ECON 4350: Growth and Investment
Lecture note 1

Department of Economics, University of Oslo

Lecturer: Kåre Bævre (kare.bavre@econ.uio.no)

Spring 2007

1 Introduction

1.1 Information on the course

- Syllabus and reading list
- Lectures
- Seminars
- Term-paper
- Exam

1.2 Motivation for the course

Required reading: BSiM:Introduction

1.2.1 Growth 1870–2000

- **Norway:**
  - Sustained growth (Average annual growth rates: 1865–1939: 1.8 per cent, 1950–2000: 3.1 per cent)
  - Substantial improvements in GDP per capita: 1865: NOK 11 967, 1950: NOK 50 337, 1999: NOK 220162 (1990 prices)
- Similar in other developed countries

- **Growth matters**:
  - Suppose the growth rate in Norway was 2 per cent instead of 3 per cent during 1950–2000. Then the actual GDP per capita in 2000, y^a_{2000} is:
    $$\ln y^a_{2000} = \ln y_{1950} + 0.03 * 50$$
    while the hypothetical figure y^h_{2000} is
    $$\ln y^h_{2000} = \ln y_{1950} + 0.02 * 50$$
    which gives:
    $$\ln y^h_{2000} - \ln y^a_{2000} = (0.02 - 0.03) * 50 \Longrightarrow$$
    $$y^h_{2000}/y^a_{2000} = \exp((0.02 - 0.03) * 50) = 0.6$$
    That is: productivity in 2000 would have been 40 per cent lower.
  - Growth phenomena dwarfs business cycles
- Sustained growth of this magnitude is a quite recent phenomenon (last 200 years)

1.2.2 A broader international perspective

- Enormous differences cross-country differences in productivity. (U.S/Tanzania=69, U.S./India:14)
- Growth experiences varies substantially
- Growth miracles and growth disasters
- Convergence among some, the poor fall behind
- 1800-2000: Divergence, big time
- Little persistence in growth rates
- Poverty worldwide is declining
- World income distribution.
  - Across individuals: improving (?)
  - Across countries: worsening

1.2.3 The large questions

1. Why growth?
2. Why are some countries so rich and some so poor?
3. Why growth miracles and disasters?
4. Is growth alleviating poverty?
5. How is the world's income distribution evolving?
6. How can policy affect growth? Enormous possibilities?

Lucas (1988): "I do not see how one can look at figures like these without seeing them as representing possibilities. Is there some action the government of India could take that would lead the Indian economy to grow like Indonesia's or Egypt's? If so, what exactly? If not, what is it about the "nature of India" that makes it so? The consequences for human welfare involved in questions like these are simply staggering: Once one starts to think about them, it is hard to think about anything else."

2 Measuring growth

Required reading: Nordhaus (1998)
Secondary reading: Bresnahan and Gordon (1997), Summers and Heston (1991)

2.1 What do we want to measure?

- Focus on GDP
  - Obvious limitations. Economic growth versus development.
  - Substantial shortcomings: Conceptual and practical.
  - The best we have?
  - The single most interesting entity?
  - Defines the subject?
- GDP per capita: Welfare?
- GDP/worker: Productivity? (Ideally:GDP/work hour)

- The choice between per worker/per capita:
  - Participation rates varies and are hard to measure
  - Demographic changes
  - GDP does not capture nonmarket production
  - No clear-cut answers, depends on what we want to study
  - In our models: We will not make a distinction. Workforce=population.

2.2 Growth rates

Theory

- Let $Y(t)$ be GDP at time t
- Let $y(t) = Y(t)/L(t)$ be GDP per capita/worker
- We will usually suppress the time notation, ie. $Y(t) = Y$ etc.
- Time-derivatives (annual changes) will be denoted

$$\frac{dY(t)}{dt} \equiv \dot{Y}(t) \equiv \dot{Y}, \quad \frac{dL(t)}{dt} \equiv \dot{L}(t) \equiv \dot{L} \quad \text{etc.}$$

- Growth is usually characterized by roughly constant growth *rates*

$$\frac{\dot{y}}{y} = g, \quad \frac{\dot{L}}{L} = n$$

- This corresponds to exponential growth:

$$y(t) = y(o)e^{gt}, \quad L(t) = L(0)e^{nt} \quad \text{etc.}$$

- Therefore: When using a logarithmic scale when plotting, the slope translates to a growth rate.
- More generally, for any entity measured over time $X(t)$

$$d(\ln X)/dt = \dot{X}/X$$

- Define the growth rate operator as

$$\gamma_X = \dot{X}/X$$

- Growth rate arithmetic:

$$\gamma_{XY} = \gamma_X + \gamma_Y$$

$$\gamma_{X/Y} = \gamma_X - \gamma_Y$$

$$\gamma_{X^\alpha} = \alpha\gamma_X$$

- Proof: Take the logs of the compound expressions before taking the time derivative. E.g.

  $$\gamma_{XY} = d(\ln XY)/dt = d(\ln X + \ln Y)/dt = d(\ln X)/dt + d(\ln Y)/dt = \gamma_X + \gamma_Y$$

  It is highly recommended to learn and understand these rules!

- **Exercise:** Express the growth rate of the compound expression

  $$\frac{K^\alpha L^\eta}{TL}$$

  as a linear function of the growth rates of K, L and T

  Empirical growth rates

- Estimating growth rates

- Average annual growth rate between t_0 and t_1:

$$g = \frac{\ln y_{t_1} - \ln y_{t_0}}{t_1 - t_0}$$

- Using endpoints?
- Regressions?
- Does growth possess a stationary trend? (Probably)
- Average annual growth rates (discrete time). Not identical, but almost if g is small. (**Exercise:** Show this by using the approximation $\ln(1 + x) \approx x$ when x is small.)

2.3 Comparisons of GDP over time and across countries (CPI,PPP)

- We are interested in measures of *real* production, not nominal
- Making comparisons over time requires adjusting for changes in the price level (CPI)
- Making comparisons across countries requires constructing price indexes suitable for such comparisons (Purchasing power parity, PPP)
- Note: It might be best to use the CPI while looking at growth, PPP when looking at levels

2.4 Problems of measurement

- Major differences in construction of national accounts across countries. Both in method and reliability.
- International comparison program (ICP) has developed System of national accounts (SNA). Has primarily dealt with making measures comparable over time. Only recently has there been work on *space*-time SNA (S-T SNA).
- Fundamental problems about getting the CPI and the PPP indexes right
- Likely to be systematic biases.
- Boskin commission: increase in the U.S. CPI is overestimated by 1.1 per cent annually (range: 0.8-1.6)

- Implies that growth is underestimated
- Why do we overestimate the price increase:
  - Laspeyer index gives substitution bias (Boskin: 0.15+0.25+0.1 per cent)
  - Quality changes (Nordhaus: the conventional price of light has risen by a factor of about 1000 relative to the true price)
  - New products
  - Health improvements?
  - The importance of services in consumption
- The scope of the task is enormous
- What are new goods? Phasing in of new goods.
- Use of hedonic methods
- Quality Engel-curves
- Large and important challenges (with relevance far beyond growth research)

2.5 Some important data sources

- Maddison: 1870(20)-2000
- The Penn World Tables 1950-2000, many countries (PPP)
- World Bank
- Barro-Lee, many covariates
- Links are on the course web-site. We will use data from these sources, so play around with them.

3 The Solow model

Required reading: BSim:Ch. 1, Jones (2000)
The classics: Solow (1956), Swan (1956)

3.1 The model

3.1.1 Assumptions

- Aggregate production function

$$Y(t) = F(K(t), L(t))$$

- Idea: growth in output (Y) only possible from growth in inputs (K, L)
- Labor: $\dot{L}/L = n$ (exogenous)
- Labor homogeneous. (No human capital?)
- K produced by same technology as Y
- K a reproducible input
- One sector production of homogeneous good that can be
  - consumed, $C(t)$
  - or invested, $I(t)$, to create more capital $K(t)$
- Closed economy. Saving equals investments
- Growth in K from investment (saving):

$$\dot{K}(t) = I(t) - \delta K(t) \tag{1}$$

  where δ denotes the rate of depreciation of the capital stock.
- Robinson Crusoe like economy (Firms/households and market structure are 'behind the scene', will deal with this later)
- Exogenous savings rate, s: $S(t) = sY(t)$
- No behavior

3.1.2 The neoclassical production function

- The production function satisfies the following three properties (the time notation is suppressed):
  1. Positive and diminishing marginal products

$$\begin{array}{ll} \frac{\partial F}{\partial K} > 0 & \frac{\partial^2 F}{\partial K^2} < 0 \\ \frac{\partial F}{\partial L} > 0 & \frac{\partial^2 F}{\partial L^2} < 0 \end{array}$$

2. Constant returns to scale (CRS)

$$F(cK, cL) = cF(K, L), \quad \text{for all } c \geq 0$$

3. The Inada conditions:

$$\begin{aligned} \lim_{K \to 0} F_K = \lim_{L \to 0} F_L &= \infty \\ \lim_{K \to \infty} F_K = \lim_{L \to \infty} F_L &= 0 \end{aligned}$$

where $F_K = \frac{\partial F}{\partial K}$ and $F_L = \frac{\partial F}{\partial L}$

- CRS implies that the production function can be written in intensive form

$$Y = F(K, L) = LF(K/L, 1) = Lf(k) \Rightarrow y = f(k)$$

where $y \equiv Y/L$, $k \equiv K/L$ and the function $f(k) \equiv F(k, 1)$

- $y = f(k)$. Only capital-intensity k matters for prosperity (i.e. y). CRS $\approx$ neutrality with respect to scale.

- **Exercise 1**: Show that

$$\begin{aligned} \frac{\partial Y}{\partial K} &= f'(k) \\ \frac{\partial Y}{\partial L} &= f(k) - kf'(k) \end{aligned}$$

and that the Inada conditions tranlates into:

$$\begin{aligned} \lim_{k \to 0} f'(k) &= \infty \\ \lim_{k \to \infty} f'(k) &= 0 \end{aligned}$$

- **Exercise 2** (Harder): Show that the conditions above implies that each factor is **essential** to production, that is $F(0, L) = F(K, 0) = 0$ for all K, L

- **Exercise 3**: Show that a Cobb-Douglas production function $Y = BK^{\alpha}L^{1-\alpha}$ satisfies these neoclassical properties.

- **Exercise 4**: Does the following production functions satisfy the neoclassical properties?

$$Y = AK \tag{2}$$
$$Y = AK + BK^{\alpha}L^{1-\alpha} \tag{3}$$
$$Y = A\left\{a(bK)^{\psi} + (1-a)((1-b)L)^{\psi}\right\}^{1/\psi} \tag{4}$$

3.1.3 The solution of the model

- Inserting for fixed savings rate, (1) becomes

$$\dot{K} = sF(K, L) - \delta K \Rightarrow \dot{K}/L = sf(k) - \delta k \tag{5}$$

- It is easy to verify that $\dot{k} = \dot{K}/L - nk$ when labor grows at the fixed rate $\dot{L}/L = n$.

- Thus, the model is characterized by a single dynamic equation for k (the fundamental equation for the model)

$$\dot{k} = sf(k) - (n + \delta)k \tag{6}$$

- The fundamental equation is unchanged by integrating perfectly competitive markets. Consumers own inputs and financial assets. Inputs are supplied inelastically. Firms hire inputs and sell the product. **Study BSiM 1.2.3.** Understanding this argument will make it easier to see the link to the model with endogenous savings behavior.

- The workings of the model is well described by a phase-diagram

- The system converges towards the state where $\dot{k} = 0$. We call this a *steady-state*.

- The steady state k^* is (implicitly) determined by:

$$sf(k^*) = (n + \delta)k^*$$

which gives a constant level of production per capita:

$$y^* = f(k^*)$$

- Thus in the long run, when the economy has converged to the steady state, there is *no growth in production per capita*. Thus the model *can not explain perpetual growth in production per capita*.

- In the steady state Y,K and L all grow at the same rate n. Hence we are on a balanced growth path.

- Note that the central mechanism that ensures convergence to the steady state is the declining marginal product with respect to the accumulated factor (i.e. capital).

- The role of the savings rate: An increased savings rate increases the *level* of output per capita in the long run but not the growth in the long run.

- Since the savings rate is bounded above by 1, continually increasing the saving rate can not give perpetual growth.

- Policy can not affect growth in long run.

- You should study BSiM 1.2.5 to make sure you understand the concepts of *the golden rule of capital accumulation* and *dynamic inefficiency*

3.1.4 The transitional dynamics

- It is illustrative to consider the dynamics in a (k, γ_k) diagram, that is, where we can read off the growth rate $\gamma_k = \dot{k}/k$ vertically.

- We plot the transformed version of equation (6).

$$\dot{k}/k = sf(k)/k - (n + \delta)$$

where $f(k)/k$ is the average productivity of capital, which is declining in k (Why?)

- This illustrates a very important implication of the model: If the two countries have the same steady states, the poorer country will grow faster.

3.1.5 Technological progress

- We can include exogenous technological progress into the model. Since this progress is unexplained, we do not learn much new about the sources of growth. But it is a useful exercise because we 1) can use the model to do growth accounting, 2) see how technological progress affects the dynamics.

- We rewrite the production function

$$Y(t) = F(K(t), L(t), T(t))$$

where $T(t)$ is a shift parameter that captures technological progress. $T(t)$ grows at a constant rate $\gamma_T = \dot{T}/T = x$.

- We can classify three stylized cases of technological change:

  1. Neutral (Hicks neutral): $Y = TF(K, L)$,
     where the ratio $\frac{\partial Y}{\partial K}/\frac{\partial Y}{\partial L}$ will remain constant for a given value of $k = K/L$.
  2. Labor-augmenting (Harrod neutral): $Y = F(K, TL)$,
     where the ratio $K\frac{\partial Y}{\partial K}/L\frac{\partial Y}{\partial L}$ will remain constant for a given value of the ratio Y/K.
  3. Capital-augmenting (Solow neutral): $Y = F(TK, L)$,
     where the ratio $K\frac{\partial Y}{\partial K}/L\frac{\partial Y}{\partial L}$ will remain constant for a given value of the ratio Y/L.

- **Exercise:** Show these properties, and draw the isoquants for different values of T. (Note that BSiM are sloppy with their notation on pp. 52-53, their F_K and F_L should be replaced by $\frac{\partial Y}{\partial K}$ and $\frac{\partial Y}{\partial L}$ respectively.)

- We observe that the relative input shares ($K\frac{\partial Y}{\partial K}/L\frac{\partial Y}{\partial L}$) do not show any trend over time (although they fluctuate quite a bit in the short run). We also observe that Y/K is fairly stable.

- It can be shown that technological progress must be labor augmenting for the model to exhibit a balanced growth path.

- Growth patterns in certain developed countries are steady, and suggestive of balanced growth path behavior.
- More recent research suggests (theoretically) that profit-maximizing firms will in the long run choose to conduct a type of research that leads to labor augmenting technical change. (Acemouglu, 2004).
- For these reasons technological progress is modeled to be of the labor augmenting type, i.e.

$$Y(t) = F(K(t), T(t)L(t)) \tag{7}$$

- We now instead of dividing all quantities by the stock of labor L, divide by the stock of labor measured in efficiency units, TL. (Define $\hat{k} \equiv K/TL$, $\hat{y} \equiv Y/TL$.) The model is structurally the same as before, the only change is that the term n is now replaced by $n + x$ (Why?)
- Thus we reach a steady state $\hat{k}^*$ as before and hence also a steady state level of production per effective worker, $\hat{y}^*$. On the balanced growth path we thus have

$$\gamma_{\hat{y}} = \gamma_{Y/AL} = 0 \Rightarrow \gamma_{Y/L} = \gamma_A = x$$

  i.e. GDP per capita grows at the rate of exogenous technological change (x).
- This implies:
  1. No long-run growth in GDP per capita without technological progress
  2. Growth in GDP per capita in the long run is due to unexplained technological progress

  The statements are symmetric and effectively equivalent, but the first one places the emphasis more appropriately.

3.2 Predictions and results

3.2.1 Qualitative predictions

- The savings rate and population growth:
  - Affects growth in GDP per capita in the short run, but not in the long run

- Affects the steady-state *level* y^*.

- Conditional convergence
- Only productivity growth drives growth in GDP per capita in the long run
- Policy is unimportant for growth in the long run

3.2.2 A special case: Cobb-Douglas

- When working with the quantitative predictions of the Solow-model it is very convenient (and common) to use the special case with a Cobb-Douglas production function.
- With a CD production function it is straightforward to find the steady state value y^* of $y = Y/L$ (Make sure you are able to do this)

$$\left(\frac{Y(t)}{L(t)}\right)^* = T(t)\left(\frac{s}{n+x+\delta}\right)^{\frac{\alpha}{1-\alpha}} \tag{8}$$

or log-linearizing

$$\ln(Y(t)/L(t)) = \ln(T(0)) + xt + \frac{\alpha}{1-\alpha}\ln(s) - \frac{\alpha}{1-\alpha}\ln(n+x+\delta) \tag{9}$$

- The model is characterized by a single differential equation (6). With a general production function this equation can not be solved explicitly. However, in the Cobb-Douglas case a simple transformation cast this equation as a linear differential equation, which is easy to solve. Transforming back we have an explicit solution for the path of $k(t)$ and thus $y(t)$. (For details see Jones (2000), and BSiM pp. 44-45).
- The solution gives us some extra insights into the dynamics of the model, and it is well worth knowing this solution. In addition it delivers quantitative predictions much more readily than does the general model.
- The solution is

$$y(t) \equiv Y(t)/L(t) = \left(\frac{s}{n+x+\delta}(1-e^{-\beta t}) + \left(\frac{Y(0)}{L(0)A(0)}\right)^{\frac{1-\alpha}{\alpha}} e^{-\beta t}\right)^{\frac{\alpha}{1-\alpha}} T(t) \tag{10}$$

- Where the key parameter $\beta \equiv (1-\alpha)(n+x+\delta)$ determines the rate at which the the economy converges to its balanced growth path.

3.2.3 Quantitative predictions

Differences in savings rates and population growth

Let two countries N and S be equal except that they have saving rates s_N and s_S respectively, and population growth n_N and n_S respectively.

Using (8), we then find that the predicted relationship between the steady state values of GDP per capita, y_N^* and y_S^* is

$$\frac{y_N^*}{y_S^*} = \left(\frac{s_N}{s_S}\right)^{\frac{\alpha}{1-\alpha}} \cdot \left(\frac{n_N + x + \delta}{n_S + x + \delta}\right)^{-\frac{\alpha}{1-\alpha}} \tag{11}$$

We can also see the same relationship directly from (9), which translates to

$$dy^*/y^* = (\alpha/(1-\alpha))[ds/s - d(n+x+\delta)/(n+x+\delta)]$$

after differentiation. Thus, a 1 per cent increase (decrease) in the saving rate (population growth) increases the steady state level of income per capita by $\alpha/(1-\alpha)$ per cent.

The speed of convergence

We remember from (10) that the parameter $\beta = (1-\alpha)(n+x+\delta)$ determines how quickly $y(t)$ is approaching it steady-state value y^*.

We will later examine this more closely and give a more precise definition of this parameter as *the speed of convergence.*

Note that the savings rate does not affect this measure (can you guess why). Apart from that, we see that the same parameters are essential also for this quantitative measure.

Differences in rates of returns to capital

The marginal product of capital is

$$R = f'(k)$$

or

$$R = \alpha k^{-(1-\alpha)} = \alpha y^{-\frac{1-\alpha}{\alpha}}$$

in the CD-case. Let countries N and S have GDP-per capita y_N and y_S. The model predicts that the relationship between the return to capital in these two countries should be

$$\frac{R_N}{R_S} = \left(\frac{y_N}{y_S}\right)^{-\frac{1-\alpha}{\alpha}} \tag{12}$$

The parameter α is thus crucial.

With a general production function we must replace this expression with

$$\frac{dR}{R} = -\frac{1-\alpha}{\alpha\sigma}\frac{dy}{y}$$

where σ is the elasticity of substitution between capital and labor. In the CD-case this elasticity is constant and 1. A lower elasticity will lead us to predict smaller differences in returns for given differences in GDP per capita.

3.3 Alternative production functions: Perpetual/endogeneous growth, poverty traps

- We know that declining returns to capital is essential for the results of the Solow-model.
- Let us instead consider the production function

$$Y = AK.$$

  Here A is a constant parameter, and we again assume there is no technological progress.
- Now average productivity is constant, or

$$f(k)/k = A$$

- As long as $sA > n + \delta$ we therefore get $\gamma_k = sA - (n + \delta) > 0$ and constant. It is easy to see this graphically:

- With the AK-production function we therefore get
  1. Perpetual growth from factor accumulation
  2. No convergence
- We will later study models that have solutions that are essentially as if they were AK-models.
- Note that we can get perpetual growth also with other production functions, such as

$$
\begin{aligned}
Y &= AK + BK^{\alpha}L^{1-\alpha} \\
Y &= A\left\{a(bK)^{\psi} + (1-a)((1-b)L)^{\psi}\right\}^{1/\psi}
\end{aligned}
$$

- These satisfy the condition that there is declining returns to capital, but does not satisfy the upper Inada condition. Hence the average product of capital will not go asymptotically towards 0 and we can get perpetual growth.
- Again, this can be seen graphically
- Violation of the upper-Inada condition follows when the non-reproducible factors of production (labor here) is *inessential*, i.e. we produce something also without using this factor.
- An early example of a model with the possibility for a fragile type of perpetual growth is the Harrod-Domar model.

- Yet an interesting class of models are those with poverty traps. These models have more complicated patterns for $f(k)/k$ which can give rise to multiple equilibria, as is easily illustarted in a graph. BSiM 1.4.2 gives an example that you should study.

References

Bresnahan, Timothy F. and Robert J. Gordon, "The Economics of New Goods: Introduction," in Timothy F. Bresnahan and Robert J. Gordon, eds., *NBER Studies in Income and Wealth*, number 58. In 'The economics of new goods.', Chicago and London: University of Chicago Press, 1997, pp. 1–26.

Jones, Charles I, "A Note on the Closed.Form Solution of the Solow Model," 2000. http://elsa.berkeley.edu/users/chad/closedform.pdf.

Lucas, Robert E. Jr., "On the Mechanics of Economic Development," *Journal of Monetary Economics*, July 1988, *22* (1), 3–42.

Nordhaus, William D., "Quality Change in Price Indexes," *Journal of Economic Perspectives*, Winter 1998, *12* (1), 59–68.

Solow, Robert M, "A Contribution to the Theory of Economic Growth," *The Quarterly Journal of Economics*, February 1956, *70* (1), 65–94.

Summers, Robert and Alan Heston, "The Penn World Table (Mark 5): An Expanded Set of International Comparisons, 1950-1988," *Quarterly Journal of Economics*, May 1991, *106* (2), 327–68.

Swan, Trevor W, "Economic Growth and Capital Accumulation," *Economic Record*, November 1956, *32*, 334–361.