ECON 4350: Growth and Investment
Lecture note 5

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10 Endogenous growth: An overview

Required reading: Romer (1990), sec. 1 -2., BSiM: 4.1-4.2

10.1 What do we mean by endogenous growth?

- In the neo-classical growth models we ended up with the following conclusion:

  1. Factor-accumulation can not give rise to growth in the long run, i.e. it can not explain perpetual growth.
  2. The only source of long-run growth in the model is if we postulate a process of changes in technology, i.e. if we introduce exogenous technological progress.

- We will now turn to models falling under the heading "endogenous growth". This terminology is somewhat misleading, since we will be focusing on models that departure from the neo-classical models with respect to either of the two conclusions above.

- Our models will follow either of four paths:

  1. Relaxing the upper Inada-condition
  2. Externalities and/or public goods (Abandoning CRS)
  3. Two-sector models, giving 'effective' increasing returns to the accumulated factors.

4. Models of research and technological progress (Endogenizing T)

- The first three gives rise to perpetual (long-run) growth from factor-accumulation. Only the last one explicitly endogenizes something that was exogenous in the neo-classical model (i.e. technological progress).
- It is thus not a clear distinction in terms of exogeneity/endogenity between these models and the neo-classical models.
- The term endogenous growth theory is, however, fairly well established as a caption for the full array of models.

10.2 Inessential non-reproducible factors

- We consider the case with only two inputs to production, characterized by the production function

$$Y = F(K, L)$$

- Note that in the models we have looked at so far there is the following important distinction between the two inputs:
  1. Capital, K, is reproducible, i.e. it can be made from Y.
  2. Labor, L, is non-reproducible, i.e. it can not be produced/taken from Y.
- In our previous models also human capital has been reproducible, and more specifically been produced by the same technology as Y and K.
- Remember that

$$\dot{k}/k = sf(k)/k - (n + \delta)$$

and we have the graph in $(\hat{k}, k)$ space

In order to have perpetual growth in k (and hence in y) we must have

$$\lim_{k \to \infty} [f(k)/k] > (n + \delta)/s \tag{1}$$

For this to hold we must have that the average productivity of capital, $f(k)/k$, approaches a positive number when k approaches infinity. (Note that since $f(k)/k = Y/K$ this also corresponds to $\lim_{K \to \infty}[Y/K] > (n + \delta)/s$ for any given L).

- Since $\lim_{k\to\infty} f(k) = \infty$ (Show this!) we must use l'Hopitals rule on the limit in expression (1), that is

$$\lim_{k\to\infty} [f(k)/k] = \lim_{k\to\infty} f'(k)$$

  We hence see that when the upper Inada-condition holds, i.e.

$$\lim_{k\to\infty} f'(k) = 0$$

  condition (1) cannot hold and we cannot have perpetual growth.

- Hence the upper Inada condition for the reproducible input, is the property of the neo-classical production function that is fundamental in securing convergence to a steady state, and thereby preventing long-run growth.

- It is also illustrating to see this in the familiar graph in $(\dot{k}, k)$ space

- That is: When we relax the upper Inada condition we can get perpetual growth by. This is a form of endogenous growth as explained above.

- Remember from Lecture 2 that the upper Inada-conditions ensures that capital and labor are essential for production, i.e. $F(0, L) = F(K, 0) = 0$ for all K, L.

- Thus we can conclude that

  - *When the non-reproducible factors of production are inessential to production we can get perpetual growth.*

10.2.1 Two examples

- Let us therefore investigate some cases where $\lim_{K\to\infty} F_K = \mu > 0$.
- These are cases where only the reproduced factor of production (K) is essential to production, meaning $F(K, 0) > 0$ for all $K > 0$, while $F(0, L) = 0$ for all L.
- Consider a Constant Elasticity of Substitution (CES) production function:

$$Y = F(K, L) = A\left\{a(bK)^{\psi} + (1-a)[(1-b)L]^{\psi}\right\}^{1/\psi} \tag{2}$$

  where $a < a < 1$, $0 < b < 1$ and $\psi < 1$. It is easy to show that the production function exhibits constant returns to scale (do this!). When $\psi \to -\infty$ the production function approaches $Y = \min[bK, (1-b)L]$ i.e. a limitation law as in the Harrod-Domar case, while it approaches a cobb-Douglas, $Y = CK^{a}L^{1-a}$ when $\psi \to 0$. For $\psi = 1$ the function is linear in L and K.

- For simplicity we let A be a constant. Dividing by L we then get output per capita:

$$y = f(k) = A\left\{a(bk)^{\psi} + (1-a)(1-b)^{\psi}\right\}^{1/\psi} \tag{3}$$

  and hence

$$f'(k) = Aab^{\psi}\left\{ab^{\psi} + (1-a)(1-b)^{\psi}k^{-\psi}\right\}^{(1-\psi)/\psi} \tag{4}$$

  and

$$f(k)/k = A\left\{ab^{\psi} + (1-a)(1-b)^{\psi}k^{-\psi}\right\}^{1/\psi} \tag{5}$$

- Consider the case with a high degree of substitutability between labor and capital, that is, $0 < \psi < 1$. Then the limits of the expressions above are

$$\lim_{k\to\infty}[f'(k)] = \lim_{k\to\infty}[f(k)/k] = Aba^{1/\psi} > 0$$

  and

$$\lim_{k\to 0}[f'(k)] = \lim_{k\to 0}[f(k)/k] = \infty$$

- What happens if $\psi < 0$? We will get convergence, but can have quite problematic solutions. Consider what happens with $sAba^{1/\psi}$ as an exercise. This resembles what we have seen for the Harrod-Domar model.

- Hence the CES-production function gives an example of perpetual growth even if we have constant returns to scale.
- As another example consider the production function

$$Y = F(K, L) = AK + BK^\alpha L^{1-\alpha}$$

or in per capita terms

$$y = f(k) = Ak + Bk^\alpha$$

Note that this is a mixture of a AK-production function, and a Cobb-Douglas. The function exhibits constant returns to scale. In the limit the latter part becomes relatively unimportant.

- We now have

$$f(k)/k = A + Bk^{-(1-\alpha)}$$

which approaches A as k tends to infinity. We are thus approaching AK-like behavior and have perpetual growth.

10.3 Externalities and public provision

- Consider an economy characterized by a continuum of representative agents/workers. (For simplicity we keep their number fixed, i.e. there is no population growth). Each agent's output is given by a constant returns to scale production function

$$y = F(k, El)$$

where for each agent, k is the stock of available capital and l is the input of labor. E is the efficiency of each worker and is common to all workers.

- Assume that the total amount of capital in the economy has a positive impact on the productivity of each worker, i.e.

$$E = A(K) \tag{6}$$

where $A(K)$ is increasing in K.

- We will later look more closely at examples where we justify this assumption based on externalities in the use of capital (learning by doing/social knowledge) or by public provision of services (the level depending on K) that improve productivity and are provided as public goods.

- The relationship (6) adds to

$$Y = F(K, A(K)L)$$

and the growth rate is then

$$\dot{Y}/Y = (F_K + F_L A'(K)L)\dot{K}/Y$$

- With a constant savings rate $s = \dot{K}/Y$, we get

$$\dot{Y}/Y = sF_K + sF_L A'(K)L$$

- Due to the usual upper Inada-condition the term involving F_K vanishes when K approaches infinity, however $\hat{Y}/Y$ might still be positive if $sF_L A'(K)L$ is bounded away from zero.

- Since we have CRS, the partial derivative $F_L(K, A(K)L)$ is homogenous of degree zero, so we have

$$F_L(K, A(K)L)A'(K)L = F_L(1, A(K)L/K)A'(K)L$$

- Assume that

$$\lim_{K\to\infty} A'(K)L = b \tag{7}$$

then (since $\lim_{K\to\infty} A'(K) = c \Rightarrow \lim_{K\to\infty} A(K)/K = c$) we also have that

$$\lim_{K\to\infty} \frac{A(K)}{K} \cdot L = b$$

and hence

$$\lim_{K\to\infty} F_L(1, \frac{A(K)}{K}L)A'(K)L = F_L(1, b)b$$

giving

$$\lim_{K\to\infty} \frac{\dot{Y}}{Y} = sF_L(1, b)b > 0$$

and perpetual growth.

- Hence given the framework described by (6) condition (7) is a sufficient condition for growth in the long run.

- We will be considering conditions (6) and (7) in detail in the more specific models to follow under topic 11 and 12.

10.4 Two-sector approaches

- We now consider another approach which takes a minimum departure from the neo-classical model. We show that

  - *long-run growth is possible even with a neo-classical production function if at least one of the accumulated factors are produced only by use of reproducible factors.*

- To see an example of this consider an economy with the following two-sector structure.

- Output C of the consumption good is produced by a Cobb-Douglas production function

  $$C = K_C^{\alpha} L^{1-\alpha}$$

  where K_C is the amount of capital used in this production sector.

- The sector producing capital, the investment sector, uses only capital (K_I) but exhibits constant returns to scale

  $$\dot{K} = aK_I$$

  and where $K = K_C + K_I$.

- Assume that a constant fraction of capital goes to the investment sector, $K_I = \phi K$ (and hence $K_C = (1-\phi)K$). Note that this replaces the assumption of a fixed savings rate.

- For simplicity we keep population constant. Then we see that

  $$\frac{\dot{C}}{C} = \alpha \frac{\dot{K}_C}{K_C} = \alpha \frac{\dot{K}}{K} = \alpha a \phi$$

  So the growth rate of consumption is constant and positive. We will see long-run growth even if we have not introduced any departures from constant returns to scale.

- The most interesting example of this model is one with a separate sector for production of human capital. We will return to this in more detail as topic 13.

10.5 Why is explaining technological progress so hard?

- The final approach we will follow is to try to explain the growth process of T where T is interpreted as a stock of knowledge.
- A typical trait of these models is that previous inventions (increases in the level of T) fuels back on the cost of future investment.
- A central feature of these models is a production function for knowledge (i.e. characterizing the R&D-sector)

$$\dot{T} = G(T, K, L)$$

- A serious problem for the models is that it is very hard to have a good idea about the nature of this production function (in particular the role of T as an input). This is serious since results tend to be strongly dependent upon parameters of the production function.
- Why is it so difficult to explain such research and development activities? At this point we should note two things:
  1. There are several reasons to expect the ideas/knowledge embodied in T to have the character of a public good. It is non-rival in its use (as reflected by the production function $Y = F(K, TL)$) and to a certain extent non-excludable. (Cf. the excellent discussion in Romer (1990), sec. 1 -2.
  2. Due to constant returns to scale in the rival inputs K and L we know that if we have a perfectly competitive market structure we have

     $$RK + wL = Y$$

     Hence, there is no income left to remunerate production of T.
- Hence, under perfect competition private agents/firms will have no incentives to do research, and we will not get growth in A.
- Therefore we must seek models that introduces markets with imperfect competition.
- In topic 14 and 15, we will pursue two such paths
  1. Monopolistic competition in a market with a variety of products.
  2. Quality ladders with temporary monopoly power for the use of the newest technology.

10.6 Endogenous growth: Does it matter?

- An important lesson of our study of the neo-classical growth model is that there is basically no room for policy in promoting growth. This is so for two reasons
  1. The decentralized solution is pareto-optimal
  2. Policy can not affect the growth in the long run
- We now briefly sketch why this in general will change in models with endogenous growth.
- Consider the AK-model where

$$Y = AK$$

- The AK-model is in many ways the reduced form of several of the models we will encounter. The parameter A then summarizes various parameters (e.g. those related to policy), and should be thought of more broadly as characterizing the level of technology.
- An important rationale for the AK-model is that it can be seen as an alternative representation of a model with

$$Y = F(K, H)$$

and CRS in K and H, i.e. CRS in broad capital. See BSiM 4.2 for the details.

- Using our usual setup for household preferences, household behavior in this technological environment is characterized by the Euler-equation

$$\frac{\dot{c}}{c} = (1/\theta)(r - \rho) \tag{8}$$

- But we now have that the rental price must bee a constant $R = A$. Hence, the interest rate is also constant

$$r = A - \delta \tag{9}$$

- So we have

$$\frac{\dot{c}}{c} = (1/\theta)(A - \delta - \rho) \tag{10}$$

- The growth rate of consumption is therefore constant (and we assume $A - \delta > \rho$, so that it is positive).

- Capital is evolving according to

$$\dot{k} = (sA - \delta - n)k - c \tag{11}$$

- It is easy to show (cf. BSiM p. 207) that in the steady state c/k must be fixed, so k must grow at the same (constant) rate as c. Further, if we investigate the transversality condition more carefully (see BSiM 4.1.4) we see that (not surprisingly) there is no transitional dynamics in this model, and

$$\frac{\dot{c}}{c} = \frac{\dot{c}}{c} = (1/\theta)(A - \delta - \rho) = \text{Constant} \tag{12}$$

  at all times.

- We can also see this in the phase diagram.

- Notice that in the AK-model changes in parameters such as A, δ and ρ will affect both levels **and** growth in the long run.

- Remembering that A might in turn depend on parameters of policy, we see that in this framework there is a much more important role for policy.

References

Romer, Paul M., "Endogenous Technological Change," *Journal of Political Economy*, October 1990, *98* (5), S71–102. Part 2.