

# ECON4510 Finance Theory

## Lecture 9



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# Empirical Tests of the Capital Asset Pricing Model

- **Traditional Tests of the CAPM**
- **Black, Jensen, and Scholes Study**
- **Fama and MacBeth Study**
- **Roll's Critique of Tests of the CAPM**
- **Additional Evidence**

## **Traditional Tests of the CAPM**

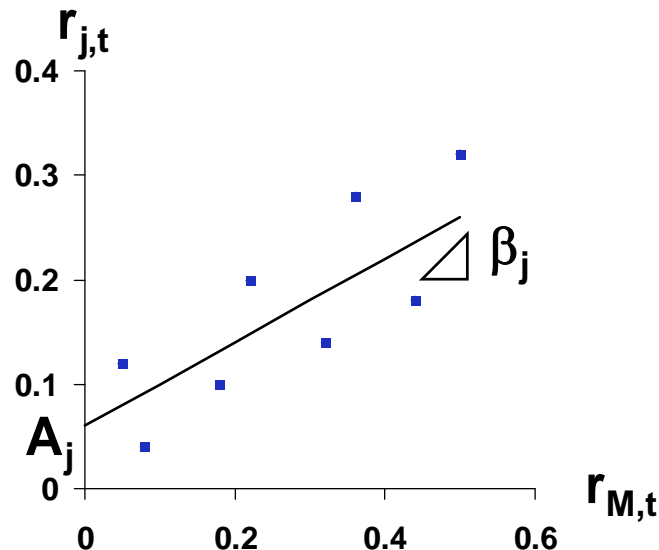
- **The CAPM predicts that all investors hold portfolios that are efficient in expected return, standard deviation space. Therefore, the Market Portfolio is efficient. To test the CAPM, we must test the prediction that the Market Portfolio is positioned on the efficient set.**
- **Initial testing of the CAPM did not test directly the prediction, “The Market Portfolio is efficient.” Instead, researchers followed the theme, “If a linear positive relationship exists between portfolio return and beta, the Market Portfolio must be efficient.”**

## **Traditional Tests of the CAPM (Continued)**

- **Two classical traditional studies:**
  - **Black, F., Jensen, M. C., & Scholes, M. “The Capital Asset Pricing Model: Some Empirical Tests,” in Ed. Jensen, M. C. *Studies in Theory of Capital Markets*. New York: Praeger, 1972.**
  - **Fama, E. F., & MacBeth, J., “Tests of Multiperiod Two Parameter Model,” *Journal of Political Economy* (May 1974).**

# Black, Jensen, and Scholes Study

- Sample: All stocks on the NYSE (1926 - 1965).
  - Market Index: Equally weighted portfolio of all stocks on the NYSE.
- Outline of the Study
1. Estimate a beta for each stock using monthly returns during the period, 1926 - 1930, (i.e., 60 months):



$$r_{j,t} = A_j + \beta_j r_{M,t} + \varepsilon_{j,t}$$

2. Rank order all of the stock betas, and form 10 portfolios:
  - Top 10% with the highest betas comprise portfolio #1, next 10% portfolio #2, . . . and so forth until portfolio #10 contains the bottom 10% with the smallest betas.
3. Compute each of the portfolio's returns for each of the 12 months in 1931:

$$r_{p,t} = \frac{\sum_{j=1}^n r_{j,t}}{n}$$

#### 4. Repeat Steps 1 through 3 many times:

Estimate Stock Betas and  
Form 10 Portfolios

1927 - 1931

1928 - 1932

.

1960 - 1964

Compute Monthly  
Portfolio Returns

1932

1933

.

1965

#### 5. Results of the above process - A series of monthly returns for 10 portfolios:

Portfolio 1/31 2/31 3/31 . . . 12/65

1 Thirty five years of monthly returns

2 (i.e.,  $35 \times 12 = 420$  monthly returns for

. each of the 10 portfolios)

10

6. For the entire 35 year period, calculate the mean monthly return, and estimate the beta coefficient for each of the 10 portfolios:

$$\bar{r}_p = \frac{\sum_{t=1}^{420} r_{p,t}}{420}$$

$$r_{p,t} = \alpha_p + \beta_p r_{M,t} + \varepsilon_{p,t}$$

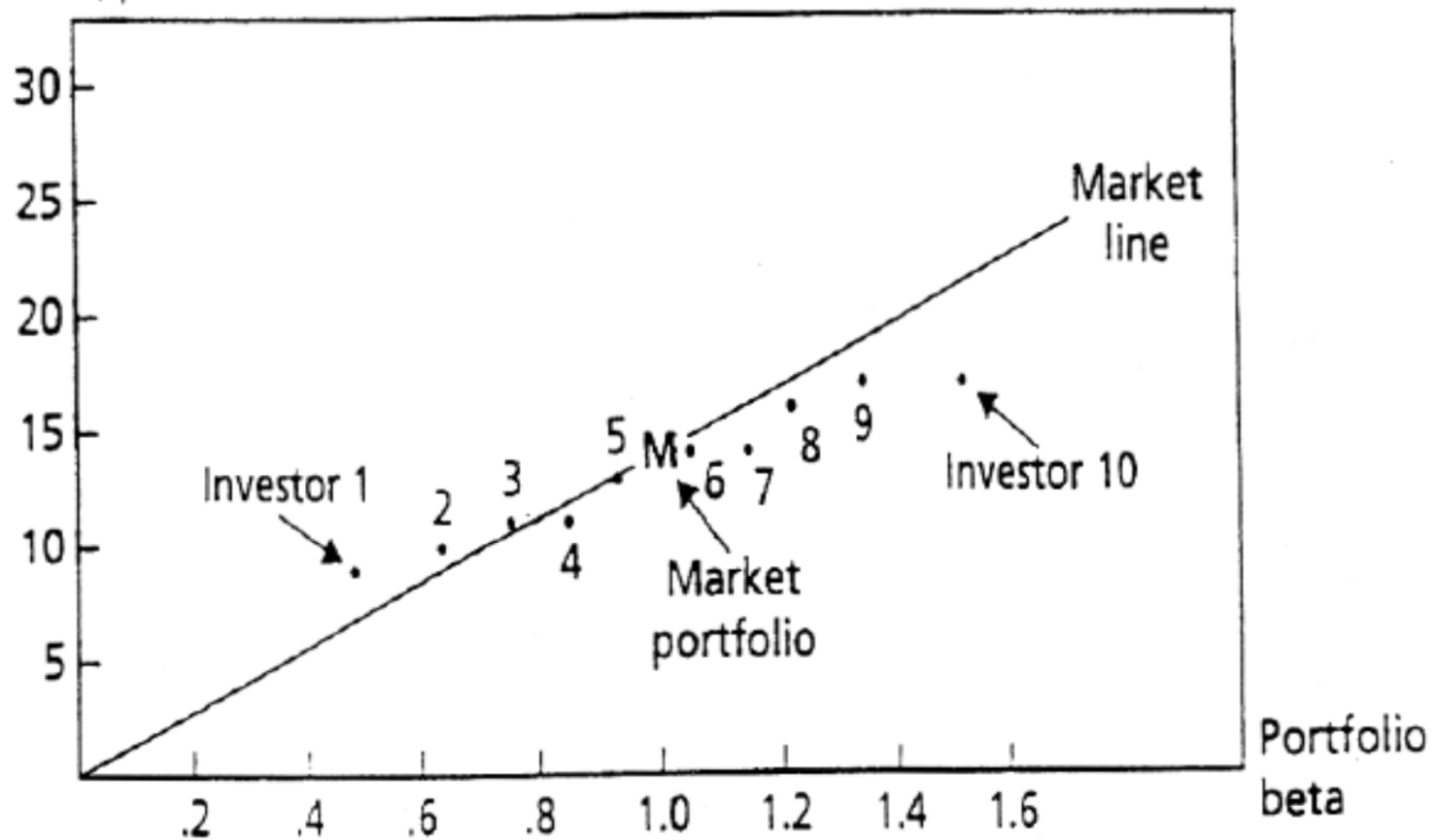
420datapoints

7. Regress the mean portfolio returns against the portfolio betas (i.e., estimate the ex post Security Market Line). See the following graph.



# BJS Ex Post Security Market Line (SML)

Average risk premium,  
1931-1991, percent

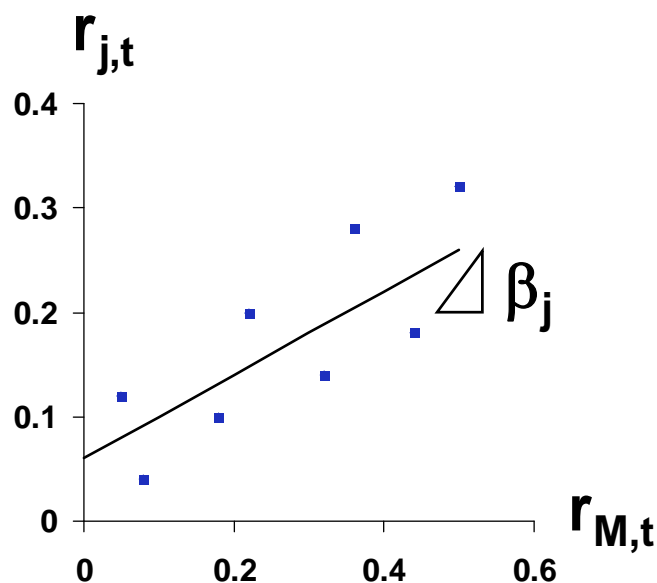


## Results of the BJS Study

- **The results of the study appeared to be consistent with the zero beta version of the Capital Asset Pricing Model (CAPM):**
  - **The intercept of the SML was greater than the interest rate on risk-free bonds.**
  - **The slope of the SML, which was highly significant was linear and positive.**
- **Also note that Black, Jensen, and Scholes, estimated the ex post Security Market Lines in various subperiods. In general, the results were similar.**

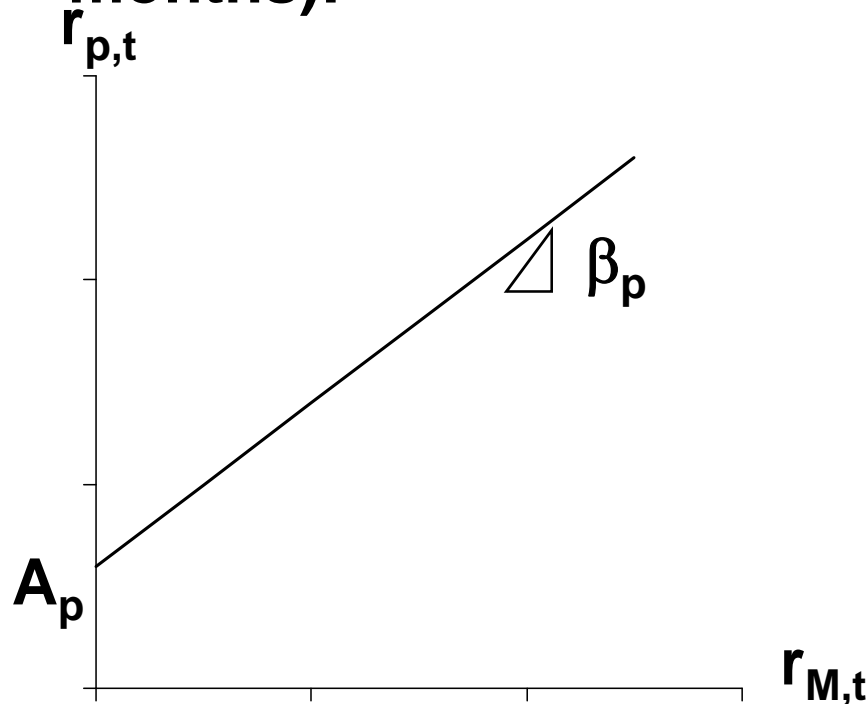
# Fama - MacBeth Study

- Sample: All stocks on the NYSE (1926 - 1968)
- Market Index: Equally weighted portfolio of all stocks on the NYSE.
- Outline of the Study
  1. Estimate a beta for each stock using monthly returns during the period, 1926 - 1929, (i.e., 48 months):



$$r_{j,t} = A_j + \beta_j r_{M,t} + \varepsilon_{j,t}$$

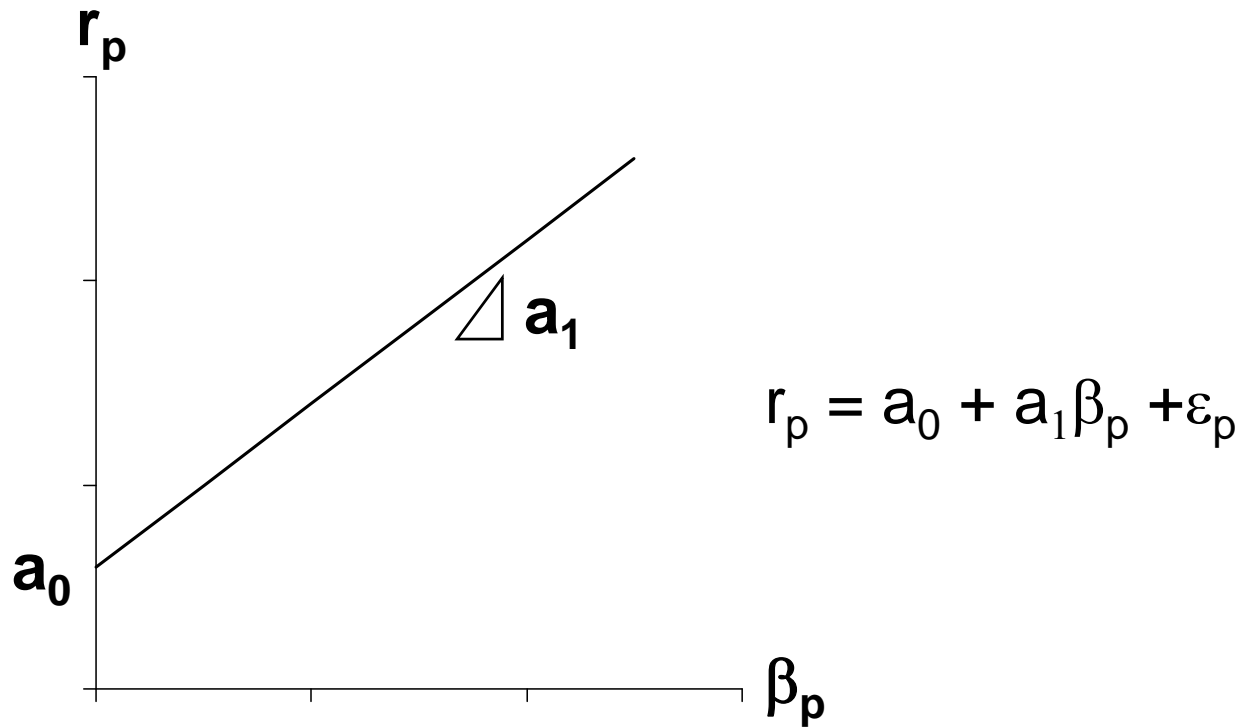
2. Rank order all of the stock betas, and form 20 portfolios. The top 5% with the highest betas are in portfolio #1, . . . etc. . . the bottom 5% with the smallest betas are in portfolio #20.
3. Estimate the beta of each of the portfolios by regressing portfolio monthly returns against the market index during the period, 1930 - 1934, (i.e., 60 months):



$$r_{p,t} = A_p + \beta_p r_{M,t} + \varepsilon_{p,t}$$

**4. For each of the months during the period, 1935-1938 estimate the ex post SML by regressing portfolio returns against portfolio betas.**

**Note: 48 SMLs will be estimated (one for each month).**



## Summary of Steps 1 Through 4

<u>1926 - 1929</u>	<u>1930 - 1934</u>	<u>1935 - 1938</u>
Used to estimate stock betas and form 20 portfolios	Used to estimate beta of each of the 20 portfolios	Used to estimate 48 SMLs (One for each month)

5. For each of the months during the period, 1935 - 1938, estimate two additional equations:

$$r_p = a_0 + a_1\beta_p + a_2\beta_p^2 + \varepsilon_p \text{ to test for nonlinearity}$$

$$r_p = a_0 + a_1\beta_p + a_2\beta_p^2 + a_3RV_p + \varepsilon_p \text{ to test whether the residual variance of stocks affects portfolio return}$$

$$\text{where: } RV_p = \frac{\sum_{j=1}^m \sigma^2(\varepsilon_j)}{m} = \text{Average residual variance of the stocks}$$

## 6. Repeat Steps 1 through 5 many times:

Estimate stock betas and form 20 portfolios 1930 - 1933 1934 - 1937 Etc.	Estimate the beta of each of the portfolios 1934 - 1938 1938 - 1942 Etc.	Estimate each equation for each month 1939 - 1942 1943 - 1946 Etc.
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## 7. Results of the above process:

390 sets of the following three equations:

$$r_p = a_0 + a_1\beta_p + \varepsilon_p$$

$$r_p = a_0 + a_1\beta_p + a_2\beta_p^2 + \varepsilon_p$$

$$r_p = a_0 + a_1\beta_p + a_2\beta_p^2 + a_3RV_p + \varepsilon_p$$

8. For each equation, compute the mean value for each of the coefficients, and test whether the means are significantly different from zero:

$$\bar{a} = \frac{\sum_{t=1}^{390} a_t}{390}$$

**Results of the FM Study** The mean values of the coefficients are shown below. (\*) indicates that the mean was significantly different from zero.

$$r_p = a_0 + a_1\beta_p + \varepsilon_p$$

.0061\* .0085\*

$$r_p = a_0 + a_1\beta_p + a_2\beta_p^2 + \varepsilon_p$$

.0049\* .0105\* -.0008

$$r_p = a_0 + a_1\beta_p + a_2\beta_p^2 + a_3RV_p + \varepsilon_p$$

.0020 .0114\* -.0026 .0516



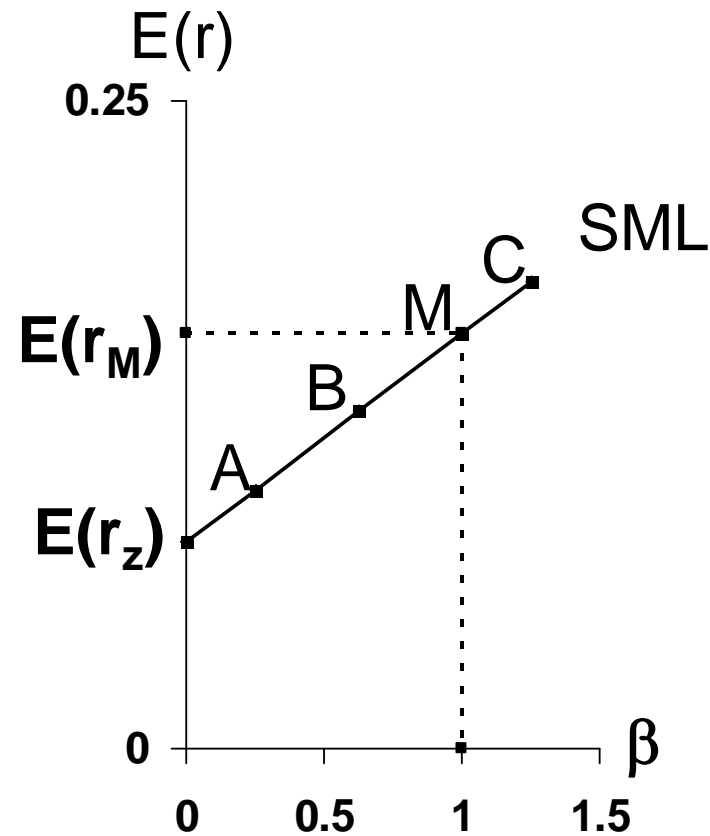
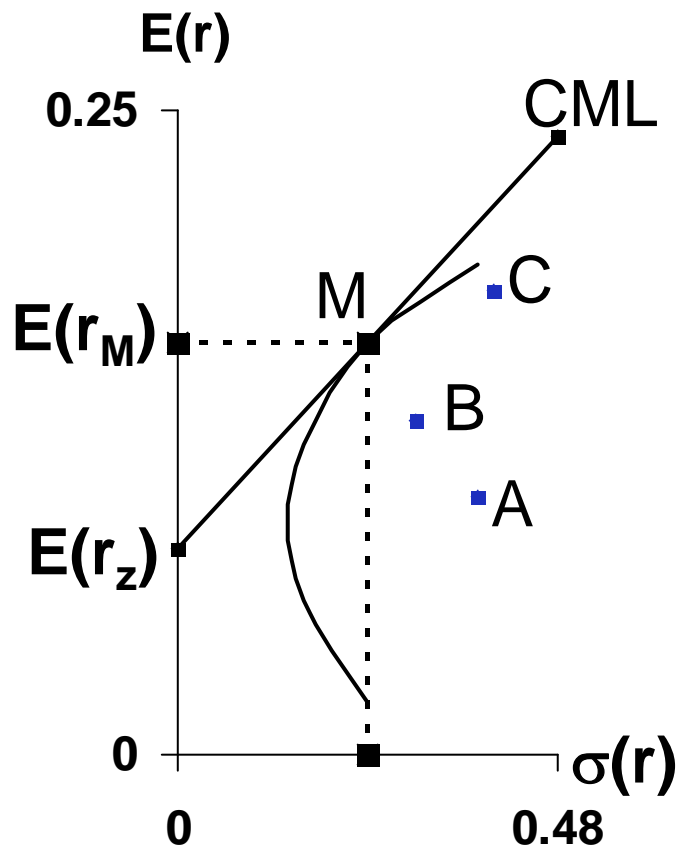
## **The FM results appeared to be consistent with the zero beta version of the CAPM:**

- $a_0$  was significantly greater than the mean of the risk-free interest rate.
  - $a_1$  was significantly different from zero. The slope of the SML was positive.
  - $a_2$  and  $a_3$  were not significantly different from zero (i.e., there was no evidence of nonlinearity or of residual variance affecting returns).
- **Difference between BJS and FM Studies**
- In BJS, betas and average returns were computed in the same periods.
  - In FM, betas in one period were used to predict returns in a later period.

## Roll's Critique of Tests of the CAPM

- **Tests like those of BJS and FM are tautological (not necessary). Results like those reported could be obtained irrespective of how securities were priced relative to risk. (e.g., pulling numbers out of a hat would produce the same results). Therefore, the CAPM was not really tested. Furthermore, since we cannot operationally define the market portfolio, the CAPM can never be tested.**

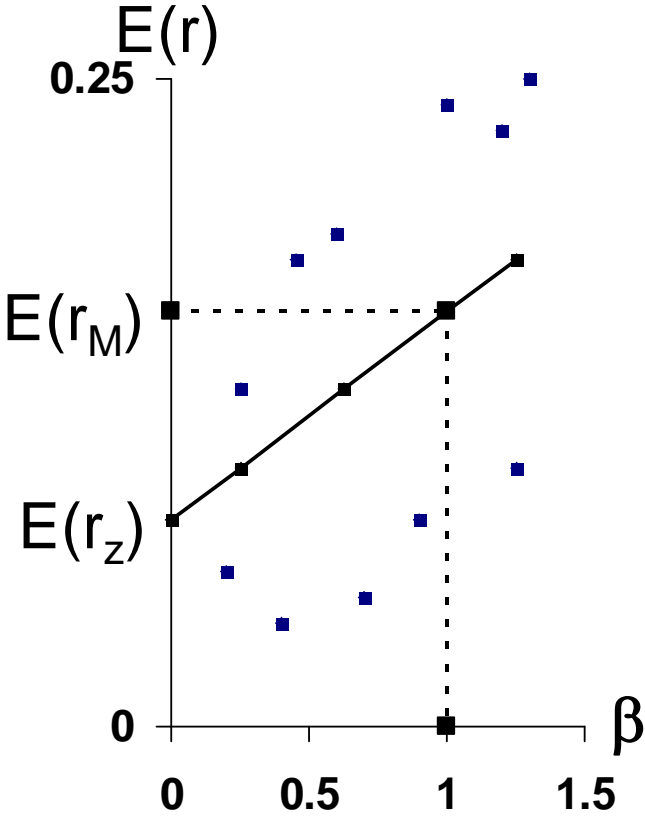
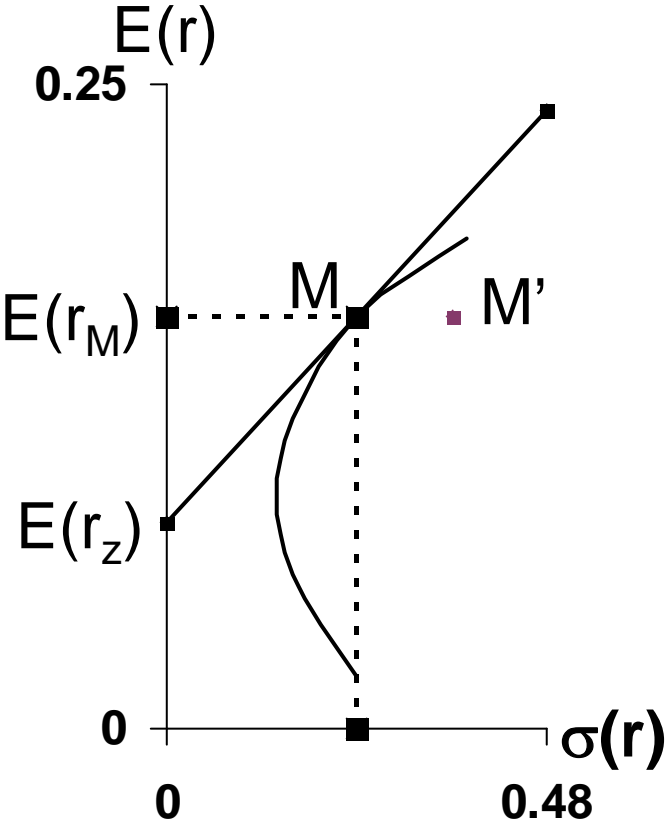
**It is true that if the market portfolio is efficient, the relationship between expected return and beta will be perfectly linear and positively sloped:**



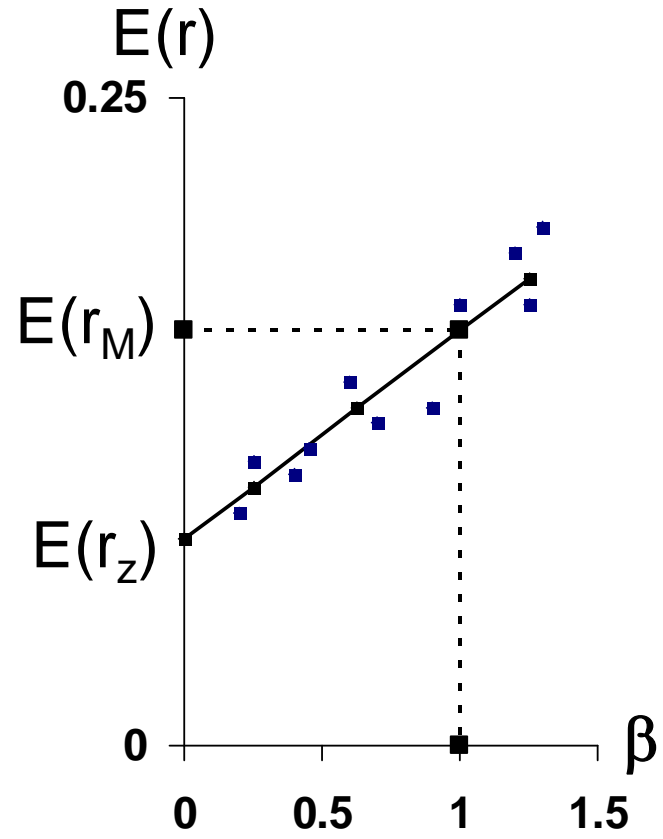
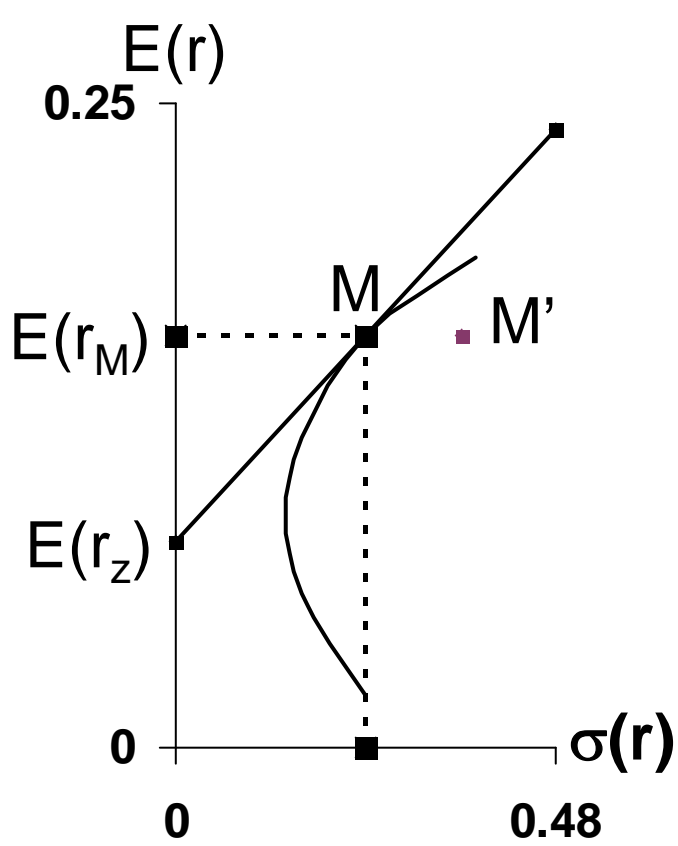
However, given a linear relationship between portfolio return and beta, it does not necessarily follow that the market portfolio is efficient. Therefore, we have not tested the CAPM.

- For example, if betas are computed with reference to an index portfolio inside the minimum variance set, the relationship between security betas and expected returns will not be linear. Portfolios, however, will plot closer to the SML because the security residuals will be averaged in the portfolios. Therefore, using an inefficient market portfolio (M'):  
(See the graphs that follow)

# For Individual Securities



# For Portfolios



# Some Recent Controversial Evidence

## ■ Fama/French Study

- Small firms tend to outperform large firms.
- Stocks with low ratios of price to book value tend to outperform stocks with high ratios of price to book value.
- The relationship between return and beta was essentially flat to negative.

## **On the Other Hand . . . Some Argued . . .**

- **Fama and French study suffers from “survival bias.”**
  - **Many stocks with low ratios of price to book value are in financial stress and wind up failing. These stocks were excluded from the Fama and French data. This produced upward bias in the performance of low price to book value stocks as a class.**



## **Still Others Argued . . .**

- **The impact of survival bias does not fully explain the relationship found between performance and the price to book value ratio.**
- **Irrespective of the price to book value ratio, survival bias cannot be used to explain the flat to negative relationship between return and beta.**

# Comment on Testing the CAPM

The evidence discussed above does not prove that the CAPM is invalid since only stocks were included in the analyses. The “Market Portfolio” contains all of the capital assets in the universe. We will never be able to observe the returns on the “true” Market Portfolio. Therefore, the CAPM is simply not a testable theory.

## Some Additional Evidence

- Estimated betas are very sensitive to the market index being used.
- In risk-return space, indices can be close to each other, and close to the efficient set, and still produce different relationships (positive and negative) between return and beta.
- One study suggested using a dual beta approach to adjust for risk differences in bull and bear markets.