Derivative assets

- By *derivative assets* we mean assets that derive their values from the values of other assets, called the *underlying assets.*
- For short called *derivatives* (*derivater*).
- The underlying assets could be many different things, such as
  - Shares of stock in a company
  - Commodities
  - A stock exchange index (interpreted as a money amount)
  - Bonds
- Main types of derivatives to be discussed here: Forward contracts (*terminkontrakter*) and options (*opsjoner*).
  - There exist something similar to forward contracts, called futures contracts. (Both are classified as *terminkontrakter* in Norwegian.) In the chapters on your reading list Hull sometimes uses the term "futures contracts," but to the extent that they differ, he really only considers forward contracts. When future riskless interest rates are constant, the two are identical (see link, bottom of p. 111).[1]
- A forward contract is an agreement between two parties to make a trade at a specified date in the future.
  - The forward contract specifies carefully what is to be sold/bought (quantity, quality, place) and the price.
  - The price is fixed (in nominal terms), but not paid until the delivery date.

[1] In the seventh edition, there is no link, but the whole argument is given in pages 126–127.

Investment asset versus consumption assets

- For discussion of valuation of derivative assets, need to distinguish between two types of underlying assets.
- An *investment asset* is an asset which is held for investment purposes by a significant number of investors.
- Securities (financial assets) and some precious metals (gold, silver) are investment assets.
- Other assets are often referred to as *consumption assets.*
- Reason to distinguish between the two types: Will assume that there is a market equilibrium for each investment asset with investors on the demand and supply side of the market.
- When this is the case, will assume that the value today of receiving one unit of the asset in the future is simply the spot market value today. Those investors who buy today, do it in order to have it available in the future, and for no other purpose.
  - For financial investment assets, need to correct for possible payouts like interest and dividends.
  - For physical investment assets, may correct for storage costs.
- For consumption assets, the expected price appreciation is typically too low for anyone to be willing to buy for investment purposes. The value today of receiving one unit of the asset in the future is then lower than the spot market value today.
- Some firms may be storing consumption assets (e.g., aluminium oxide) in order to secure a steady supply into production (e.g., of aluminium); not to gain from price appreciation.

Forward contracts

- Let K denote the agreed-upon price, written into a forward contract, to be paid upon delivery.
- The person who is obliged to pay K "owns" the contract, also called "has a long position" in the contract.
- The person who is obliged to deliver the underlying asset "has a short position" in the contract.
- Let S_T be the value of the underlying asset at delivery date T.
- When T is reached, the value of owning the forward contract is $S_T - K$, since the owner has the obligation to pay K and will receive something which is worth S_T at that time.
- In a diagram, the payoff at T as a function of S_T:
- Assume there is a well-functioning market for underlying asset.
- Then there is no reason to deliver the underlying asset. The contract parties may just as well hand over the payoff of the contract, in one direction or the other.

Riskless arbitrage

- In remainder of course: Many results follow from arbitrage
- More precisely, absence of *riskless arbitrage:*

  A set of transactions which gives us a net gain now, and with certainty no obligation to pay out a net positive amount at any future date.

- Example:
  - Two riskless bonds with interest rates $r_1 < r_2$
  - Leads to an arbitrage opportunity, i.e., riskless arbitrage
  - Invest I in bond 2
  - Short-sell bond 1 in the amount $I(1+r_2)/(1+r_1)$
  - Receive net gain now,

$$I\frac{1+r_2}{1+r_1} - I = I\left(\frac{1+r_2}{1+r_1} - 1\right) = I \cdot \frac{r_2 - r_1}{1+r_1} > 0$$

  - One year later, value in bond 2 is $I(1+r_2)$
  - Sufficient to cancel short sale at $[I(1+r_2)/(1+r_1)](1+r_1)$
- Arbitrage opportunities lead to infinite demand for some securities, infinite supply of others
- Thus, cannot exist in equilibrium
- Will use this to find exact values of some assets
- In other cases, will find inequality relations

Valuation of forward contracts

- What is equilibrium forward price F_0 for investment asset which has price S_T at T, and which for sure gives no payout between now (time zero) and T? (Assume zero storage costs.)
- The value of a claim to receiving S_T at T is S_0.
- Consider the value at time zero of entering into a forward contract with delivery at T.
- The value of having to pay F_0 at T is (minus) F_0e^{-rT}, with $r =$ nominal riskfree interest rate, using continuous compounding (see Sydsæter *EMEA,* sect. 10.2 or (Norw.) *MA1,* sect. 8.2).
- Since no payment is made at time zero, and the contract is voluntary, the net value of entering into it must be zero, so $F_0e^{-rT} = S_0$.
- If not, one could make a risk free arbitrage, buying the cheaper, selling the more expensive:
- If $F_0e^{-rT} > S_0$; buy underlying asset, sell bonds (i.e., borrow) in amount F_0e^{-rT}, sell forward contract, make net positive profit now (time zero) equal to $F_0e^{-rT} - S_0 > 0$. At delivery date; deliver underlying asset, receive F_0, pay back loan, with no net payout and no remaining obligations.
- If $F_0e^{-rT} < S_0$; do the opposite.
- In both cases there is thus an arbitrage opportunity.

Valuation of forward contracts when there are payouts

- If the underlying asset gives payouts, the formula needs to be modified.
- Payout could be dividends (from shares) or interest (coupon payments from bonds).
- For consumption assets something similar called convenience yield; more about this later.
- Modification necessary because owner of forward contract has no claim to payout.
- When payout between time zero and T: Value today of claim to S_T is less than S_0.
- S_0 is the value today of claim to both the payout and S_T.
- Let the valuation of payout(s) be $= I$; then the valuation today of S_T is $S_0 - I$, and the equilibrium forward price is given by $F_0 e^{-rT} = S_0 - I$.
  - The arbitrage argument will now include buying or selling a claim to the payouts. OK if the payouts are known for sure, or if there is a market for claims to them. But if the payouts are unknown and cannot be related to anything which is traded in a market, the argument does not work.
- Another possibility: A continuous payout stream qS_t proportional to the price of the underlying asset.
- Similar to a reduced interest rate in continuous compounding, $F_0 e^{-rT} = S_0 e^{-qT}$.

Valuation of pre-existing forward contracts

- Consider now a forward contract which was entered into some time before date zero.
- The underlying asset is still assumed to be an investment asset.
- K is the price written into the contract, while F_0 is the equilibrium forward price now.
- S_0 and expectations of S_T will (typically) have changed since those that determined K.
- We can consider $K \neq F_0$ as a kind of mispricing of the contract, which means that owning the contract now has a positive or negative value.
- The value is $f = (F_0 - K)e^{-rT}$, and there is no reason to believe this is zero.
- For the case of an investment asset without payouts, $F_0 e^{-rT} = S_0$, and $f = S_0 - Ke^{-rT}$, which is the valuation of S_T minus the valuation of the obligation to pay K.

Storage costs

- Precious metals like gold and silver require storage cost, U.
- Like negative payout, so $F_0e^{-rT} = S_0+U$. Arbitrage argument:
- If $F_0e^{-rT} > S_0 + U$; buy underlying asset, pay the storage, sell bonds (i.e., borrow) in amount F_0e^{-rT}, sell forward contract, make net positive profit now (time zero) equal to $F_0e^{-rT} - S_0 - U > 0$. At delivery date; deliver underlying asset, receive F_0, pay back loan, with no net payout and no remaining obligations.
- If $F_0e^{-rT} < S_0 + U$; do the opposite. More precisely, this includes that those who own the asset and a storage facility start by selling the asset and renting out their storage facility for the period until T, to receive $S_0 + U$, at the same time buying a forward contract and bonds. This gives the arbitrage profit now equal to $S_0 + U - F_0e^{-rT} > 0$. At time T the forward contract will imply getting back the asset, which is then put into storage.

Valuation of forward contracts on consumption assets

- For a consumption asset, those who own it will generally not be willing to do the second arbitrage mentioned above, even if they observe that $F_0e^{-rT} < S_0 + U$. They are not indifferent between having the asset available throughout the period $(0, T)$ and having a claim to receiving it at T, even when storage is taken care of.
- For a consumption asset, then, we cannot rule out $F_0e^{-rT} < S_0 + U$, but we can rule out $F_0e^{-rT} > S_0 + U$ through the first arbitrage argument above.

Storage costs as continuous stream; convenience yield

- Hull introduces the idea that storage costs may be a continuous stream proportional to the asset price, uS_t.
- For an investment asset, $F_0e^{-rT} = S_0 + U$ is replaced by $F_0e^{-rT} = S_0e^{uT}$.
- This must be seen as a simplifying assumption (only), hardly realistic.
- The left-hand side, F_0e^{-rT}, is what one must pay today in order to get a unit of the asset at time T through the forward market.
- The right-hand side, S_0e^{uT}, is what one must pay to get a unit at T through the spot market, including storage cost.
- For a consumption asset we found $F_0e^{-rT} \leq S_0e^{uT}$.
- This means that the cost of buying the asset in the spot market and paying for storage (the right-hand side) exceeds the cost of buying the asset in the forward market.
- When someone nevertheless buys today, this is explained by the fact that they have some extra gain or benefit from having the asset available in the period $(0, T)$. This extra is called the convenience yield, defined as a continuous yield yS_t, proportional to the asset price, so that $F_0e^{-rT} = S_0e^{(u-y)T}$.
- This means that the convenience yield is exactly enough to compensate for the too high cost of spot-buying and storing the asset.

Financial options

A *call option* (*kjøpsopsjon*) is a security — issued by "X" — which gives its owner the right to buy a specified asset from "X" at a specified price, either *at* a given date (a *European* call option) or at *any time before* a given date (an *American* call option).

A *put option* (*salgsopsjon*) is a security which gives its owner a similar right to *sell* an asset to "X".

- Such options have positive value since they give a right, but no obligation.
- Issuer (here called "X") normally *sells* the option (— alternatively it could be a gift or a compensation for some service).
- Subsequent owners may sell option onwards.
- Market for options.
- Specified asset: *Underlying asset.* (*Underliggende aktivum.*)
- Specified price: *Exercise price, striking price, strike price*, K. (*Utøvingspris, kontraktpris.*)
- Specified date: *Maturity date, expiration date.* (*Bortfallsdato, forfallsdato.*)
- "European" and "American" are jargon, only.
- When liquid markets: No need for issuer of call to own underlying. May as well settle in cash.

Financial options, contd.

- Original issuer called *writer* of option.
- The right for the option owner is vis-a-vis that writer, irrespective of subsequent trading of the option.
- The right (to buy or sell) implies no obligation. (Different from forward or futures contracts.)
- To use option called to *exercise* option.
- Define these values:

	Before expiration	At expiration
Market value, underlying asset	S	S_T
Value of American call option	C	C_T
Value of American put option	P	P_T
Value of European call option	c	c_T
Value of European put option	p	p_T

Call option at expiration date

- Consider first call option *at* expiration date.
- Then European options equal to non-exercised American.
- If exercise call: Receive S_T, pay K.
- Exercise if and only if $S_T > K$. Else: Value = zero.
- Value is $C_T = \max(0, S_T - K)$.
- Gross value, not subtracting purchase price for option.
- Called "payoff" in Hull, fig. 9.5; as opposed to "profit," figs. 9.1–9.4.[2]
- Increasing in S_T, although not strictly.
- Owner of option protected against downside risk.

[2] The corresponding figures in the seventh edition are 8.5 and 8.1–8.4.

Put option at expiration date

- If exercise put: Receive K, give up S_T.
- Exercise if and only if $K > S_T$. Else: Zero.
- Value is $P_T = \max(0, K - S_T)$.
- Gross value, not subtracting purchase price for option.
- Decreasing in S_T, although not strictly.