Final Exam ECON4510 «Finance Theory» Spring 2024

(10 points) 1. *Investing in bonds*

Suppose you purchase a 30-year, zero-coupon bond with a yield to maturity of 6%. You hold the bond for five years before selling it. (Hint: first compute the price at year 0, then at year 5.)

(a) If the bond's yield to maturity is 6% when you sell it, what is the internal rate of return of your investment?

(b) If the bond's yield to maturity is 7% when you sell it, what is the internal rate of return of your investment?

(c) If the bond's yield to maturity is 5% when you sell it, what is the internal rate of return of your investment?

(d) Even if the bond has no chance of default, is your investment risk free if you plan to sell it before it matures? Explain.

(10 points) 2. *The CAPM*

(a) Widget shares have a beta of 1.5. The expected risk-free rate is 3%, and the expected returns on the market portfolio are 8%. In equilibrium, supposing that the CAPM holds, what should be the expected return on Widget shares?

(b) Superwidget shares have a beta of 1.2. You invest half of your money in Widget and half of your money in Superwidget. What are your expected returns?

(c) You buy a portfolio of 5 risk-free bonds with a price of NOK 950 each and 10 shares in Widget with a current price of NOK 120. What is the expected return on the portfolio?

(d) What is the beta of the portfolio?

(10 points) 3. *The lognormal model and aggregate puzzles*

Assume utility is additively separable CRRA and consumption growth is lognormally distributed such that the pricing kernel is

$$m = \delta \left(\frac{c_1}{c_0}\right)^{-\alpha} \quad \text{where} \quad \frac{c_1(z)}{c_0} = e^z, \quad \text{with} \quad z \sim \mathcal{N}(\kappa_1, \kappa_2)$$

and equity levered claim on consumption; dividend growth is a function of consumption growth for some arbitrary value of λ

$$d^e(z) = \left[\frac{c_1(z)}{c_0}\right]^{\lambda}$$

then

$$R_f = \frac{1}{\delta} e^{\alpha\kappa_1 - \alpha^2\kappa_2/2} \quad \text{and} \quad \frac{\mathbf{E}[R_m]}{R_f} = e^{\lambda\alpha\kappa_2}$$

so

$$\log R_f = -\log\delta + \alpha\kappa_1 - \frac{1}{2}\alpha^2\kappa_2 \quad \text{and} \quad \log\mathbf{E}[R_m] - \log R_f = \lambda\alpha\kappa_2$$

Using the two equations on the last line, please *briefly* explain the aggregate asset pricing puzzles and how the model with additively separable utility and lognormal consumption growth cannot quantitatively account for asset pricing movements, and the tensions between the "equity premium puzzle" and "risk-free rate puzzle".

(20 points) 4. *From Gordon growth to valuation multiples*

In this question, we will investigate how to translate the value from the Gordon growth model into multiples, including EV/EBITDA, P/E, and EV/Sales.

One of the simplest valuation techniques is the Gordon growth model:

$$\text{Enterprise value} = \frac{\text{Free cash flow}}{\text{Cost of capital} \; - \; \text{growth}}$$

Enterprise value (EV) of the firm equals the equity market capitalization plus debt and other liabilities, minus cash.

Free cash flow (FCF) is the cash a firm generates that is free to be distributed to the holders of debt and equity. Formally, it equals net operating profit after taxes (NOPAT) minus investments in future growth. These investments include changes in working capital and capital expenditures.

The cost of capital is the weighted average cost of capital (WACC) for companies financed with both debt and equity. Growth captures the expected increase in FCF over time.

In class, we decomposed free cash flow (FCF) as follows:

$$\text{FCF} = \text{EBIT}(1-t) - (\text{capital expenditures} - \text{depreciation}) - \Delta \cdot \text{working capital}$$

where EBIT stands for earnings before interest and taxes and t stands for tax rate. The first term on the right side of the equation is net operating profit after taxes (NOPAT) and the second two terms are investments.

(a) When

$$\text{EBITDA} = \text{EBIT} + \text{depreciation}$$

and

$$\text{Cost of capital} = \text{WACC}$$

show that the multiple enterprise value (EV) over EBITDA can be expressed as

$$\frac{\text{EV}}{\text{EBITDA}} = \frac{\text{Depreciation} \cdot t/\text{EBITDA}}{\text{WACC} \; - g} + \frac{1-t}{\text{WACC} \; - g} - \frac{\text{capital expenditures/EBITDA}}{\text{WACC} \; - g} - \frac{\Delta \text{ in working capital/EBITDA}}{\text{WACC} \; - g}$$

Assume we have the following numbers:

Sales	500
EBIT	100
Depreciation	25
EBITDA	125
Tax rate	15.0%
Capital expenditures	31.25
Working capital	0
WACC	8.5%
Growth	4.0%

(b) Use the numbers above to compute EV/EBITDA

(c) Value (EV) can also be expressed

$$\text{EV} = \frac{\text{EBITDA}(1-t) + \text{depreciation} \cdot t - \text{capital expenditures} - \Delta \cdot \text{working capital}}{\text{WACC} - \text{growth}}$$

Compute EV using this equation, and then check that this number is consistent with your estimate in the previous question by multiplying the EV/EBITDA ratio with EBITDA

(d) Then compute the value-to-sales multiple. Since we know the warranted EV/EBITDA multiple, you may obtain EV/Sales by multiplying with EBITDA/Sales,

(e) Lastly compute the P/E multiple. In this case where dere is no debt, NOPAT and earnings are the same, and earnings equals EBIT$\cdot(1-t)$. As usual, you may presume value and price are synonymous.

(20 points) 5. *Investments in risky technologies*

(a) Consider the following investment case ("Case A") with a highly risky technology:

- Cost setting up a production facility: 100M
- Construction time: 1 year
- After the facility has been constructed, the owners will know whether the technology is a success or not:
  - With 50% probability the production facility will generate a net cash flow of 25M in perpetuity
  - With 50% probability the production facility will generate a net cash flow of 1.5M in perpetuity
- The ordinary tax rate is 20% on net cash flows.
- Given the riskiness of the project, the required rate of return is 10%

What is the expected after-tax NPV of the project? Should the investors build the production facility or should they shelve their plans for good?

(b) Hypothetical outcome: the entrepreneurs decided to build the production facility, and the technology turned out to be a success. After one year of operations, the government observes that the business is highly profitable, and that, since all investments have already been made, it is not physically mobile. They suddenly announced that they would levy an additional 40 percentage points "ground rent tax" on all future net cash flows on this facility.

i. Will the business continue its operation?

ii. If it had been known *ex ante* that the government would levy an additional 40 percentage points "ground rent tax" if the project was a success, what would the expected NPV have been? Would the facility have been built? You may assume that the required rate of return on capital was unchanged.

(c) Consider another investment case, a few years later, ("Case B") with another highly risky technology. They have observed the political fallout after "Case A" and the additional tax that was levied on that industry.

- Cost setting up a production facility: 150M
- Construction time: 1 year
- After the facility has been constructed, the owners will know whether the technology is a success or not:
  - With 50% probability the production facility will generate a net cash flow of 40M in perpetuity
  - With 50% probability the production facility will generate a net cash flow of 1M in perpetuity
- The ordinary tax rate is 20% on net cash flows.

- The entrepreneurs assess the political risk and assume that with 50% probability they will be levied an additional 40 percentage points additional "ground rent tax" if the technology turns out to be a success
- In absence of political risk, the required rate of return would still have been 10%

First ignoring political risk, what is the expected NPV?

(d) Compute the additional risk premia investors would require when there is political risk. (Hint: the additional risk premium may be computed analogously to how yields are computed on risky bonds, ie. bonds with a strictly positive default probability).

(30 points) 6. *Asset pricing, options, and the CAPM*

Use a one-period binomial model with two future states at time 1, up and down. The market index, which at time 0 has value $S_0 = 500$, takes the values $S_u = 624$ and $S_d = 468$ in the states "up" and "down", respectively. Assume that the gross risk-free interest rate is $R^f = 1.04$.

(a) Check whether the state prices, q_u and q_d for states up and down, respectively, in this model are

$$q_u = \frac{25}{78}, \quad q_u = \frac{25}{39}$$

A put option on the market index with exercise price K has time-1 payoff $\max\{K - S_1, 0\}$, where $S_1 = S_u$ in state up, and $S_1 = S_d$ in state down. Assume that $K = 507$.

(b) Calculate P_0, the time-0 value of the put option from the state prices.

Let m_u and m_d be the value of the stochastic discount factor in the states up and down, respectively.

(c) Explain why

$$q_u = \pi m_u, \qquad \text{and} \qquad q_d = (1 - \pi) m_d$$

where π is the probability for state up.

Assume that $\pi = \frac{5}{13}$.

(d) Calculate the expected gross return $\mathbf{E}[R_m]$ of the market index.

(e) Calculate m_u and m_d.

(f) Calculate the time-0 price of the put option from the formula

$$P_0 = \mathbf{E}[mX]$$

where $m = m_u$ in state up, and $m = m_d$ in state down. Also, $X = \max(K - S_1, 0)$.

(g) If the stochastic discount function can be written as a linear function of the market return, the CAPM is equivalent to the state price/stochastic discount factor approach. I.e, if we can find constants a and b such that

$$a + bR_m^u = m_u$$

and

$$a + bR_m^d = m_d$$

then the CAPM produces the same prices as the other two approaches. Please compute a and b.

Denote the time-0 price of the put option calculated by CAPM by q.

In the following four questions, the answers should be stated in terms of q.

(h) Calculate the gross return R_i of the put option as a function of the unknown time-0 price q.

(i) Calculate $\operatorname{Cov}(R_m, R_i)$, the covariance between R_m and R_i, and $\operatorname{Var}(R_m)$, the variance of the return of the market index.
Reminder 1: $\operatorname{Var}(X) = \mathbf{E}[X^2] - (\mathbf{E}[X])^2$.
Reminder 2: $\operatorname{Cov}(X, Y) = \mathbf{E}[XY] - \mathbf{E}[X]\mathbf{E}[Y]$.

(j) Calculate the β of the put option.
Reminder:

$$\beta = \frac{\operatorname{Cov}(R_m, R_i)}{\operatorname{Var}(R_m)}$$

(k) Calculate $\mathbf{E}[R_i]$, the expected return of the option from CAPM.
Reminder: CAPM states that

$$\mathbf{E}[R_i] = R^f + \beta \left[\mathbf{E}[R_m] - R^f\right]$$

By definition, $\mathbf{E}[X] = q\mathbf{E}[R_i]$, where $X = \max(K - S_1, 0)$ is the (random) payoff of the put option.

(l) Calculate q by equating $\mathbf{E}[R_i] = \frac{\mathbf{E}[X]}{q}$ from the above equation with the expression for $\mathbf{E}[R_i]$ from CAPM in (k). Comment whether $q = P_0$.