = pR-(1+f)+(1+f)-\frac{p(1+f)}{m} \\ &= pR-(1+f)-(1+f)\left[\frac{p}{m}-1\right] \end{aligned}$$

The classical standard investement criterion under full information; $pR-(1+f)$, is made stricter when being adjusted with a term like:

$$(12)\quad (1+f)\left[\frac{p}{\alpha p+(1-\alpha)q}-1\right] = (1+f)(1-\alpha)\frac{p-q}{m} = \frac{p}{m}(1-f)(1-\alpha)\frac{p-q}{p}$$

On using that $\frac{m}{p} = 1-\Lambda$, we have that a good borrower's net payoff under asymmetric information can be written as:

$$(11)'\qquad V^B(good) = \underbrace{pR-(1+f)}_{std\ NPV} - (1+f)\frac{\Lambda}{1-\Lambda} \leq pR-(1+f)$$

Hence the good borrowers are inflicted an additional cost due to the presence of bad borrowers in the market. The inability for banks to distinguish between good and bad projects is translated into a too high rate of interest charged on loans, and hence a lower expected payoff for the good entrepreneurs.