Part 1

Disyatat's model

- Firms (borrowers), banks, households (depositors)
- Everyone is risk-neutral
- Each firm can only borrow from one bank (not important?)
- Each household lends to only one bank (not mentioned)
- Open sector

Firms

- Firms pay labor in one period, sell output in the next
- Borrow to finance the wage bill
- Limited liability

Production function

$$y = N^{\beta}(1+\varepsilon) \qquad 0 < \beta < 1 \tag{1}$$

ε = aggregate productivity shock,

$$Pr(\varepsilon = 0) = \theta, \; Pr(\varepsilon = -1) = 1 - \theta$$

- Everything is lost in bad state
- If one firm fails, all fail

Firms

Expected profits (P=price, P_N = wage rate):

$$\Pi^e = \theta P \left[N^\beta - (1 + r_L) P_N N \right] \tag{2}$$

Optimal employment, from 1.o.cond.:

$$N = \left[\frac{P\beta}{(1 + r_L) P_N} \right]^{1/(1-\beta)} \tag{3}$$

Borrowing from bank (L = wage bill):

$$L = P_N N \tag{4}$$

Borrowing depends negatively on r_L
Risk of failure independent of r_L

Initial balance sheet

$$w = A \tag{5}$$

w = net assets, D = deposits, A = Other assets
Balance sheet after lending to firm

$$w = A + (L - D), \qquad D = L$$

Gross return on A is $1 + u$, with u uniformly distributed over $[\underline{u}, \overline{u}]$, $-1 < \underline{u} < 0$
Density $g(u) = \frac{1}{\overline{u} - \underline{u}}$
If firms fail, bank is able to repay depositors only if $u > u^*$ defined by

$$w(1 + u_*) = L(1 + r_d) \Longleftrightarrow u^* = \frac{L}{w}(1 + r_d) - 1 \tag{6}$$

Default probabilities

The probability that a bank fails given that borrowers fail is

$$Pr(u < u^*) = \frac{u^* - \underline{u}}{\overline{u} - \underline{u}} \tag{7}$$

The unconditional probability of bank default is then

$$\Pi = (1 - \theta)\frac{u^* - \underline{u}}{\overline{u} - \underline{u}} \tag{8}$$

Expected value of failed bank's assets

$$\psi^e = w \int_{\underline{u}}^{u^*} (1+u) \frac{1}{u^* - \underline{u}} du = w \left[1 + \frac{1}{2}(u^* + \underline{u}) \right] \qquad (9)$$

If we insert for u_*we get

$$\psi^e = \frac{1}{2} \left[L(1 + r_d) + w(1 + \underline{u}) \right]$$

Households

Investment alternatives:
Government bond, risk free $1 + r_f$
Bank deposit $1 + r_d$ if bank can pay
Risk neutral households require equal expected returns

$$1 + r_f = (1 - \Pi)(1 + r_d) + \Pi\left(\frac{1}{D}\psi^e - c\right)$$

Alternative expression

$$1 + r_f = (1 - \Pi)(1 + r_d) + \Pi\frac{w}{L}\left[1 + \frac{1}{2}(u^* + \underline{u}) - c\right]$$

Insert $u^* = \frac{L}{w}(1 + r_d) - 1$ and solve for $1 + r_d$:

$$1 + r_d = 1 + r_f + \frac{\Pi}{2 - \Pi}\left[1 + r_f - \frac{w}{L}(1 + \underline{u})\right]$$

- Margin increasing in r_f and L, decreasing in w

Bank lending rate

Competition means the expected values of what banks receive and pay at the end of period must be equal

From firms they receive $(1+r_L)L$ with probability θ

To depositors they pay

$(1+r_d)L$ with probability $1-\Pi$

$w(1+u)$with probability Π

Equality in expectations means

$$\theta(1+r_L)L = (1-\Pi)(1+r_d)L + \Pi\psi^e$$

$$(1+r_L) = (1+r_d) + \frac{1}{2}\Pi\left[\frac{w}{L}(1+\underline{u}) - (1+r_d)\right]$$

Overview of expected returns

Ourcome	No default	Firm defaults	Bank defaults	Alt. cost
Probability	θ	$1-\theta-\Pi$	Π	
Bank	$(r_L - r_D)L$	$-(1-r_D)L$	$-\psi^e$	0
Household	$(1+r_D)L$	$(1+r_D)L$	$\psi^e - cL$	$(1+r_f)L$
Sum	$(1+r_L)L$	0	$-cL$	$(1+r_f)L$

Table:

Since both are risk-neutral

$$\theta(1+r_L)L - \Pi cL = (1+r_f)L$$

Solution for loan rate is

$$1+r_L = \frac{1}{\theta}(1+r_f) + \frac{\Pi}{\theta}c$$

Division of the margin

Deposits over safe asset:

$$r_D - r_f = \frac{\Pi}{1-\Pi}\left[(1+r_f) - (\psi^e/L) + c\right]$$

$$r_L - r_f = \frac{1-\theta}{\theta}(1+r_f) + \frac{\Pi}{\theta}c$$

Loan supply

$$1 + r_L = \frac{1}{\theta}(1 + r_f) + \frac{\Pi c}{\theta}$$

$$\Pi = (1 - \theta)\frac{u^* - \underline{u}}{\overline{u} - \underline{u}}$$

$$u^* = \frac{L}{w}(1 + r_D)$$

$\Pi$is increasing in u^*,which is increasing in L. Problem $r_D$also depends on$L$

Part 2

Bianchi model

Background

Bianchi (2011) AER: Overborrowing and systemic externalities in the business cycle

- Sudden stops:
  - Capital inflow suddenly turns around
  - Current account deficit turns to surplus
  - Boom turns to deep recession
  - Example: Asian crisis 1997-98, Greece, Portugal, Spain
- RBC-model
  - Technology shocks
  - Consumption smoothing
  - CA surplus in booms, deficits in recession

Bianchi's explanation

- Incomplete markets, only safe bond, no insurance
- Moral hazard, difficulty collecting debt payments
- Lenders limit borrowing relative to income
- Consumers sometimes borrow up to the limit
- If a bad shock hits, the limit is reduced and consumers forced to save
- Increased saving reduces the price of non-traded goods $\rightarrow$Even lower limit
- Overborrowing: One person's borrowing contribute to reduce the limit for all

Consumers maximize

$$\mathbb{E}_0 \left\{ \sum_0^\infty \beta^t u(c_t) \right\} \tag{10}$$

where

$0 < \beta < 1$

$u'(c_t) > 0,\ u''(c_t) < 0, u'''(c_t) > 0$

$c_t = w(c_t^T, c_t^N)$, where v is homogeneous of degree 1

Impatience, risk aversion precaution

Constraints

Budget

$$b_{t+1} + c_t^T + p_t^N c_t^N = b_t(1+r) + y_t^T + p_t^N y_t^N \tag{11}$$

Borrowing

$$b_{t+1} \geq -\kappa \left(y_t^T + p_t^N y_t^N \right) \tag{12}$$

$\beta(1+r) < 1$ Assumption: Impatience beats real interest rate
Debt limited relative to income

Shocks

$$
\begin{aligned}
\ln y_t^T &= \rho_{11} \ln y_{t-1}^T + \rho_{12} \ln y_{t-1}^N + \varepsilon_t^T \\
\ln y_t^N &= \rho_{21} \ln y_{t-1}^T + \rho_{22} \ln y_{t-1}^N + \varepsilon_t^N
\end{aligned}
$$

Calibration: $\rho_{11} = 0.9$, $\rho_{22} = 0.2$, time unit one year

Optimality conditions

Intertemporal condition

$$\text{Either} \quad b_{t+1} > -\kappa\left(y_t^T + p_t^N y_t^N\right) \quad \text{and} \quad \frac{\partial u}{\partial c_t^T} = \beta(1+r)\mathbb{E}_t \frac{\partial u}{\partial c_{t+1}^T} \tag{13}$$

$$\text{or} \quad b_{t+1} = -\kappa\left(y_t^T + \kappa^N p_t^N y_t^N\right) \quad \text{and} \quad \frac{\partial u}{\partial c_t^T} \geq \beta(1+r)\mathbb{E}_t \frac{\partial u}{\partial c_{t+1}^T} \tag{14}$$

Intratemporal condition

$$\frac{w_2'(c_t^T, c_t^N)}{w_1'(c_t^T, c_t^N)} = p_t^N$$

Budget constraint

$$b_{t+1} + c_t^T + p_t^N c_t^N = b_t(1+r) + y_t^T + p_t^N y_t^N$$

The intratemporal condition

$$\frac{w_2'(c_t^T, c_t^N)}{w_1'(c_t^T, c_t^N)} = p_t^N$$

Because of homogeneity:

$$\frac{w_2'(c_t^T, c_t^N)}{w_1'(c_t^T, c_t^N)} = \frac{w_2'(1, c_t^N/c_t^T)}{w_1'(1, c_t^N/c_t^T)} = p_t^N$$

With constant elasticity of substitution, $1/(\eta + 1) > 0$:

$$\left(\frac{c_t^T}{c_t^N}\right)^{(1+\eta)} = p_t^N \iff c_t^N = c_t^T \left(p_t^N\right)^{-1/(1+\eta)} \qquad (15)$$

Equilibrium

Condition

$$c_t^N = y_t^N \tag{16}$$

Hence:

$$p_t^N = \left(\frac{c_t^T}{y_t^N}\right)^{(1+\eta)}$$

$$b_{t+1} \geq -\kappa\left(y_t^T + p_t^N y_t^N\right) = -\kappa\left[y_t^T + \left(y_t^N\right)^{\eta}\left(c_t^T\right)^{1+\eta}\right]$$

- More consumption raises borrowing limit of everybody

Some intuition

- Impatience $\beta < 1$: Consume early, acquire debt up to limit
- Risk aversion $u'' < 0$: Smooth consumption, save in good times, use in bad times
- Precaution $u''' > 0$: Better to save too much than too little, save early. if bad luck, rebuild.
- Credit constraint
  - Prevents full consumption smoothing
  - Activates precautionary saving with infinite horizon
- Graphs will be shown

Policy conclusion

- Tax debt to internalize externality, even if it is a pecuniary externality
- Optimal tax structure complicated

Alternative views on sudden stops

- Model can produce sudden stops
- What was the technology shock that hit Thailand? in 1997?
- How come that so many other Asian countries were hit?
- Simpler explanation: It started with a loss of confidence in the fixed parities to dollar.
- Domestic investors had borrowed in dollars financing investment boom in non-traded industries
- Foreign investors had lent in bath.
- Panic resulted when parity questioned
- Policy conclusion: Capital controls to limit inflows of hot money

Relevance for banking

- No bank in model, no defaults
- Too much debt is bad! or
- Living too close to borrowing constraint = Danger!
- Main lesson: Amplifying of negative shocks